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Analysis of the ^{208}Pb(p,t) Reaction[*]

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Differential cross sections for the (p,t) reaction leading to many levels in ^{206}Pb have been calculated. Three of the transitions are essentially insensitive to the detailed nuclear description, and are used to confirm the treatment of the reaction dynamics. One is therefore in a position to use the reaction to test nuclear wave functions. It is found that agreement between observed and calculated cross sections for many levels is within the assigned errors and confirms the True-Ford wave functions for those levels.

I. INTRODUCTION

TWO-NUCLEON transfer reactions, as has already been emphasized, are highly sensitive to the type of correlations that exist between the transferred pair in their nuclear state. For this reason they should provide a very important means of testing detailed nuclear descriptions. The theory of these reactions can be formulated in such a way as to concentrate the dependence on the nuclear structure in one set of amplitudes, and the dynamical description of the reaction in another.[1] Before the full potential of this type of reaction can be realized, one has to know that the dynamic part can be handled satisfactorily. There has been little opportunity to investigate this point so far. Recently, however, differential cross sections for the reaction ^{208}Pb(p,t) leading to a number of states in ^{206}Pb were reported.[2] Not only are the nuclear wave functions in these cases as well known, or better, than in any other nuclei, but the dependence of some of the transitions on the nuclear description is minimal, as we explain in the next section. In addition, and of vital importance, is the absence of strongly enhanced transitions in the inelastic scattering channels. This permits an interpretation of the transfer reaction in terms of the simple direct mechanism as contrasted with the complication attendant on core excitation.

This article is devoted first to the question concerning the dynamics of the two-nucleon transfer reaction. It is found in fact that the probability for a direct transfer, calculated in the distorted-wave Born approximation, gives a very good account of the angular distributions. Thus encouraged to trust the calculation of the dynamical parts of the transfer amplitude we proceed to a detailed calculation of the cross sections implied by the shell-model description of ^{206}Pb given by True and Ford.[3]

II. THEORY

For each transition multipole (L,S,J), the information carried in the nuclear wave functions that is relevant to the reaction can be concentrated into a set of structure amplitudes G_{NLSJ} in a way described in our earlier work.[1] Here N refers to the number of nodes in the radial function describing the center of mass of the transferred pair, while L,S,J refer to the orbital, spin, and total angular momentum of the pair with respect to the nuclear center. In general, several multipoles (L,S,J) can contribute to a given transition. However for (p,t) transitions connecting even-even nuclei, only one is allowed, the one with $S=0$ and $L=J$ equal to the spin of the excited state. Thus for each transition we need to specify one set of G_N, $N=1, 2, \cdots$. From our knowledge of the ^{207}Pb spectrum, we know the single-particle spectrum.[4] From it we can conclude fairly safely that for the low-lying *positive*-parity states of ^{206}Pb the dominant configurations will involve only the $3p_{1/2}$, $2f_{5/2}$, and $3p_{3/2}$ single-particle states, which all belong to the same oscillator shell. The significance of this is that for a state having the favored correlations, *one* member of the set of amplitudes G_N will dominate.[5] For such a state, that value of N will be

$$\begin{aligned} N &= \tfrac{1}{2}(2\mathfrak{N}-L)+1 \\ &= 6-\tfrac{1}{2}L, \quad \text{(for Pb)} \end{aligned}$$

where the oscillator quantum number of the single-particle state n, l is defined as $\mathfrak{N}=2(n-1)+l$. We have thus argued that, for a strong transition to any low-lying positive-parity level of ^{206}Pb, we know the radial state from which the pair is taken and therefore can calculate the angular distribution (though not the magnitude) without a detailed knowledge of the nuclear

TABLE I. Optical-model parameters. (Energy is in MeV; length in F.)

	V	W	W_D	r_V	r_W	r_c	a_V	a_W
p	51	8	0	1.2	1.428	1.2	0.65	0.704
	49	0	18.1	1.21	1.23	1.2	0.77	0.551
t	160	20	0	1.1	1.6	1.4	0.75	0.75

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[*] This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] N. K. Glendenning, Phys. Rev. **137**, B102 (1965); *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, California, 1963), Vol. 13, p. 191; Argonne National Laboratory Report No. ANL-6848, 1964 (unpublished).

