Level Structure of Ca^{45} Investigated by Deuteron Stripping*

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The level structure of Ca^{45} below 6.3-MeV excitation has been studied by the $Ca^{44}(d,p)Ca^{45}$ reaction. Twenty-eight of the observed 88 transitions showed stripping angular distributions and were analyzed in terms of the distorted-wave Born approximation to obtain l_n values and spectroscopic factors. A sum-rule analysis of the spectroscopic factors is given. A level at 1.886 MeV was excited by an $l_n=2$ transition and is interpreted as a $1d_{3/2}^{-1}$ hole state in Ca^{45}. This state is further discussed in terms of core excitation in Ca^{44}. The observed Ca^{45} level scheme is compared to calculated level schemes and to the level schemes of the Sc^{46} and Ti^{47} isotones.

I. INTRODUCTION

THE $Ca^{44}(d,p)Ca^{45}$ reaction has been investigated at an incident deuteron energy of 7.00 MeV. The level scheme of Ca^{45} up to 3.4-MeV excitation has been reported by Braams[1] using the same reaction; this work was later extended by Cobb and Guthe[2] who obtained angular distributions, assigned l_n values, and extracted relative yields for these transitions. The work in Refs. 1 and 2 was carried out on the MIT single-gap spectrograph.[3]

The present measurements were made using the MIT multiple-gap spectrograph,[4] which allowed the simultaneous measurement of the reaction protons at 23 scattering angles located every 7.5 deg from 7.5 to 172.5 deg in the laboratory system. A proton yield several times larger than that of Ref. 2 was obtained, thus permitting angular-distribution measurements for the weaker transitions. Data were obtained for 88 levels below 6.3-MeV excitation. The experimental procedures and results are discussed in Sec. II.

Stripping reactions at 7.0-MeV bombarding energy on the other stable even isotopes of calcium, $Ca^{40,42,46,48}$, have been previously reported from this laboratory[5–8]. These experiments and analyses were carried out in a manner similar to those reported here. A distorted-wave Born-approximation (DWBA) analysis is presented in Sec. III, and a discussion of the results including a sum-rule analysis, the effect of core excitation, and a comparison of the observed with calculated level schemes is given in Sec. IV. The deduced ground-state wave functions for even-A calcium isotopes are discussed in Sec. IV E.

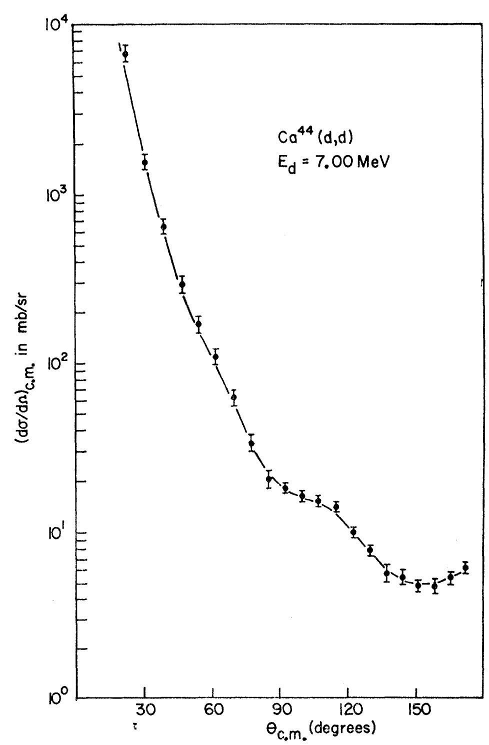

FIG. 1. Angular distribution of deuterons scattered elastically by Ca^{44} at 7.00 MeV. The error flags indicate statistical uncertainties. The solid curve is an optical-model fit to the data using the deuteron parameters given in Table II.

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[1] C. M. Braams, Ph.D. thesis, University of Utrecht, Utrecht, Netherlands, 1956 (unpublished).

[2] W. R. Cobb and D. B. Guthe, Phys. Rev. **107**, 181 (1957).

[3] C. P. Browne and W. W. Buechner, Rev. Sci. Instr. **27**, 899 (1956).

[4] H. A. Enge and W. W. Buechner, Rev. Sci. Instr. **34**, 155 (1963).

[5] T. A. Belote, A. Sperduto, and W. W. Buechner, Phys. Rev. **139**, B80 (1965).

[6] W. E. Dorenbusch, T. A. Belote, and Ole Hansen, Phys. Rev. **146**, 734 (1966).

[7] T. A. Belote, H. Y. Chen, Ole Hansen, and J. Rapaport, Phys. Rev. **142**, 624 (1966).

[8] E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner, Phys. Rev. **135**, B865 (1964).

