

Quasi-Elastic (p,n) Scattering from Scandium and Titanium Isotopes*

C. D. GOODMAN

Oak Ridge National Laboratory, Oak Ridge, Tennessee

AND

J. D. ANDERSON AND C. WONG

Lawrence Radiation Laboratory, University of California, Livermore, California

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The differential cross sections have been measured for quasi-elastic (p,n) scattering on a sequence of targets ^{45}Sc and $^{46,47,48,49,50}\text{Ti}$ which have neutron excesses from 2 to 6. Except for ^{47}Ti and ^{49}Ti , the cross sections are proportional to the neutron excess. The measured Q values for the reaction are interpreted as measurements of the Coulomb energy difference for the isobars. The results show that the effective charge radius increases less rapidly than $A^{1/3}$ as neutrons are added to the titanium isotopes.

1. INTRODUCTION

THE excitation of isobaric analog states by the (p,n) reaction can be explained with an optical model that includes an isospin term.¹⁻⁴ Although the agreement between the calculations and the measurements is not overly impressive, the shapes of the angular distributions and the magnitudes of the cross sections are reproduced reasonably well by the model²; the required strength of the isospin potential is in agreement with other estimates of the symmetry term in the optical potential.^{5,6}

Briefly, the model consists of adding a term proportional to $(\mathbf{t} \cdot \mathbf{T})$ in the optical potential. Here t signifies the isobaric spin of the incident nucleon and T signifies the total isobaric spin of the target nucleus. The $(t+T^-)$ part of this interaction gives rise to the (p,n) quasi-elastic scattering.

A simple prediction of this formulation which is particularly amenable to experimental verification and which is relatively insensitive to optical parameters is the dependence of the isobaric (p,n) cross section on the neutron excess: The cross section is proportional to the neutron excess.

Using the Livermore 90-in. cyclotron facility, we have measured the angular distributions of the quasi-elastic (p,n) reaction for 15.25-MeV protons on ^{21}Sc , ^{22}Ti , ^{47}Ti , ^{48}Ti , ^{49}Ti , and ^{50}Ti in approximately 15° intervals between 3° and 153° . For the even-even Ti isotopes and for ^{45}Sc , the measured cross section is proportional to the neutron excess. The ^{47}Ti and ^{49}Ti cross sections are 60% and 25% higher, respectively, than the expected values.

Good resolution (± 40 keV) isobaric Q value measurements were made on $f_{7/2}$ shell nuclei (^{21}Sc , $^{22,46,47,48,49,50}\text{Ti}$, ^{23}V , ^{24}Cr , ^{25}Mn , and $^{26,56,58}\text{Fe}$) using 14.25-MeV protons. The results are compared with the recent (p,d) Coulomb displacement energy data⁷ and with previous (p,n) results.⁸

2. EXPERIMENTAL DETAILS

A. Geometry and Electronics

The electronic systems and experimental geometries have been described in detail elsewhere^{9,10} and are only briefly summarized here.

For angular-distribution measurements, the Livermore time-of-flight facility has available 10-m flight paths at 3° , 30° , 60° , 90° , 120° , and 135° . With an additional bending magnet and auxiliary equipment (see Fig. 3, Ref. 9), the proton beam can be doubly bent so that its incidence angle at the target is 18° to the

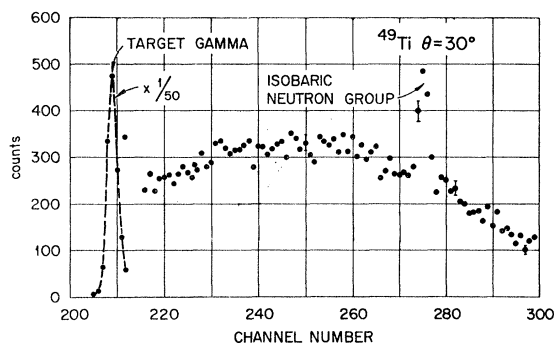


FIG. 1. Neutron time spectrum from $^{49}\text{Ti}(p,n)^{49}\text{V}$ recorded for angular-distribution data. This is one of six such spectra recorded simultaneously in 100-channel subgroups of the analyzer.

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¹ J. D. Anderson, C. Wong, J. W. McClure, and B. D. Walker, Phys. Rev. **136**, B118 (1964).

² G. R. Satchler, R. M. Drisko, and R. H. Bassel, Phys. Rev. **136**, B637 (1964).