[2] G. M. Reynolds, J. R. Maxwell, and Norton M. Hintz, Phys. Rev. **153**, 1283 (1967).

[3] W. W. True and K. W. Ford, Phys. Rev. **109**, 1675 (1958).

[4] D. E. Alburger and A. W. Sunyar, Phys. Rev. **99**, 695 (1955); N. H. Lazar and E. D. Klema, *ibid.* **98**, 710 (1955).

[5] For a given configuration the G_N are generally monotonic increasing to the maximum allowed N. [This follows from the behavior of the overlap integral on the relative motion contained in G_N (see Ref. 1).] When all configurations belong to the same oscillator shell, each has the same maximum N. A correlated state will have such phases that the contribution of all configurations to this last G_N will be coherent.

wave function. Thus transitions to the lowest $J=0^+$, 2^+, and 4^+ levels are characterized as $6s$, $5d$, and $4g$, respectively. This is the meaning of our statement in the Introduction that the nuclear structure enters the description of some of the ^{208}Pb(p,t) transitions in a minimal way.

III. RESULTS

A. Enhanced Transitions: Test of the Reaction Theory

For each of the lowest states of spin $J=0^+$, 2^+, and 4^+ the angular distribution for the (p,t) reaction has been calculated. In each case the radial function, as we argued in the previous section, is essentially known, aside from its normalization. Its asymptotic behavior should be determined by the energy required to remove the pair, leaving the residual nucleus in the energy state under consideration. In the interior region we represent it by a harmonic-oscillator function. If the single-particle states have the oscillator parameter ν $(=m\omega/\hbar)$, then the center of mass of a pair of nucleons has the parameter 2ν. We use the same value as True and Ford,[3] $\nu=0.185$ F^{-2}. In Ref. 1 we suggested two possible ways of handling the bad asymptotic behavior of the harmonic-oscillator functions. The first is the one used here. The other consists in using single-particle states of a Woods-Saxon potential. This will yield a wave function for the center of mass of the pair which is improved over a pure harmonic-oscillator function in the sense that it does not not decay so rapidly, and hence will yield improved results for the calculated angular distribution as was emphasized recently by Drisko and Rybicki.[6] However, this function still does not have the asymptotic behavior associated with the separation energy of the pair. We therefore prefer our first prescription.

We calculate cross sections using the distorted-wave Born approximation with an interaction between the proton and center of mass of the neutrons having zero range. The optical-model parameters are taken from the literature.[7] They are shown in Table I. The first set of proton parameters was used throughout, except for comparison with the second as discussed later.

The calculated angular distributions for the lowest state of each spin $J=0^+$, 2^+, and 4^+ are compared in Fig. 1 with the data of Reynolds, Maxwell, and Hintz. The agreement in each case is excellent and tends to confirm the theoretical description of the two-nucleon transfer reaction. (It should be noted that some authors have used a point-triton approximation.[8] This is not equivalent to our assumption of a zero-range interaction, but is additional to it and in particular leads to quite different radial functions for the center of mass of the transferred pair. This fact has also been emphasized by Broglia and Riedel.[9] However, the reaction is concentrated at the surface to such an extent that both radial functions would yield almost the same angular distribution, though they would, in general, lead to different cross sections.)