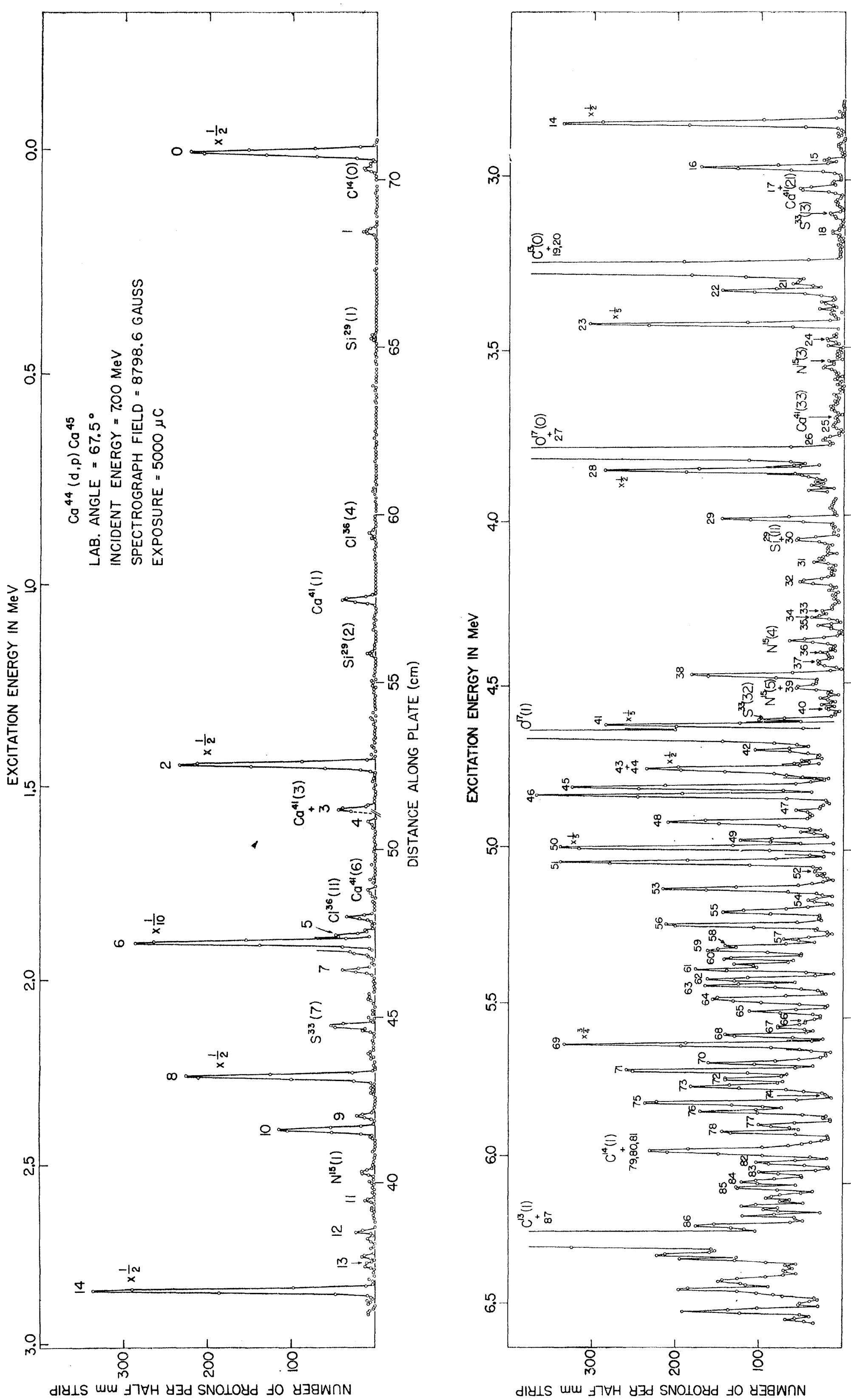

FIG. 2. Proton spectrum from $Ca^{44}(d,p)Ca^{45}$ observed at a laboratory angle of 67.5 deg. Proton groups corresponding to levels of Ca^{45} are labeled with the numbers used to identify these states in Table I. Several contaminant groups are identified.

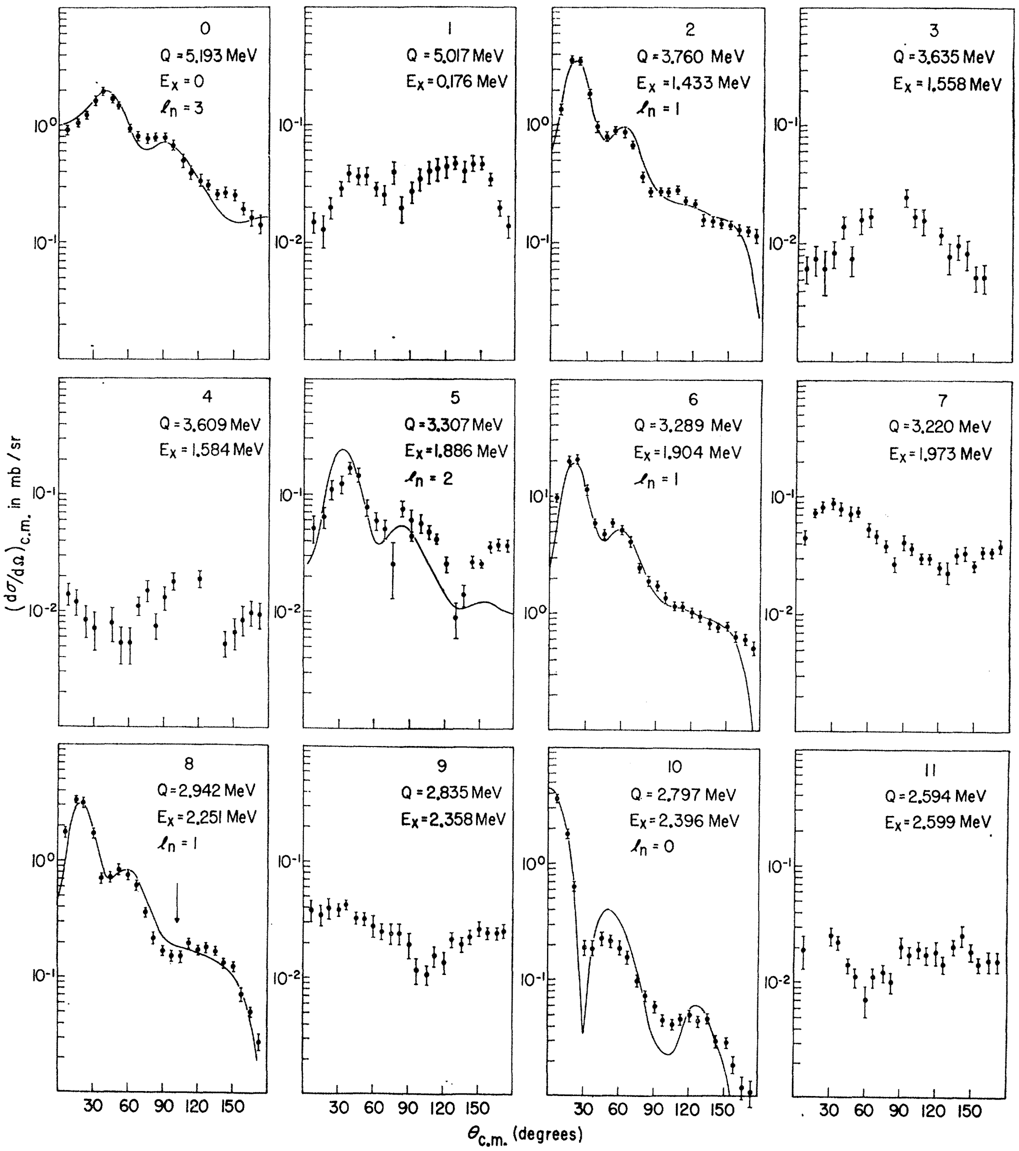

FIG. 3. Angular distributions of protons from $Ca^{44}(d,p)Ca^{45}$. The distributions are labeled with the numbers used to identify the corresponding states in Table I. Statistical uncertainties are indicated by error flags on the data points. The solid curves are the DWBA predictions using the parameters given in Table II.

II. EXPERIMENTAL PROCEDURES AND RESULTS

A. Target

The target used in the present experiment was prepared by vacuum evaporation of $CaCO_3$, 98.6% enriched in Ca^{44}, onto a carbon and Formvar foil approximately 15 $\mu g/cm^2$ thick. The calcium target had a thickness of 15.3 $\mu g/cm^2$. The enriched material was obtained from the Stable Isotopes Division, Oak Ridge National Laboratory, and the isotopic abundances were Ca^{44}, 98.6%; Ca^{42}, 0.08%; and Ca^{40}, 1.26%.

B. Elastic Scattering

Elastic scattering of deuterons from Ca^{44} was observed at 7.00- and 3.0-MeV bombarding energy. The 3.0-MeV scattering was used to establish an absolute cross-section scale, assuming Rutherford scattering. The 7.00-MeV results, together with an optical-model calculated curve, are shown in Fig. 1. The error on the absolute cross-section scale is estimated to be $\pm 20\%$. Relative errors are represented by error flags on the data points.