³ A. M. Lane, Phys. Rev. Letters **8**, 171 (1962).

⁴ A. M. Lane, Nucl. Phys. **35**, 676 (1962).

⁵ F. G. Perey, Phys. Rev. **131**, 745 (1963).

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⁷ R. Sherr, B. F. Bayman, E. Rost, M. E. Rickey, and C. G. Hoot, Phys. Rev. **139**, B1272 (1965).

⁸ J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. **138**, B615 (1965).

⁹ B. D. Walker, J. D. Anderson, J. W. McClure, and C. Wong, Nucl. Instr. Methods **29**, 333 (1964).

¹⁰ J. D. Anderson and C. Wong, Nucl. Instr. Methods **15**, 178 (1962).

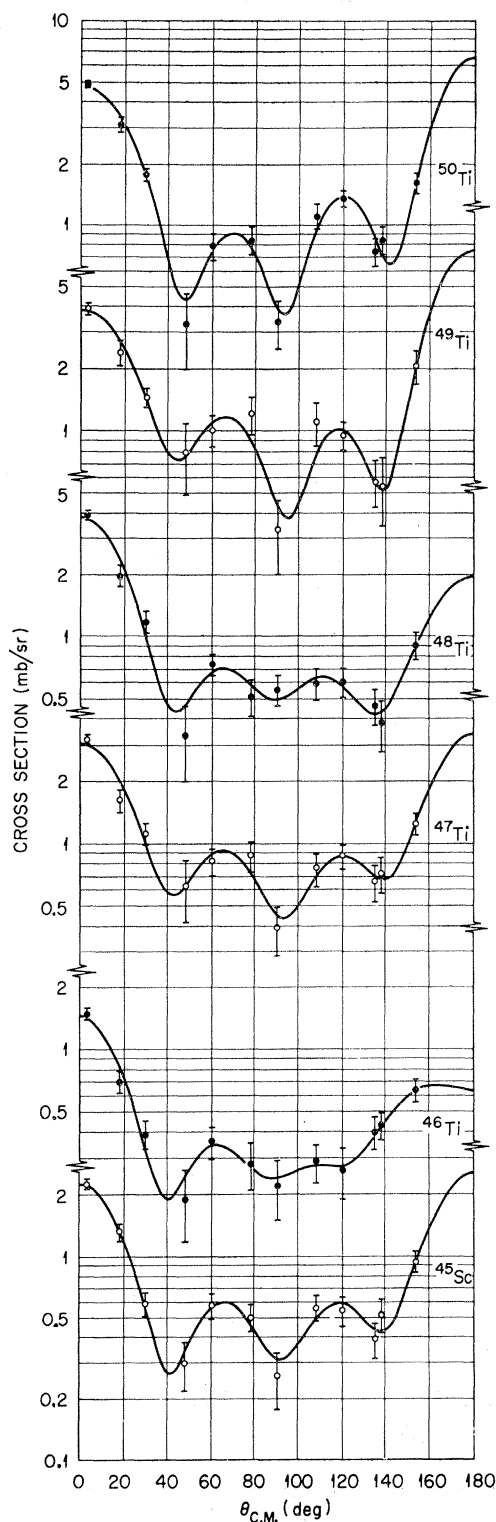


FIG. 2. Angular distributions of the neutron groups leading to the isobaric analogs of the target states. The solid curves are polynomial fits to the data used for estimating the total cross sections.

usual beam line. This doubles the number of angles for which measurements can be made. The 10-m flight paths are collimated so that the detectors view directly only a few inches around the target area. This reduces the background from the "beam catcher" and sweeping slits.

A 23° flight path that can be varied from 8.7 to 15 m (Fig. 6, Ref. 10) was used to measure the isobaric (p, n) Q values (Coulomb displacement energies). Proton-electron pulse shape discrimination was used in this measurement to reduce the background from gamma radiation.¹⁰

B. Targets

The isotopic Ti targets¹¹ were rolled foils of thicknesses corresponding to 50–100-keV energy loss for the incident beam and 1 in. in diam. The target thicknesses, determined by weighing, and mass analysis are listed in Table I. The Sc target was a rolled foil of 5.2 mg/cm² thickness. The beam spot was diffuse and covered most of the target area.