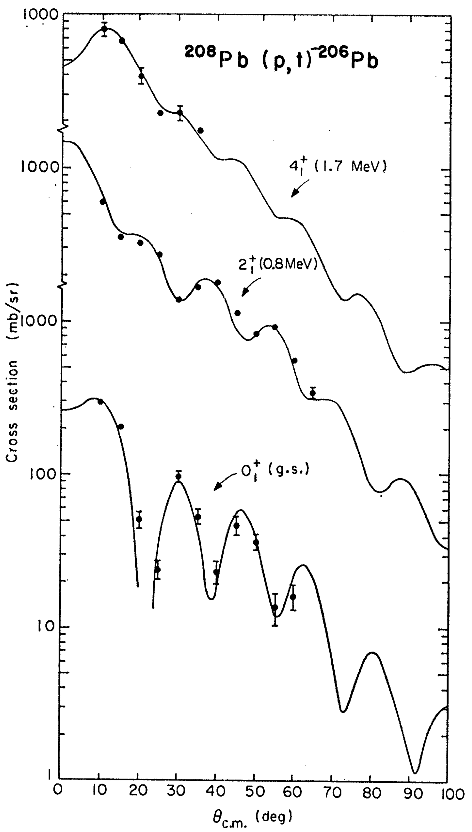

FIG. 1. Differential cross sections for the first 0^+, 2^+, and 4^+ states in ^{206}Pb excited by 40-MeV protons in the (p,t) reaction. Solid lines are calculated as described in Sec. III A. Data are from Reynolds, Maxwell, and Hintz, Ref. 2 (g.s.≡ground state). Calculations are independently normalized to the data. See Table II for absolute comparison.

B. Cross Sections: Test of the Nuclear Model

Having confirmed the theory of the reaction mechanism for those levels for which the nuclear structure enters in a minimal way in determining the angular distribution, we now consider the cross sections to all levels. The wave functions for ^{206}Pb have been obtained through a shell-model calculation by True and Ford.[3] Since ^{208}Pb is doubly magic, its ground-state wave function, to excellent approximation, will be the pure closed-shell wave function. The structure amplitudes based on these wave functions were calculated by Reynolds, Maxwell, and Hintz. Even without a calculation of the

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[6] R. M. Drisko and F. Rybicki, Phys. Rev. Letters **16**, 275 (1966). All publications based on our work since 1963 have treated the asymptotic region properly, and consequently are not subject to the criticism implied by Drisko and Rybicki.

[7] The proton parameters are taken from M. P. Fricke and G. R. Satchler, Phys. Rev. **139**, B567 (1965). The triton parameters are attributed to G. R. Satchler by S. Hinds *et al.*, Nucl. Phys. **83**, 17 (1966).

[8] J. R. Rook and D. Mitra, Nucl. Phys. **51**, 96 (1964); R. N. Glover, A. D. W. Jones, and J. R. Rook, *ibid.* **81**, 289 (1966).

[9] R. A. Broglia and C. Riedel, Nucl. Phys. **A92**, 125 (1967).

transfer amplitudes one is able, as these authors did, to draw conclusions about some relative intensities. However we have proceeded to calculate the cross sections to all levels computed by True and Ford to lie below about 3 MeV. Some of these have small cross sections and were not observed. Of the observed levels, angular distributions to the higher-lying levels are shown in Fig. 2. In two cases, the observed peak was known to contain a doublet, and for these, we compare the summed computed angular distributions. Also shown are the separate angular distributions in case subsequent experiments resolve the levels. The integrated cross sections for all natural-parity levels (which alone can be excited in this reaction), predicted to be in the energy range considered, are listed in Table II, together with the observed cross sections. Our calculation does contain, for each level, a common factor which we do not calculate, but evaluate by normalizing to the 4_1^+ level.

Surveying the angular distributions shown in Figs. 1 and 2 reveals excellent agreement in most cases, the most notable exception being the unresolved doublet at 2 MeV, thought to contain the 4_2^+ and 7_1^- states. As is well known, however, the angular distribution for a process in which the transfer of angular momentum occurs mainly in the surface region is not particularly sensitive to the details used to describe the reaction, when a single multipole, alone, is allowed. Of course each multipole generally has its own angular distribution. Therefore the angular distribution, in cases where several multipoles are allowed, will depend upon the details, since they determine the weight with which the individual multipoles contribute. However that is not the situation for the reaction considered here, so we have to look to the absolute cross sections to provide a more stringent test of the nuclear wave functions. Table II shows that levels for which small cross sections are calculated are either not observed, or only weakly excited. Of the more strongly excited levels, the agreement is generally good. Undoubtedly some uncertainty should be attached to the calculated cross sections due to ambiguities in the optical-model parameters, neglect of finite-range effects, etc. An uncertainty of 30% is often quoted for *single*-nucleon transfer reactions although relative cross sections should be better determined. There is also an uncertainty in the measured cross sections of ±20%. The experiment and theory agree within the combined errors in all cases. Agreement within such a large error is perhaps more significant than one would at first consider, inasmuch as transition rates are a rather sensitive probe of the wave functions.