C. The (d,p) Reactions

The reaction protons were recorded at 23 angles in the MIT multiple-gap spectrograph,[4] in 50-μ Eastman Kodak NTA nuclear emulsions that were covered with a layer of aluminum foil sufficiently thick to stop the elastically scattered deuterons. Exposures of 5000 and 500 μC were taken, and the proton spectrum obtained at 67.5 deg is shown in Fig. 2. The energy resolution was 12 keV. The proton groups corresponding to a residual mass of 45 were identified from their kinematic

TABLE I. Summary of results for the $Ca^{44}(d,p)Ca^{45}$ reaction for $E_d = 7.0$ MeV.

Level	Q^a (MeV)	$E_x{}^b$ (MeV)	$(d\sigma/d\Omega)_{max}{}^c$ (mb/sr) c.m.	l_n	$(2J_f+1)S_{l,j}{}^c$	$J^{\pi d}$
0	5.193	0	1.97	3	3.36	$\frac{7}{2}^-$
1	5.017	0.176	0.04			$\frac{5}{2}^-$
2	3.760	1.433	3.36	1	0.47	$\frac{3}{2},(\frac{1}{2})^-$
3	3.635	1.558	0.02			
4	3.609	1.584	0.015			
5	3.307	1.886	0.17	2	0.15	$\frac{3}{2}^+$
6	3.289	1.904	20.6	1	2.56	$\frac{3}{2},(\frac{1}{2})^-$
7	3.220	1.973	0.087			
8	2.942	2.251	3.32	1	0.36	$\frac{1}{2},(\frac{3}{2})^-$
9	2.835	2.358	0.04			
10	2.797	2.396	3.60	0	0.11	$\frac{1}{2}^+$
11	2.594	2.599	0.02			
12	2.510	2.683	0.05			
13	2.427	2.766	0.04			
14	2.346	2.847	4.02	1	0.46	$\frac{3}{2},(\frac{1}{2})^-$
15	2.240	2.953	0.05			
16	2.220	2.973	0.37			
17	2.158	3.035	0.02			
18	2.042	3.151	0.02			
19	1.946	3.247	1.83	1	0.12	$\frac{1}{2},(\frac{3}{2})^-$
20	1.915	3.278	0.02			
21	1.894	3.299	0.1			
22	1.871	3.322	0.25			
23	1.771	3.442	6.75	1	0.79	$\frac{1}{2},(\frac{3}{2})^-$
24	1.730	3.463	0.018			
25	1.480	3.713	0.03			
26	1.440	3.753	0.040			
27	1.407	3.786	1.18	(1)	0.16	$(\frac{3}{2},\frac{1}{2}^-)$
28	1.348	3.845	2.23	1	0.26	$\frac{3}{2},(\frac{1}{2})^-$
29	1.200	3.993	0.40	3	0.65	$\frac{5}{2}^-$
30	1.145	4.048	0.040			
31	1.078	4.115	0.04			
32	1.016	4.177	0.12			
33	0.935	4.258	0.12			
34	0.907	4.286	0.28			
35	0.881	4.312	0.37	(1)	0.03	$(\frac{1}{2},\frac{3}{2}^-)$
36	0.805	4.388	0.2			
37	0.772	4.421	0.25			
38	0.729	4.464	1.15	1	0.09	$(\frac{1}{2},\frac{3}{2})^-$
39	0.682	4.511	0.91	(1)	0.05	$(\frac{1}{2},\frac{3}{2}^-)$
40	0.634	4.559	0.15			
41	0.571	4.622	4.95	1	0.45	$(\frac{1}{2},\frac{3}{2})^-$
42	0.498	4.695	0.25			
43	0.443	4.750	2.11			
44	0.431	4.762				
45	0.383	4.810	1.25	1	0.11	$(\frac{1}{2},\frac{3}{2})^-$
46	0.356	4.837	1.42	2	0.20	$\frac{5}{2}^+$
47	0.308	4.885	0.12			
48	0.274	4.919	2.88	0	0.05	$\frac{1}{2}^+$
49	0.212	4.981	0.25			
50	0.188	5.005	5.34	1	0.50	$(\frac{1}{2},\frac{3}{2})^-$
51	0.146	5.047	2.1			
52	0.114	5.079	0.02			
53	0.065	5.128	0.74			
54	0.029	5.164	0.13			
55	−0.008	5.201	1.14			
56	−0.050	5.243	0.84	1	0.07	$(\frac{1}{2},\frac{3}{2})^-$
57	−0.092	5.285	0.20			

TABLE I (*continued*).

Level	Q^a (MeV)	$E_x{}^b$ (MeV)	$(d\sigma/d\Omega)_{max}{}^c$ (mb/sr) c.m.	l_n	$(2J^\pi+1)S_{l,j}{}^c$	$J^{\pi d}$
58	−0.116	5.309	0.15			
59	−0.131	5.324	0.42	2	0.07	$\frac{5}{2}^+$
60	−0.159	5.352	1.40			
61	−0.197	5.390	0.25	(4)	0.69	$(\frac{9}{2}^+)$
62	−0.224	5.417	0.46			
63	−0.247	5.440	0.44			
64	−0.286	5.479	1.49			
65	−0.328	5.521	0.20			
66	−0.358	5.551	0.40			
67	−0.376	5.569	0.36			
68	−0.405	5.598	0.70			
69	−0.436	5.629	1.16			
70	−0.494	5.687	0.6			
71	−0.523	5.716	0.55			
72	−0.549	5.742	0.3	2	0.05	$\frac{5}{2}^+$
73	−0.571	5.764	0.35	3	0.48	$\frac{5}{2}^-$
74	−0.599	5.792	0.07			
75	−0.625	5.818	0.83	1	0.07	$(\frac{1}{2},\frac{3}{2})^-$
76	−0.653	5.846	0.20			
77	−0.699	5.892	0.28	2	0.04	$\frac{5}{2}^+$
78	−0.722	5.915	0.50			
79	−0.755	5.948	0.24			
80	−0.774	5.967	0.30			
81	−0.797	5.990	0.45	2	0.07	$\frac{5}{2}^+$
82	−0.825	6.018	0.15			
83	−0.858	6.051	0.2			
84	−0.884	6.077	0.3			
85	−0.913	6.106	0.50			
86	−1.041	6.234	0.5			
87	−1.108	6.301	0.6			

[a] The estimated uncertainty is 10 keV for levels No. 0 through 23; 12 keV for the other levels.
[b] The estimated uncertainty is 4 keV for levels No. 1 and 2; 10 keV for levels No. 3; 6 keV for levels No. 4 through 23; 10 keV for the other levels.
[c] The estimated error in the absolute measured cross sections is ±20%. This uncertainity is also assigned to the strengths. For the $l_n=0$ cases, the cross section at a laboratory angle of 7.5 deg is given.
[d] The preferred spin assignments are based on shell-model systematics. The $l_n=1$ preferences based on tentative "dip" assignments are discussed in the text. Levels No. 5 and 10 are assigned $1d_{3/2}$ and $2s_{1/2}$, respectively. Other $l_n=2$ and 0 transitions are assigned as $2d_{5/2}$ and $3s_{1/2}$.

energy shift with angle. Up to an excitation energy of 6.1 MeV, 86 transitions were observed. Above level No. 85, only two relatively strong groups were analyzed.