The ⁵⁸Fe target used for the measurement of Coulomb displacement energy was a 2 mg/cm² self-supporting, electroplated foil. The elemental targets used have been described elsewhere.^{1,8}

3. RESULTS

A. Angular-Distribution Measurements

The 30° time-of-flight spectrum resulting from 15.25-MeV proton bombardment of ⁴⁹Ti is shown in Fig. 1. The principal background obscuring the neutrons corresponding to the isobaric state is a continuum which we assume to be due to evaporation neutrons from compound nuclear processes. Angular distribution data (Fig. 2) were taken in two steps. Data at 3°, 30°, 60°, 90°, 120°, and 135° were obtained simultaneously in 100-channel subgroups of an 800-channel pulse-height analyzer. Then the proton beam was doubly bent to strike the target at 18° to the normal beam line and the 18°, 48°, 108°, 138°, and 153° data were obtained.

The Ti cross sections (Table II) were corrected for the isotopic impurities, typically a 15 to 30% correction. The largest corrections were for ⁴⁶Ti where the cross section is relatively small (see Fig. 2) and for ⁵⁰Ti where the relative abundance was poor (see Table I).

TABLE I. Isotopic composition of titanium targets.

Target	Thickness (mg/cm ²)	Mass analysis, % atoms				
		46	47	48	49	50
⁴⁶ Ti	4.88	84.3	2.8	10.8	1.0	1.1
⁴⁷ Ti	3.60	1.9	80.0	15.8	1.2	1.1
⁴⁸ Ti	5.78	0.2	0.3	99.2	0.2	0.1
⁴⁹ Ti	2.39	2.0	1.9	18.9	75.4	1.8
⁵⁰ Ti	4.74	3.1	2.4	23.1	2.0	69.4

¹¹ The targets were prepared by the Oak Ridge National Laboratory Isotopes Division.

TABLE II. Analog-state angular distributions for the (p, n) reaction from 15.25-MeV protons.^a

θ_{Lab}	⁴⁶ Sc	⁴⁶ Ti	⁴⁷ Ti	⁴⁸ Ti	⁴⁹ Ti	⁵⁰ Ti
3	2.25±0.11	1.49±0.08	3.16±0.13	3.93±0.14	3.90±0.20	4.79±0.18
18	1.32±0.11	0.69±0.08	1.62±0.20	1.98±0.20	2.42±0.32	3.10±0.23
30	0.59±0.08	0.39±0.06	1.12±0.13	1.19±0.14	1.47±0.15	1.76±0.12
48	0.30±0.08	0.19±0.07	0.62±0.20	0.33±0.13	0.79±0.30	0.33±0.13
60	0.58±0.08	0.36±0.06	0.82±0.12	0.73±0.08	1.02±0.18	0.79±0.12
78	0.51±0.08	0.28±0.07	0.87±0.14	0.51±0.10	1.23±0.25	0.85±0.13
90	0.26±0.08	0.22±0.07	0.39±0.10	0.55±0.09	0.33±0.13	0.34±0.09
108	0.57±0.08	0.29±0.06	0.76±0.14	0.59±0.10	1.13±0.27	1.12±0.15
120	0.56±0.09	0.26±0.07	0.87±0.11	0.60±0.09	0.97±0.15	1.37±0.12
135	0.40±0.08	0.40±0.07	0.66±0.13	0.46±0.09	0.58±0.15	0.75±0.11
138	0.53±0.10	0.43±0.06	0.72±0.14	0.38±0.10	0.55±0.20	0.86±0.13
153	0.96±0.10	0.64±0.08	1.25±0.14	0.91±0.13	2.09±0.34	1.65±0.19

^a Units = mb/sr.

The errors are computed from the statistical errors and the errors due to uncertainties in line shape. The uncertainty in line shape produces no appreciable error when the neutron group is prominent (near 0°) but accounts for most of the error near minima in the angular distributions.

B. Isobaric (p, n) Reaction Q -Value Measurements

The 23° time-of-flight spectrum resulting from 14-MeV proton bombardment of ²²⁵⁰Ti is shown in Fig. 3. The target γ rays appear twice since a double display is used—one converter stop pulse for every two beam pulses.¹⁰ The neutron groups correspond to (A) configuration states,^{12,13} i.e., states in the residual nucleus having the same shell-model orbital configuration as the target nucleus but different isotopic spin ($\Delta T=1$), and (B) isobaric state ($\Delta T=0$).