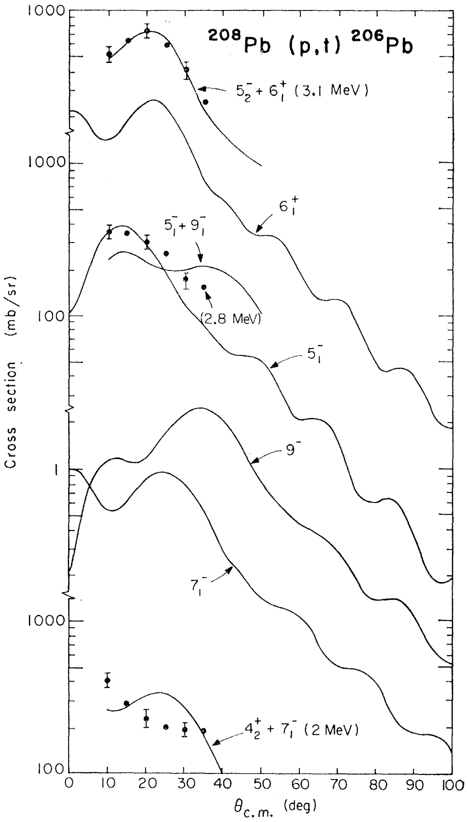

FIG. 2. Differential cross sections for higher levels in Pb^{206} excited by 40-MeV protons in the (p,t) reaction. Solid lines are calculated as described in Secs. III A and III B.

TABLE II. Natural-parity states of ^{206}Pb that lie below about 3 MeV. Experimental cross sections are compared with calculated values. Also listed are total calculated cross sections.

Level (MeV)			Cross section (mb)		
			Integrated over observed range[b]		Total
Calc.	Obs.[a]	Spin	Obs.[a]	Calc.	calculated[c]
0	0	0_1^+	140	94	131
0.725	0.8	2_1^+	500	410	550
1.36		0_2^+	≤15	10	14
1.39		2_2^+	≤15	23	28
1.68	1.68	4_1^+	300	300	650
1.77		2_3^+	small	0.2	0.8
2.06		0_3^+	not seen	11	32
2.01	2.	$\left.\begin{matrix}4_2^+\\7_1^-\end{matrix}\right\}$	180	$\left\{\begin{matrix}49\\230\end{matrix}\right.$	110
2.17					550
2.19		2_4^+	not seen	6	18
2.53		2_5^+	not seen	10	27
2.53[d]		3_1^-	not seen	...	...
2.60		9_1^-	?	$\left\{\begin{matrix}98\\120\end{matrix}\right.$	530
2.81	2.8	5_1^-	210		250
2.98		7_2^-	?	$\left\{\begin{matrix}16\\58\\76\\280\end{matrix}\right.$	42
3.01		4_3^+	?		120
3.06	3.1	$\left.\begin{matrix}5_2^-\\6_1^+\end{matrix}\right\}$	450		150
3.15					570

[a] Groups observed in Ref. 2.

[b] The first four entries are results of integrating from 10°–60° and remainder from 10°–30°. Theory is normalized to the 4_1 state.

[c] Calculated cross section integrated from 0°–180° with normalization to the 4_1 state.

[d] Not seen in experiment of Ref. 2, nor calculated by True and Ford. See *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Acadamey of Sciences–National Research Council, Washington, 25, D. C.).

The greatest discrepancy seen in Table II occurs for the ground-state transition. It is actually to *this* transition that the neglected ground-state correlations consisting of excited pairs in the neutron shell above $N=126$

would make their contribution. While such admixtures may be small, they are coherent.