Table I gives the Q values and excitation energies for the levels observed in Ca^{45} up to an excitation energy of 6.3 MeV. Because of uncertainties in the calibration of the multiple-gap spectrograph, Q values for those levels in Ca^{45} previously reported are taken from Refs. 1 and 2, recalculated using 5.3042 MeV for the energy of Po^{210} α particles. The Q values for levels previously unreported were determined either by interpolation using the known states from Ref. 1 or by reference to known contaminant Q values. The angular distributions are shown in Figs. 3 through 7, in comparison with DWBA predictions.

We have identified three levels at 1.584-, 1.886-, and 3.278-MeV excitation energy which were not seen in the earlier (d,p) works of Refs. 1 and 2. The 1.886-MeV level lies 18 keV below the strongest $l_n=1$ transition. This state has recently been observed by Ames *et al.*[9] from the study of γ rays following the β^- decay of K^{45}. The decay data suggest that this level corresponds to a $\frac{3}{2}^+$ hole state, in agreement with the present stripping results. The analysis of this level in terms of core excitation is given in Sec. IV D. Braams[1] also assigned tentative levels at 1.036- and 1.475-MeV excitation energy in Ca^{45}. These levels were not observed in the present experiment.

III. OPTICAL-MODEL AND DWBA ANALYSIS

A. Stripping Analysis

The experimental 7-MeV elastic deuteron scattering was analyzed by using the optical-model search code ABACUS,[10] employing a least-squares criterion. The

[9] O. Ames, G. Garrett, and P. J. Vajk, Bull. Am. Phys. Soc. 11, 393 (1966).
[10] E. H. Auerbach, Brookhaven National Laboratory Report No. BNL 6562 (ABACUS-2), 1962 (unpublished).

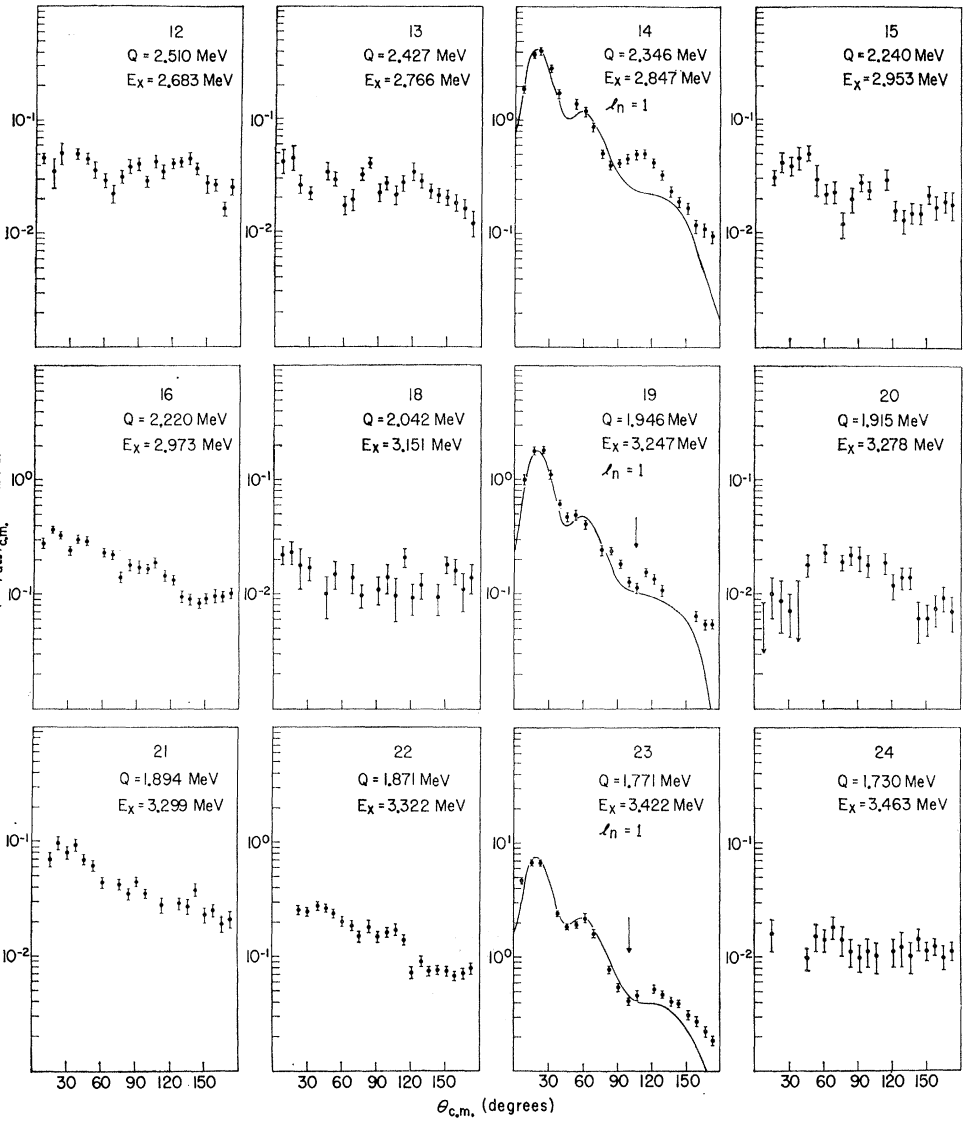

FIG. 4. Angular distributions of protons from $Ca^{44}(d,p)Ca^{45}$. The distributions are labeled with the numbers used to identify the corresponding states in Table I. Statistical uncertainties are indicated by error flags on the data points. The solid curves are the DWBA predictions using the parameters given in Table II.

parameter search was started from the parameter sets used for the (d,p) reactions on Ca^{42} and Ca^{46} at 7.00 MeV.[6,7] The resulting best fit is shown in Fig. 1, and the corresponding parameters are given in Table II. The proton parameters were obtained from the work of Perey.[11]

Reaction cross sections were calculated with the code JULIE, originated by Bassel *et al.*[12] The radial integrals were calculated in the zero-range approximation with no lower cutoff. Figures 3 through 7 show the predicted distributions in comparison with the experimental data. The values of the orbital angular momentum of the captured neutron, l_n, were assigned by comparing the predicted and observed angular-distribution shapes. The transition strengths $(2J_f+1)S_{l,j}$, given in Table I, were obtained by matching the experimental differential cross sections, summed over angles, to the calculated cross-section sums.