The energies of the neutrons are calculated from their time-of-flight,¹⁴ and the energy for the incident protons is determined by means of a differential range measurement in aluminum. The isobaric (p, n) reaction Q values (Coulomb displacement energies) are summarized in Table IV. The Q values were measured during two separate runs and the targets cycled in quick succession in order to obtain good relative Q values ($\pm 30 - \pm 50$ keV). The absolute uncertainty of ± 100 keV is due to the proton beam energy measurement.

4. DISCUSSION

A. Angular-Distribution Measurements

Lane noted that an isobaric spin dependence arises in a potential calculated from a sum of two-body forces containing Heisenberg components and averaged over a Fermi gas.³ He proposed that the optical-model potential should be of the form

$$V = V_0 + (\mathbf{t} \cdot \mathbf{T}) V_1 / A, \quad (1)$$

where V_0 is the ordinary optical potential, $\mathbf{t}$ is the

isospin of the incident particle, and $\mathbf{T}$ is the isospin of the target nucleus of mass A . The isospin term ($\mathbf{t} \cdot \mathbf{T}$) acting on an incident proton can convert it into a neutron, and turn the target into the corresponding isobaric state thus resulting in a quasi-elastic (p, n) reaction.

If the ($\mathbf{t} \cdot \mathbf{T}$) potential is treated as a perturbation, the cross section for the (p, n) reaction should be proportional to the square of the matrix element of ($\mathbf{t} \cdot \mathbf{T}$) between the final and initial states. The final state is the isobaric analog of the initial state. Thus the final nuclear wave function is by definition the normalized resultant of the T^- operator. That is,

$$\psi_{\text{analog}} = T^- \psi_i / (\langle (T^- \psi_i)^* T^- \psi_i \rangle)^{1/2}.$$

Thus the square of the nuclear matrix element is

$$\langle \psi_{\text{analog}} T^- \psi_i \rangle^2 = \langle (T^- \psi_i)^* T^- \psi_i \rangle.$$

For states of pure isobaric spin, this reduces to $T(T+1) - M(M-1)$, where M is the third component of T . If $T=M$, which is expected for the ground state of a nucleus, this further reduces to $2M$, which is the neutron excess.

In complete calculations of the (p, n) cross sections, the total optical potential influences the cross section.

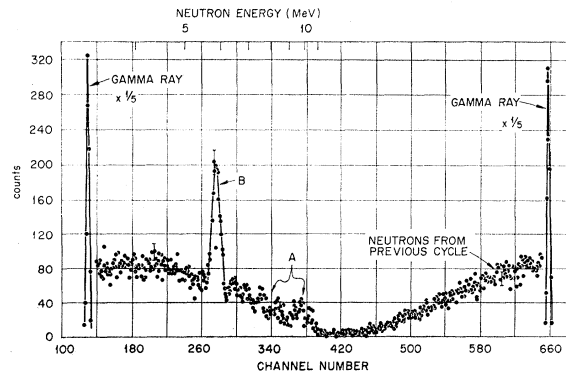


FIG. 3. Neutron time spectrum for ⁵⁰Ti(p, n)⁵⁰V obtained with the 23° flight path for Q -value measurements. The groups marked A are the configuration-state groups and the group marked B is the analog-state group.

¹² A. M. Lane and J. M. Soper, Nucl. Phys. **37**, 506 (1962).

¹³ K. Ikeda, Progr. Theoret. Phys. (Kyoto) **31**, 434 (1964).

¹⁴ J. D. Anderson, C. Wong, and J. W. McClure, Nucl. Phys. **36**, 161 (1962).

TABLE III. Total (p,n) cross sections.

Target	σ_T (mb)	$(\sigma_T)/(N-Z)$	Ratio of $(\sigma_T)/(N-Z)$ to that of ^{48}Ti
^{46}Sc	7.41	2.47	1.11 ± 0.08
^{46}Ti	4.42	2.21	0.97 ± 0.08
^{47}Ti	10.99	3.66	1.61 ± 0.10
^{48}Ti	9.10	2.27	1.00
^{49}Ti	14.33	2.87	1.26 ± 0.10
^{50}Ti	14.20	2.37	1.04 ± 0.06

However, the dominant effect over a small set of neighboring nuclei such as ^{46}Sc to ^{50}Ti is just the dependence on neutron excess. This can be seen from the set of differential cross sections obtained from a distorted wave calculation discussed in Sec. 4C, and shown in Fig. 5. The curves differ very slightly in shape, the principal difference being in the magnitude. Although the fits to the data are all rather poor, it should be noticed that the data points for ^{47}Ti lie consistently above the curve, while the magnitudes of the other cross sections are generally well represented. This suggests a departure from the expected simple dependence on neutron excess.