The 9^- level, calculated to be at 2.6 MeV, was not seen in the experiment. Since its calculated cross section is of an easily observed magnitude and since the calculated cross section for the near-lying 5^- level falls short of what is measured, we suggest that the group at 2.8 MeV actually contains both the 9^- and 5^- levels. Similarly the $4_3{}^+$ level calculated to be at 3.01 MeV probably contributes to the groups observed at 3.1 MeV, as well, possibly, as the $7_2{}^-$.

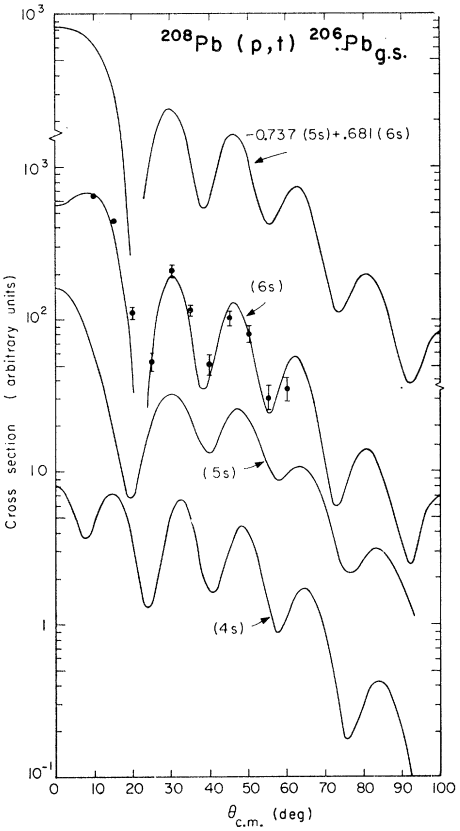

FIG. 3. Differential cross sections for the ground-state transition in ^{208}Pb(p,t) for 40-MeV protons calculated under the three different assumptions that the neutrons are transferred from a center-of-mass state $6s$, $5s$, and $4s$, respectively, and also from the mixture of $6s$ and $5s$ indicated.

In connection with Table II it should also be noted that almost the entire transition strength for each multipole is gathered into a single state. This is of course a reflection of the sensitivity of the reactions to certain correlations between the pair (in this case ^1S), so that states possessing the favored correlations are favored. That the lowest state of each spin is the one that possesses the favored correlations is a reflection of the importance of the attractive singlet-even part of the interaction in binding that state.

The last column in Table II shows the calculated total integrated cross section to each level. Summing the contributions for each multipole (shown in Table III) provides a crude measure of the validity of the classical argument for kinematically favored multipole transitions based on the momentum transfer and impact radius.

TABLE III. Total cross section going to each multipole in the ^{208}Pb(p,t) reaction at $E_p=40$ MeV.

J	0^+	2^+	4^+	5^-	6^+	7^-	9^-
Cross section (mb)	180	630	880	400	570	590	530

C. Dependence on Parameters

Nuclear description. Reactions involving complex particles, such as tritons, are fairly strongly localized in the surface region and for this reason some loss of sensitivity to the nuclear description must be expected.[10] This loss of sensitivity is registered mainly in the angular distribution rather than the magnitude of the cross section. Thus a state possessing the parentage and correlations