IV. DISCUSSION

A. Strength Function and Sum-Rule Analysis

The level structure of Ca^{45} is presented in Fig. 8 in the form of a strength function. The $l_n=3$ strength is divided between the ground-state $1f_{7/2}$ and two states at 3.993 and 5.764 MeV, which are presumably $1f_{5/2}$. A similar situation has been observed in Ca^{41}, Ca^{43}, and Ca^{47} (Refs. 5–7). The excitation energy and the non-stripping character of the 0.176-MeV level are consistent with a $\frac{5}{2}^-$ assignment.[13] The $l_n=1$ strength is

[11] F. G. Perey, Phys. Rev. **131**, 745 (1963).

[12] R. H. Bassel, D. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL 3240, 1963 (unpublished); G. R. Satchler, Nucl. Phys. **55**, 1 (1964).

[13] T. A. Belote, W. E. Dorenbusch, Ole Hansen, and J. Rapaport, Nucl. Phys. **73**, 321 (1965).

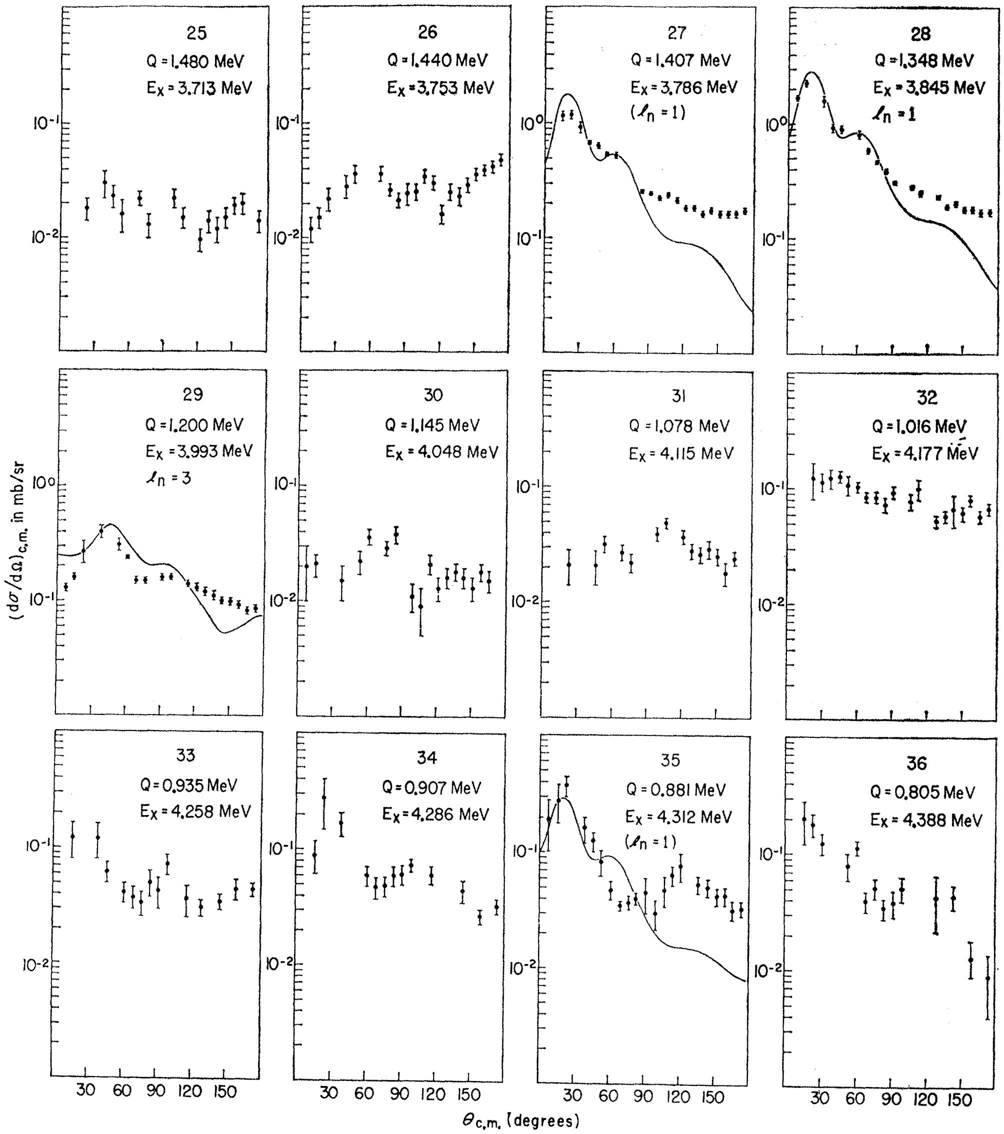

FIG. 5. Angular distributions of protons from $Ca^{44}(d,p)Ca^{45}$. The distributions are labeled with the numbers used to identify the corresponding states in Table I. Statistical uncertainties are indicated by error flags on the data points. The solid curves are the DWBA predictions using the parameters given in Table II.

distributed over the region between 1.4 and 6.0 MeV with no sharp division between $2p_{3/2}$ and $2p_{1/2}$ states. The preferred spin assignments given in Table I were based on the presence ($2p_{1/2}$) or absence ($2p_{3/2}$) of a back-angle "dip" in the measured distribution.[14] However, at 7.0 MeV, the effect is weak, and the assignments are only tentative. The angular distribution of the 2.251-MeV state (state No. 8) shows a dip near 100 deg which is also present in the angular distribution of states No. 19 and 23. These are indicated by arrows in Figs. 3 and 4. The angular distributions of states No. 2, 6, 27, and 28 showed no such dips. No spin

TABLE II. Optical-model parameters.[a]

Particle	V (MeV)	W_D (MeV)	r_o (F)	a (F)	r_o' (F)	a' (F)	r_{oc} (F)
d	114.1	12.8	1.0	0.764	1.44	0.672	1.3
p	52	10.5	1.25	0.65	1.25	0.47	1.25
n	b		1.25	0.65			

[a] The optical potential used was of the form: $V_{opt} = -V(e^x+1)^{-1} + 4iW_D(d/dx')(e^{x'}+1)^{-1} + V_c(r,r_c)$, with $x = (r - r_oA^{1/3})/a$, $x' = (r - r_o'A^{1/3})/a'$, $r_c = r_{oc}A^{1/3}$ where V_c is the Coulomb potential from a homogeneously charged sphere of radius r_c.
[b] Adjusted to give the transferred neutron a binding energy of $Q(d,p) + 2.23$ MeV.

[14] L. L. Lee, Jr. and J. P. Schiffer, Phys. Rev. **136**, B405 (1964).