Since the theory predicts that the total cross section should be closely proportional to the neutron excess, this departure can be expressed in a more succinct way through a comparison of cross sections integrated over all angles. We have obtained approximations to the integrals by fitting to the data points smooth functions composed of eight Legendre polynomials. The points and curves are shown in Fig. 2. The total cross sections and their comparison with the neutron excesses are shown in Table III. In this comparison, the cross sections for ^{47}Ti and ^{49}Ti are both larger than expected.

The anomaly suggests that either our nuclear structure assumption, i.e., that $T=T_Z$ needs modification, or an additional term contributes to the reaction cross section.

As for the first possibility, the cross section is at a minimum for $T=T_Z$ and becomes larger as components of higher T are introduced into the target wave function. To explain the large ^{47}Ti cross section would require about a 30% admixture of $T=\frac{5}{2}$ to the assumed $T=\frac{3}{2}$ wave function. This seems unlikely in view of the known positions of analog states. To estimate the positions of $T=\frac{5}{2}$ states in ^{47}Ti , we can consider the analogy with ^{47}Sc . The states of ^{47}Sc must be $T=\frac{5}{2}$ or higher. These states are expected to have analogs in ^{47}Ti in at least approximately the same sequence as in ^{47}Sc . The analog of the ground state of ^{47}Sc occurs at an excitation of about 6.5 MeV in ^{47}Ti and this is expected to be the lowest $T=\frac{5}{2}$ state in ^{47}Ti . It seems that a slight departure from this picture would not result in any appreciable $T=\frac{5}{2}$ mixing in the ground state of ^{47}Ti which is so far away in energy.

A shell-model calculation for $(f_{7/2})^n$ configurations has been carried out by McCullen *et al.*¹⁵ The two-body residual force matrix elements are chosen assuming good isobaric spin and an analog structure separated by the Coulomb energy for neighboring isobars. The calculations result in tolerable agreement with observed energy levels and, for ^{47}Ti they show the 13 possible $T=\frac{3}{2}$, spin $\frac{5}{2}^-$ states distributed over the first 10 MeV of excitation with the first $T=\frac{5}{2}$, $\frac{5}{2}^-$ state at 7 MeV.

If, however, one supposes that purity of isobaric spin breaks down only when several states can exist which are the same in other quantum numbers but differ only in T , then ^{47}Ti is in a unique position; it is the only isotope in the sequence studied in this experiment for which isobaric spin impurity might be observed if it occurred. In ^{47}Sc , three states can be constructed with spin $\frac{5}{2}$, $T=\frac{5}{2}$, and seniority 3. In ^{47}Ti , in addition to analogs of those three states, four spin $\frac{5}{2}$, $T=\frac{3}{2}$, seniority 3 states and seven other spin $\frac{5}{2}$, $T=\frac{3}{2}$ of higher seniority can be constructed.¹⁶ In the even isotopes of Ti, the only other spin-0 states of higher T differ by two units in T . In ^{46}Sc and in ^{49}Ti , the only other spin- $\frac{7}{2}$ states of higher T are the uniquely defined analogs of the ground states of ^{46}Ca and ^{49}Sc , respectively. Nevertheless, a 30% $T=\frac{5}{2}$ admixture in the ^{47}Ti ground-state wave function seems rather unlikely.

An alternative approach to explaining the anomalous cross sections is to assume that an additional interaction occurs in the reaction which is inoperative in the other cases. It has been pointed out¹⁷⁻¹⁹ that for targets of nonzero spin I , V_1 need not be a scalar and may contain even multipole moments of order $l \leq 2I$. Thus one might expect differences between even, $I=0$ and odd, $I \neq 0$ targets. Satchler *et al.*² have noted that for deformed nuclei a considerable part of the quadrupole strength for odd targets may appear as quasi-elastic transitions. If the presence of multipole moments $l \leq 2I$ in V_1 were to account for the large ^{47}Ti cross section, then one should expect to see different angular distributions for the even isotopes than for the odd and one would expect to see "quasi-inelastic" transitions in the even isotopes to the low-lying 2^+ levels. Although quasi-inelastic scattering has been seen, no pronounced excitation of low-lying 2^+ isobaric states was observed in this experiment.