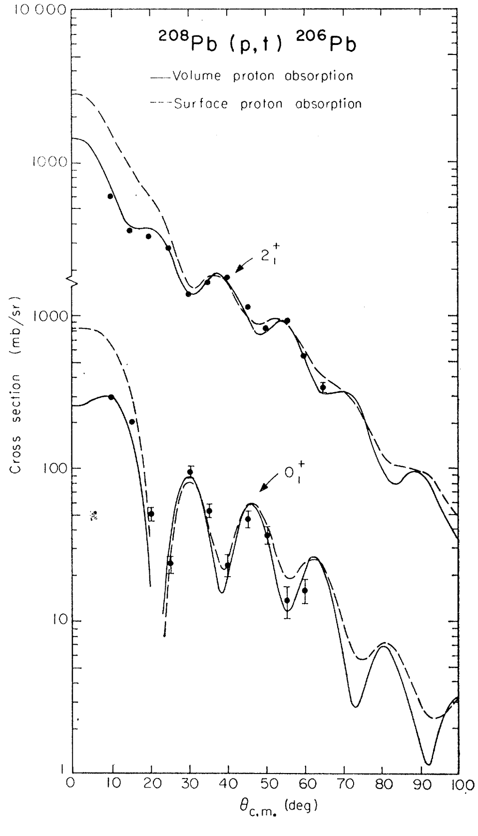

FIG. 4. Angular distributions for ^{208}Pb(p,t) leading to the ground and first-excited state of ^{206}Pb for 40-MeV protons. Solid curves correspond to volume absorption in optic parameters of proton, while dashed curves correspond to surface absorption (see Table I). Solid and dashed curves are independently normalized to the data.

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[10] N. K. Glendenning, Phys. Rev. **114**, 1297 (1959).

favorable to a given reaction will have a larger cross section, though perhaps not such a different angular distribution, than another state of the same multipolarity but unfavored, whenever only one multipole is allowed. We have calculated the cross section for four cases, supposing that the nuclear description for the ground-state transition of the ^{208}Pb(p,t) reaction was such as to yield a pure 6s, 5s, or 4s state (with unit amplitude) for the center-of-mass motion of the transferred pair and also a mixture of 5s and 6s that corresponds to the 0_2 state except for normalization. The differences between the results, shown in Fig. 3, are rather minor and demand a very close comparison with experiment in order to distinguish between them. We have shown the experimental angular distribution, and the agreement with the 6s result is so good as to permit the distinction. But the point is made, nevertheless, that the agreement must be near perfect before one can draw a conclusion based only on an angular distribution when one multipole alone is allowed by the selection rules.

Optical-model parameters. Two sets of proton parameters, one corresponding to volume and the other to surface absorption, were used. Their effect on the angular distributions for the 0_1 and 2_1 transitions is shown in Fig. 4. The volume absorption leads to substantially better results.

IV. SUMMARY

Differential cross sections to all levels below about 3 MeV in ^{206}Pb that can be excited by the (p,t) reaction have been calculated using the distorted-wave method, under the assumption of a simple direct transfer as distinguished from sequential transfers or excitation of the core. The angular distributions of several of the transitions are insensitive to the details of the nuclear wave functions and these were used to provide a test of the reaction mechanism assumed. These angular distributions are in near perfect agreement with experiment. The integrated cross sections for many levels using True and Ford's wave functions, were compared with the experimental data. The agreement at worst was about a factor of 2. Since transition rates provide quite a stringent test of the nuclear description it is felt that the True-Ford wave functions provide quite an accurate description of most of the states, particularly since some or all of the descrepancy could be blamed on the summed errors of the experiment and reaction calculation.

Of more general interest, we conclude that when there is good reason to believe that the simple direct process dominates a double transfer reaction, an analysis in terms of the theory employed here[11] can be used to test the nuclear wave functions. Such programs have already been reported by several authors.[9,12]

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[11] See Ref. 1 and also E. M. Henley and D. V. L. Yu, Phys. Rev. **133**, B1445 (1964); C. L. Lin and S. Yoshida, Progr. Theoret. Phys. (Kyoto) **32**, 885 (1964); B. Bayman, Argonne National Laboratory Report No. ANL-6878, 1964 (unpublished).

[12] J. J. Wesolowski, L. F. Hansen, J. G. Vidal, and M. L. Stelts, Phys. Rev. **148**, 1063 (1966); J. Vervier, Phys. Letters **22**, 82 (1966) and in International Conference on Nuclear Physics (unpublished); see also C. H. Hoot [in International Conference on Nuclear Physics, Gatlinburg, Tennessee, 1966 (unpublished)] who uses a diffraction model for the calculation of the transfer amplitudes and our formulation for the structure amplitudes; C. L. Lin, Progr. Theoret. Phys. (Kyoto) **36**, 251 (1966).