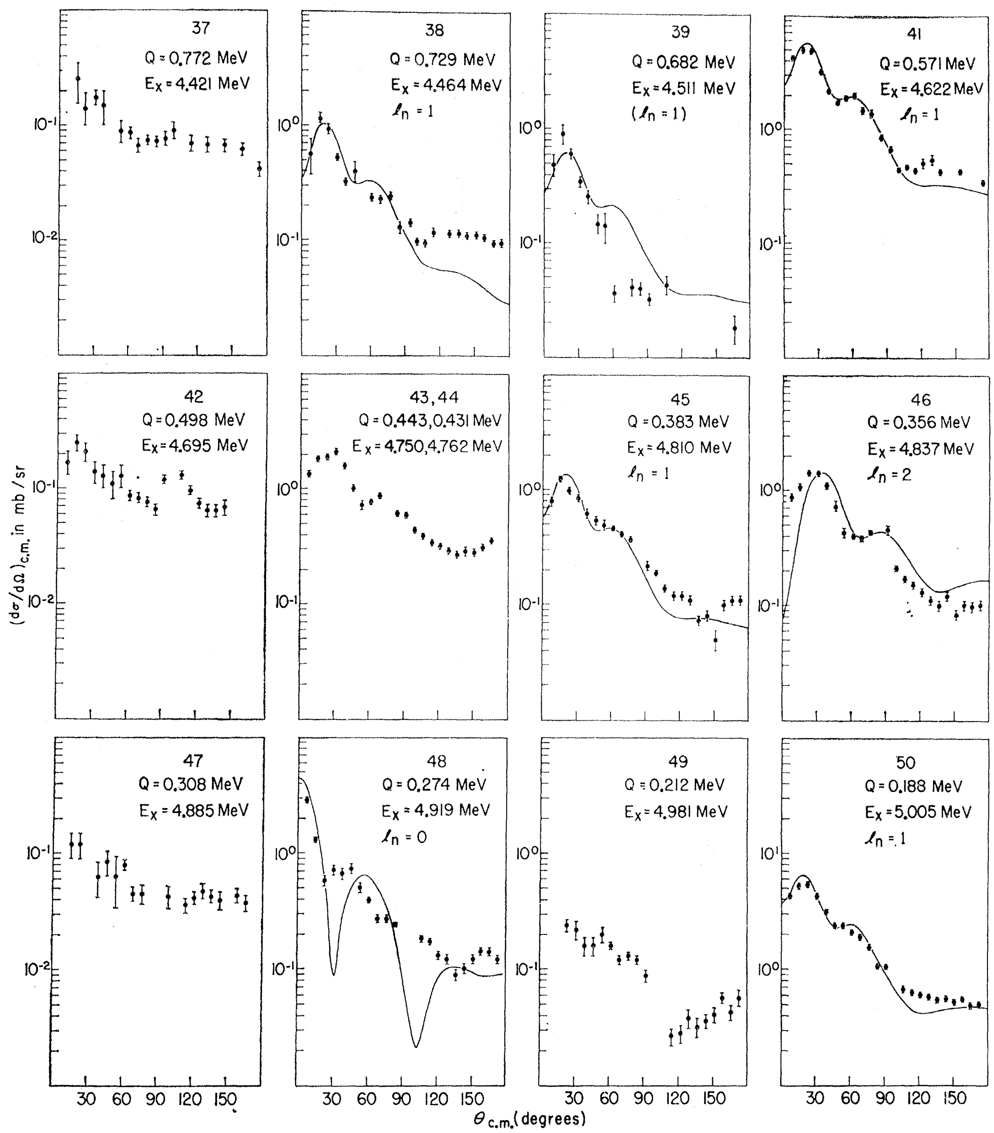

FIG. 6. Angular distributions of protons from $Ca^{44}(d,p)Ca^{45}$. The distributions are labeled with the numbers used to identify the corresponding states in Table I. Statistical uncertainties are indicated by error flags on the data points. The solid curves are the DWBA predictions using the parameters given in Table II.

assignments have been made to the higher-lying $l_n=1$ levels on this basis. Level No. 14 has been tentatively assigned $2p_{3/2}$ on the basis of its peculiar back-angle behavior which is somewhat similar to the back-angle behavior of the $\frac{3}{2}^-$ $E_x=0.594$-MeV state in Ca^{43} (see Ref. 6).

One $l_n=0$ and one $l_n=2$ transition appear below 2.5 MeV, and a single $l_n=4$ transition was observed at 5.390 MeV. The angular distribution for the doublet (states No. 43 and 44) at 4.75 MeV can be reasonably well fitted by a combination of $l_n=1$ and $l_n=2$. However, transition strengths were not extracted for this doublet.

The summed strengths of these transitions are presented in Table III and are compared with the shell-model predictions assuming Ca^{44} is a closed Ca^{40} core with four additional $1f_{7/2}$ neutrons.[15] The low-lying $l_n=0$ and $l_n=2$ transitions are assigned to $2s_{1/2}$ and

TABLE III. Summed strengths.

Orbit	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	$2p_{3/2}$	$2p_{1/2}$	$2p$	$1f_{5/2}$	$1g_{9/2}$	$2d_{5/2}$	$3s_{1/2}$
Expt.	0.11	0.15	3.36	3.91[a]	2.72[b]	6.63[c]	1.13	0.69	0.43	0.05
Theory	0	0	4	4	2	6	6	10	6	2

[a] Levels No. 2, 6, 14, 27, 28. [b] All other $l_n=1$ transitions. [c] All $l_n=1$ transitions.

[15] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. 32 567 (1960).

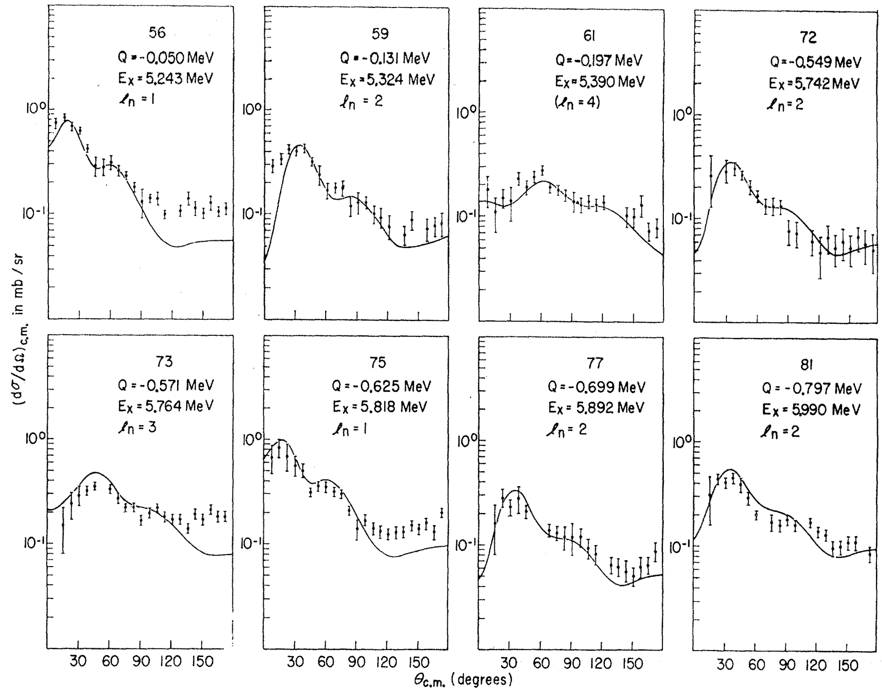

FIG. 7. Angular distributions of protons from $Ca^{44}(d,p)Ca^{45}$. The distributions are labeled with the numbers used to identify the corresponding states in Table I. Statistical uncertainties are indicated by error flags on the data points. The solid curves are the DWBA predictions using the parameters given in Table II.