Another possible additional interaction is a spin-flip term in the two-body force. In obtaining the optical isospin potential from a sum over two-body forces, Lane⁴ noted that there were two charge-exchange terms. The first is $\sum_i (\mathbf{t}_0 \cdot \mathbf{t}_i) = (\mathbf{t} \cdot \mathbf{T})$ and the second is $\sum_i (\mathbf{s}_0 \cdot \mathbf{s}_i)(\mathbf{t}_0 \cdot \mathbf{t}_i)$, where s and t refer, respectively, to

¹⁵ J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. **134**, B515 (1964).

¹⁶ B. H. Flowers, Proc. Roy. Soc. (London) **A212**, 248 (1952).

¹⁷ J. B. French and M. H. Macfarlane, Phys. Letters **2**, 255 (1962).

¹⁸ P. E. Hodgson and J. R. Rook, Nucl. Phys. **37**, 632 (1962).

¹⁹ R. M. Drisko, R. H. Bassel, and G. R. Satchler, Phys. Letters **2**, 318 (1962).

the spin and isospin of the interacting particles. Since spins s_i pair off to zero, this second term cannot become large. Although it was noted that this spin-flip term might be important for light nuclei, it was concluded that for large neutron excesses the $(\mathbf{t} \cdot \mathbf{T})$ term would dominate and the spin flip would only produce a small fluctuation in the potential. It is important to note that this second term can contribute to the excitation of the isobaric state only if $I \neq 0$. Evidence for a spin-flip interaction in light nuclei has recently been reported.²⁰ In the titanium isotopes, the spin-flip contribution should be most pronounced for ^{47}Ti since its neutron excess is the smallest ($N-Z=3$). However, until exact shell-model calculations are made, it is not clear whether this contribution is constructive or destructive, i.e., whether the isobaric cross section is increased or decreased.

B. Coulomb Displacement Energies

A summary of Coulomb displacement energies (ΔE_c) derived from the isobaric (p, n) reaction has been presented in Ref. 8. The data tabulated from this experiment are given in Table IV. In Fig. 4, the present results are plotted versus $\bar{Z}/A^{1/3}$ [where $\bar{Z}$ is the average charge, i.e., $\bar{Z} = \frac{1}{2}(Z_{\text{initial}} + Z_{\text{final}})$, and A is the mass number] and compared to ΔE_c 's derived from the (p, d) reaction.⁷ The solid line in Fig. 4 is computed from a semi-empirical description of ΔE_c based on previous (p, n) data.⁸ The dependence of ΔE_c on $\bar{Z}/A^{1/3}$ is seen to be the same for the (p, n) and (p, d) data. The absolute-energy scale would appear to differ by about 70 keV which is well within the 100-keV absolute error quoted for the (p, n) data.

For the Ti isotopes, ΔE_c was expected to vary by 2.7% between ^{46}Ti and ^{50}Ti , i.e., as $A^{-1/3}$. The measured result was $(1.0 \pm 0.4)\%$. This difference indicates the presence of a shell effect in the spatial charge distribution. If this is interpreted as a charge-radius increase of $(1.0 \pm 0.4)\%$ from ^{46}Ti to ^{50}Ti , it is similar to that found in recent electron-scattering measurements²¹ on the calcium isotopes where the addition of $f_{7/2}$ neutrons, i.e., ^{40}Ca to ^{44}Ca , increases the charge radius by 0.8% compared to the $A^{1/3}$ prediction of 3.2%.

Janecke²² took into account shell effects in his semi-empirical description of ΔE_c by assuming

$$\Delta E_c(i) = E_1(i)\bar{Z}A^{1/3} + E_2(i), \quad (2)$$

where the constants E_1 and E_2 were shell-dependent. A possible alternative description would be

$$\Delta E_c(i) = E_1\bar{Z}A^{-1/3} + E_2 + \frac{1}{2}E_3(n_i - 2n_0), \quad (3)$$

where E_1 , E_2 , and E_3 are constants, n_i is the maximum number of neutrons in the i th shell, and n_0 is the actual

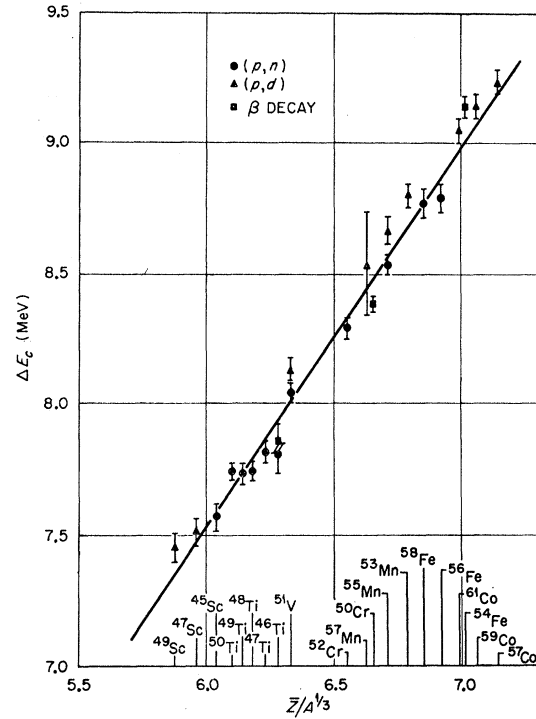


FIG. 4. Coulomb displacement energies obtained from this work and other sources. The beta-decay data are from Ref. 23.

number of neutrons in the i th shell. The titanium data yield $E_3 = -0.050$ MeV. In addition to describing the ΔE_c variation between ^{41}Sc and ^{54}Co in the $f_{7/2}$ shell,²³ this value of E_3 yields a difference in ΔE_c energies between the closing of the d shell and the beginning of the

TABLE IV. Coulomb displacement energies; $E_p = 14$ MeV.

Target	Analog state Q value (MeV)	
	This experiment ^a	Previously published ^b
^{27}Al	5.57 ± 0.05 7.74 ± 0.04	5.59 (3rd excited state ^{27}Si) 7.76
^{46}Sc	7.58 ± 0.04	
^{46}Ti	7.71 ± 0.03	7.85 ± 0.10
$^{46}\text{Ti}^c$	7.80 ± 0.04	
$^{47}\text{Ti}^c$	7.81 ± 0.04	
$^{48}\text{Ti}^c$	7.74 ± 0.03	
$^{49}\text{Ti}^c$	7.73 ± 0.03	
$^{50}\text{Ti}^c$	7.74 ± 0.03	
^{51}V	8.04 ± 0.03	8.05 ± 0.10
^{52}Cr	8.29 ± 0.03	8.40 ± 0.15
^{55}Mn	8.53 ± 0.03	
^{56}Fe	8.79 ± 0.05	8.85 ± 0.15
^{58}Fe	8.77 ± 0.05	

^a Errors are relative errors. Absolute errors are ± 0.10 MeV.

^b Absolute errors.

^c The titanium isotope Q values were measured during two separate runs and in quick succession in order to obtain good relative Q values. From ^{50}Ti to ^{46}Ti , the Coulomb displacement energy increases by $(1.0 \pm 0.4)\%$.

²³ J. H. Miller, III, thesis, Princeton University, Tech. Report No. NYO-2959 (unpublished).

²⁰ S. D. Bloom, J. D. Anderson, W. F. Hornyak, and C. Wong, Phys. Rev. Letters **15**, 264 (1965).

²¹ R. Hofstadter *et al.*, Phys. Rev. Letters **15**, 758 (1965).

²² J. Janecke, Z. Physik **160**, 171 (1960).