$1d_{3/2}$ orbitals, respectively. The observation of $1d$ and $2s$ transitions would indicate that these shells are not completely filled in the Ca^{44} ground state.

The unperturbed single-particle energies $E'_{l,j}$ have been calculated using the energy-weighted sum rule of Yoshida[16]:

$$E'_{l,j}\sum S^a_{l,j}=\sum E^a_{l,j}S^a_{l,j},$$

where $E^a_{l,j}$ and $S^a_{l,j}$ are the measured excitation energies and spectroscopic factors. The results are given in Table IV and are compared with the unperturbed single-particle energies in Ca^{41}, Ca^{43}, and Ca^{47} from Refs. 5–7.

B. Comparison between Observed and Calculated Level Structures of Ca^{45}

Figure 9 shows the observed Ca^{45} level scheme below 2.5-MeV excitation, in comparison with the results of shell-model calculations of Raz and Soga,[17] Engeland and Osnes,[18] and Federman and Talmi.[19] These calculations were carried out using effective interactions for neutrons in the $1f_{7/2}$ and $2p_{3/2}$ orbits outside an inert Ca^{40} core. All three calculations reproduce the position of the first excited $\frac{5}{2}^-$ state and predict $\frac{3}{2}^-$ states near 1.4 and 2.0 MeV. No level near the $\frac{9}{2}^-$ state at 0.7 MeV, predicted by Raz and Soga, was observed. Engeland and Osnes predict the lowest $\frac{9}{2}^-$ and $11/2^-$ states at $E_x=1.51$ and $E_x=1.74$ MeV, respectively; two non-stripping states were seen at 1.558 and 1.584 MeV that may correspond to these states. Federman and Talmi[19] predict three $\frac{3}{2}^-$ levels at 1.41, 1.93, and 2.87 MeV. These agree remarkably well with observed $l_n=1$ transitions to states at 1.433, 1.904, and 2.847 MeV.

The observed $l_n=1$ strengths to the $\frac{3}{2}^-$ states at $E_x=1.433$ and 1.904 MeV are in fair agreement with the strengths calculated from the wave functions given in Ref. 18. The observed strengths are 0.47 and 2.56, and the calculated strengths are 0.43 and 3.52.

If the procedure of Bansal and French[20] is used, a $1d_{3/2}$ hole state in Ca^{45} near 1.0-MeV excitation is expected. The present work, together with the K^{45} beta-decay study,[9] indicates a $\frac{3}{2}^+$ state at 1.886 MeV. This state is further discussed in Sec. IV E.

C. The $N=25$ Isotones: Ca^{45}, Sc^{46}, and Ti^{47}

The low-lying, negative-parity states in Ca^{45} are characterized mainly by the neutron configuration $1f_{7/2}{}^5$ relative to the Ca^{40} core. Besides Ca^{45}, there are two other nuclei, Sc^{46} and Ti^{47}, which can be excited by means of (d,p) stripping reactions and which also have five neutrons outside the $N=20$ core. In Sc^{46} the five neutrons are coupled to one proton in the $1f_{7/2}$ shell, and in Ti^{47} the five neutrons are coupled to two protons in the $1f_{7/2}$ shell. It is interesting to compare the experimental level schemes of these isotones.

The $Ca^{44}(d,p)Ca^{45}$ ground-state transition carries the full $1f_{7/2}$ strength, whereas in the $Sc^{45}(d,p)Sc^{46}$ reaction, the $1f_{7/2}$ strength is distributed over eight transitions below 1.2-MeV excitation, in agreement with the simple

[16] S. Yoshida, Nucl. Phys. **38**, 380 (1962).
[17] B. J. Raz and M. Soga, Phys. Rev. Letters **15**, 924 (1965).
[18] T. Engeland and E. Osnes, Phys. Letters **20**, 424 (1966); E. Osnes, thesis, University of Oslo, Norway, 1966 (unpublished).
[19] P. Federman and I. Talmi, Phys. Letters **22**, 469 (1966).
[20] R. K. Bansal and J. B. French, Phys. Letters **11**, 145 (1964).

TABLE IV. Unperturbed single-particle excitation energies (MeV).

Configuration	$_{20}Ca_{21}{}^{41a}$	$_{20}Ca_{23}{}^{43b}$	$_{20}Ca_{25}{}^{45}$	$_{20}Ca_{27}{}^{47c}$
$1f_{7/2}$	0	0	0	0
$2p_{3/2}$	2.07	2.19±0.09	2.16±0.25	2.15±0.10
$2p_{1/2}$	4.13	4.23±0.20	3.98±0.10	4.02±0.07
$1f_{5/2}$	5.50	>4.5	>4.7	⩾5.5
$1g_{9/2}$	>5.0	>4.0	>5.4	>5.6
$2d_{5/2}$	>6.8	>4.3	>5.3	>6.0
$3s_{1/2}$	>6.8	>4.3	>4.9	>6.0

a Reference 5. b Reference 6. c Reference 7.

shell-model prediction of eight states arising from the coupling of a $1f_{7/2}$ proton to a $1f_{7/2}$ neutron. The first excited state in Ti^{47}, with $l_n=3$ and the full $1f_{7/2}$ strength, probably has the same neutron configuration as the Ca^{45} ground state.

Below 6.0-MeV excitation energy, 16 $l_n=1$ stripping states were observed, both in Ca^{45} and in Ti^{47}; whereas 64 such states were found in Sc^{46}. This would indicate that, both in Ca^{45} and in Ti^{47}, the $l_n=1$ states have the same neutron configuration and that the two $1f_{7/2}$ protons in Ti^{47} are coupled to $J^\pi=0^+$. The total number of states in this excitation range, including the non-stripping transitions, in Ca^{45}, Sc^{46}, and Ti^{47} is 82, 168, and 107, respectively. Below 2.0-MeV excitation energy, the number of observed excited states is 7, 28, and 7, which would indicate that those states in Ti^{47} and in Ca^{45} have similar neutron configurations. In the excitation region between 2.0 and 6.0 MeV, there are approximately 25% more states in Ti^{47} than in Ca^{45}, indicating that some of the Ti^{47} levels arise from excited proton configurations.

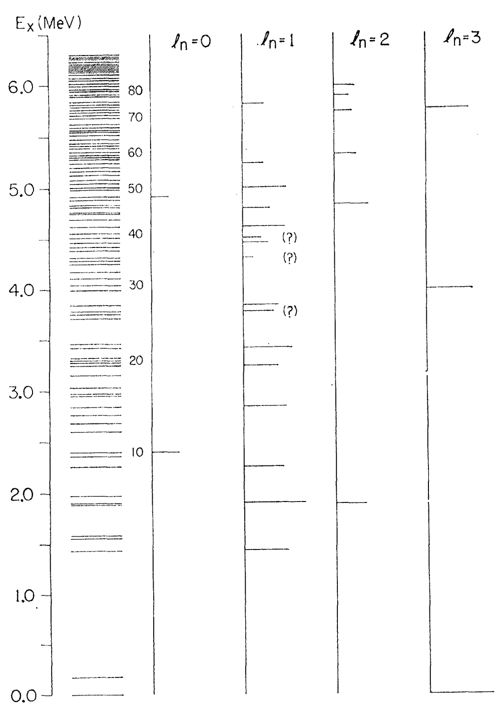

FIG. 8. Strength function for Ca^{45}. An excitation-energy scale is given on the left. The horizontal lines in the first column represent the observed levels with level numbers given to their right. The values of $(2J_f+1)S_{l,j}$ from Table I are plotted on a logarithmic scale in the last four columns. Only two proton groups above level No. 85 were analyzed.

D. The 1.886-MeV State and the Ca^{44} Ground State

Level No. 5 at 1.886-MeV excitation has previously been interpreted as a $\frac{3}{2}^+$ hole state.[9] The (d,p) transition to this state has an angular distribution (see Fig. 3) characterized by $l_n=2$, in agreement with this assignment. In accordance with the analyses of the $1d_{3/2}$ hole states, previously observed[5–7,21] in Ca^{41}, Ca^{43}, and Ca^{47}, we write the wave function for level No. 5 as

$$|Ca^{45}(5)J^\pi=\tfrac{3}{2}^+;\,T=\tfrac{5}{2},T_z=\tfrac{5}{2}\rangle = (\sqrt{6/7})^{1/2}\{1d_{3/2}{}^{-1};\tfrac{1}{2},-\tfrac{1}{2}\}\{1f_{7/2}{}^6;3,3\} - (\sqrt{1/7})^{1/2}\{1d_{3/2}{}^{-1};\tfrac{1}{2},\tfrac{1}{2}\}\{1f_{7/2}{}^6;3,2\},$$

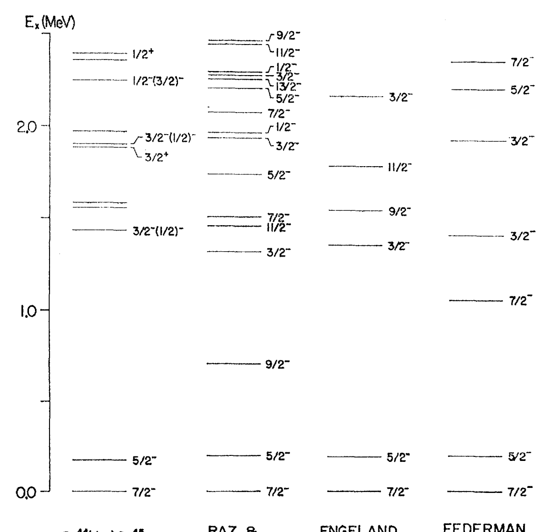

FIG. 9. Low-lying part of the Ca^{45} level scheme. The observed levels in Ca^{45} below 2.5-MeV excitation are shown, together with shell-model calculations.

21 T. W. Conlon, B. F. Bayman, and E. Kashy, Phys. Rev. 144, 941 (1966).

and the ground state of Ca^{44} as

$$|Ca^{44}(0)0^+;2,2\rangle = a\{1f_{7/2}{}^4;2,2\} + b[\{1d_{3/2}{}^{-2};T=1\}\{1f_{7/2}{}^6;T=3\}]_{T=2}.$$

The strength of the transition to level No. 5, given in Table II, yields

$$b^2=0.08\pm0.02.$$

The strength of the ground-state transition from Table II yields

$$a^2=0.84\pm0.21,$$

where we have neglected admixtures of other wave-function components into the $1f_{7/2}{}^5$ Ca^{45} ground state.

E. Ground-State Wave Functions for Even Calcium Isotopes

Figure 10 presents the results of the analyses of (d,p) transitions, carried out as indicated in Sec. IV D, from even calcium isotopes leading to $\frac{3}{2}^+$ states in the final nucleus. In the calculations, it is assumed that the ground-state wave function of the even calcium target is mainly of the form

$$Ca^{40+2n}=a|1f_{7/2}{}^{2n}\rangle+b|\{1d_{3/2}{}^{-2}\}\{1f_{7/2}{}^{2n+2}\}\rangle.$$

It is observed that the amounts of $1d_{3/2}{}^{-2}$ core excitation decrease as the $1f_{7/2}$ shell is filled; the percentages of $1d_{3/2}{}^{-2}$ admixture were found to be 60, 30, 8, and 4 in going from Ca^{40} to Ca^{46}. No evidence for $1d_{3/2}{}^{-2}$ core excitation in Ca^{48} was observed in the (d,p) experiment,[8] although Erskine *et al.*[22] report an $l_p=2$ transition in the $Ca^{48}(He^3,d)Sc^{49}$ reaction which they interpret as arising from $1d_{3/2}$ proton core excitation. Recently, Rost[23,24] has done coupled-channel calculations using a deformed-well radial wave function for stripping and pickup reactions. He finds a value of $S=0.25$ for the transition to the 2.017-MeV level in Ca^{41}. This is in good agreement with the value[5] $S=0.2$ extracted using spherical-well radial wave functions. Therefore, it is expected that the spectroscopic factors for hole-state transitions in the other calcium isotopes are not in large error.

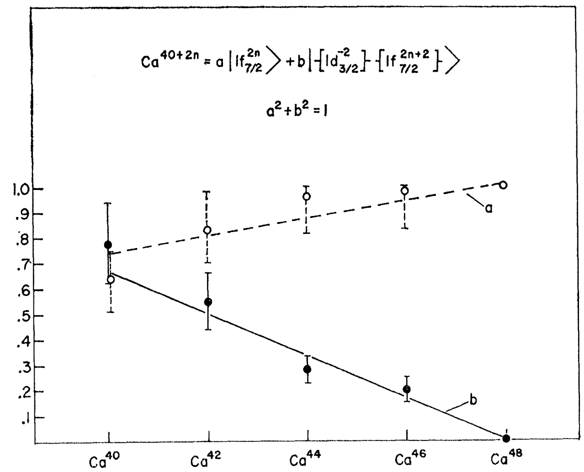

FIG. 10. Wave-function components for even-A calcium isotopes. The values of b are extracted as in Sec. IV D. The values of a given here are determined from $a^2+b^2=1$.

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[22] J. R. Erskine, A. Marinov, and J. R. Schiffer, Phys. Rev. **142**, 633 (1966).
[23] E. Rost, Phys. Letters 21, 87 (1966).
[24] E. Rost, Phys. Rev. **154**, 994 (1967).