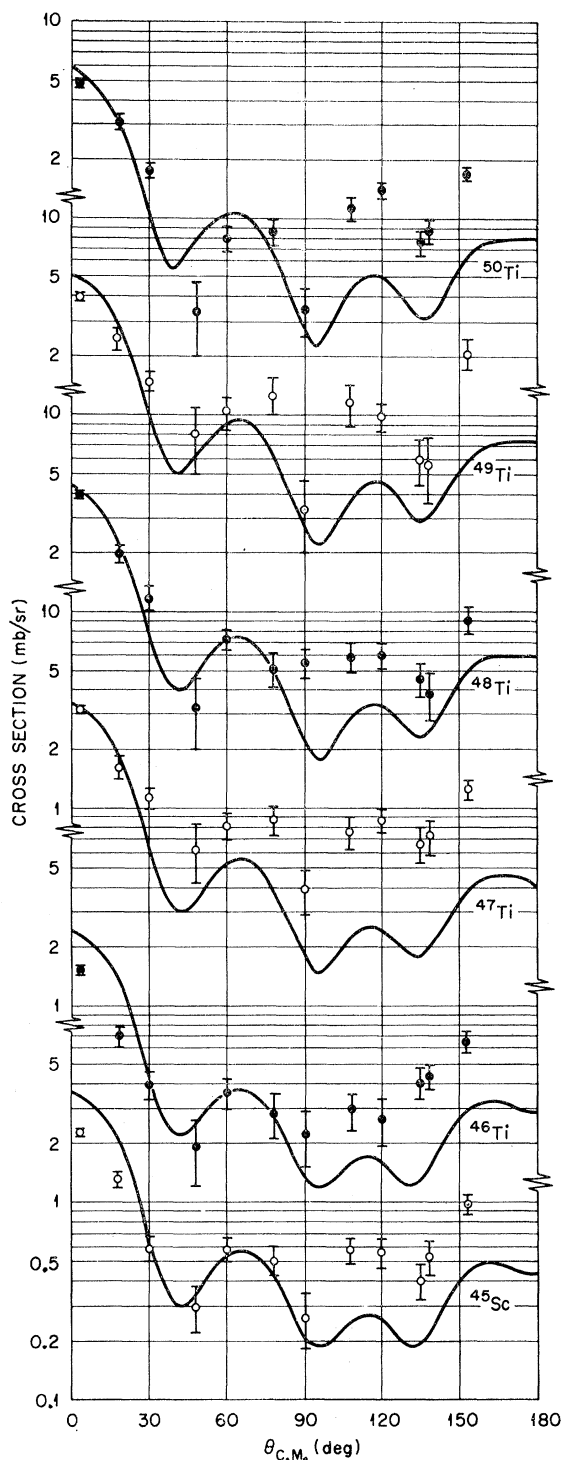


FIG. 5. Comparison of the measured (p,n) isobaric cross sections with distorted-wave calculations. The solid curves are results of calculations made with the code JULIE using the optical-model parameters given in Ref. 2.

f shell of ~ 350 keV which is in reasonable agreement with experiment.^{22,24}

C. Comparison with Distorted-Wave Calculations

In Fig. 5, data are compared with distorted-wave calculations using an isospin-dependent optical model for the isobaric reaction. The surface form of the interaction was used as described in Ref. 2, and the well parameters were taken to be identical to those used in Ref. 2.

These are, in MeV:

$$V_p = 53.3 - 0.55E_p + 0.4(Z/A^{1/3}) + 27(N-Z)/A,$$

$$V_n = 48 - 0.3E_n,$$

$$W_{Dp} = 3A^{1/3},$$

$$W_{Dn} = 96.$$

The code JULIE was used for the calculations.

D. Mass of ^{46}V

In the case of ^{46}Ti - ^{46}V isobaric pair, the analog of the ground state of ^{46}Ti is the ground state of ^{46}V and the measurement of the (p,n) Q value also provides a good determination of the mass difference of the pair. Our measured Q value yields a mass difference (^{46}V - ^{46}Ti) of 7.80 ± 0.100 MeV. Using the 1964 Mass Tables²⁵ value of 44.1226 ± 0.0023 MeV for the mass excess of ^{46}Ti , we calculate a mass excess of 37.105 ± 0.040 MeV for ^{46}V based on ^{12}C . This is in agreement with the values of -37.060 ± 0.009 MeV in the 1964 Table and in disagreement with older tables.

5. CONCLUSIONS

The excitation of isobaric analog states of Sc and Ti isotopes by the (p,n) reactions can be accounted for by an optical model with an isospin-dependent term. The even isotopes of Ti and ^{45}Sc show the expected cross-section dependence on the neutron excess. The ^{47}Ti and ^{49}Ti cross sections are larger than expected. Explanation of the larger cross sections seems to require the assumption either of isospin mixing in the target nuclei or of an additional term such as a spin-flip interaction in the reaction.

The Coulomb displacement energies measured were found to be in good agreement with previous (p,n) data⁸ and in reasonable agreement with recent (p,d) data.⁷ The Coulomb displacement energy for the Ti isotopes does not decrease as rapidly as $A^{-1/3}$. The results indicate a shell dependence of the effective charge radius.

²⁴ O. Kofoed-Hansen, Rev. Mod. Phys. **30**, 449 (1958).

²⁵ J. H. E. Mattauch *et al.*, Nucl. Phys. **67**, 1 (1965).