Elastic Scattering of Neutrons from O and Ar at 14.0 MeV*

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(Received 16 November 1966)

Differential cross sections of 14-MeV neutrons elastically scattered from liquid samples of oxygen and argon have been measured using associated-particle time-of-flight techniques. Careful attention was given to the subtraction of background neutrons scattered from the Dewar walls. The corrected data were analyzed in terms of four- and six-parameter optical-model calculations. A possible ambiguity in the imaginary potential for oxygen is discussed. The sensitivity of the results to the parameters in the spin-orbit potential is also examined.

I. INTRODUCTION

ONE important limitation in the application of an optical model for neutron scattering from nuclei has been the lack of data of a certain kind. For example, a proper situation for the application of the optical model should involve the scattering of monoenergetic neutrons from the ground state of a single isotope of the target nucleus. However, since neutron-scattering experiments frequently require the use of large quantities of target material, it has only rarely been practical to use target material with other than the isotopic abundance found in nature. It was felt then that those elements which are very nearly monoisotopic ought to be carefully studied. O^{16} and Ar^{40} are 99.5% abundant and were thus considered likely candidates for this work, particularly since scattering from argon has never been reported in this energy region. The problems associated with the use of liquefied gases were solved satisfactorily with straightforward techniques.

Analysis was performed using an optical-model potential which contained derivative surface absorption. The calculation was attempted from three separate viewpoints in an effort to isolate the effects of particular parameters on the results.

II. THE EXPERIMENTAL METHOD

The experimental arrangement used in these differential cross-section measurements is shown in Fig. 1. Neutrons were produced by means of the $T(d,n)He^4$ reaction using the 150-keV deuteron beam from a Cockcroft-Walton accelerator. The neutron producing targets were standard 2 Ci/in.^{2} tritium-loaded titanium foils on thin copper backing.

The associated-particle time-of-flight spectrometer used in this experiment has been described previously.[1] The principal differences between these measurements and those of Ref. 1 were in the nature of the scattering sample. The oxygen and argon measurements were made with liquid samples. For the oxygen measurements, a double-walled Dewar in the form of a right circular cycliner 7.0 cm high and 3.14 cm i.d. was constructed from 0.015-cm thick plastic. The spacing between the inner and outer walls was 0.022 cm. The double-walled container was then placed within a cylinder of 0.8-cm thick low-density Styrofoam which was part of a larger Styrofoam reservoir as shown in Fig. 2(a). This arrangement allowed the sample region to be filled as long as there was any liquid in the reservoir, and the volume intersected by the α-associated neutron beam was well within the inner cylinder. Any boiling which occurred due to heat leaks in the bottom or sides of the lower cylinder was confined to the small region between the double walls. Thus, no boiling took place inside the inner cylinder, so that the average density of the sample remained constant during the long counting periods.

Another Dewar of improved design [Fig. 2(b)] was used for the argon measurements. Liquid nitrogen was used to cool the liquid-argon cylinder from below. The Styrofoam insulation was replaced in favor of two dead-air spaces enclosed by very thin plastic film. Thus, scattering from the Dewar walls was greatly reduced.

The sample liquids were produced in advance by passing a regulated flow of gas from a high-pressure

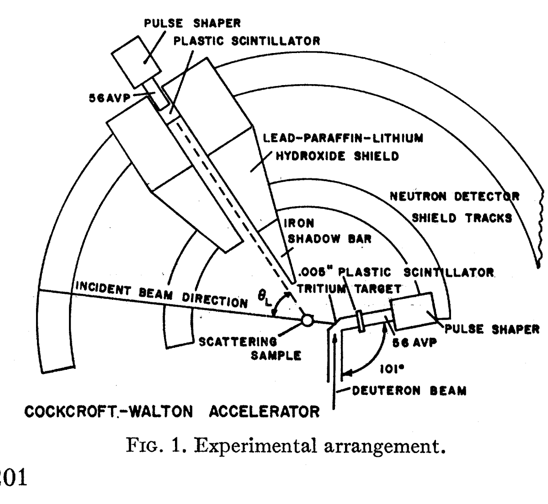

FIG. 1. Experimental arrangement.

* Work supported in part by the National Science Foundation.
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§ Supported in part by the U. S. Air Force Office of Scientific Research, Office of Aerospace Research, under AFOSR Grant No. 854-65.
[1] A. J. Frasca, Roger W. Finlay, R. D. Koshel, and R. L. Cassola, Phys. Rev. 144, 854 (1966).

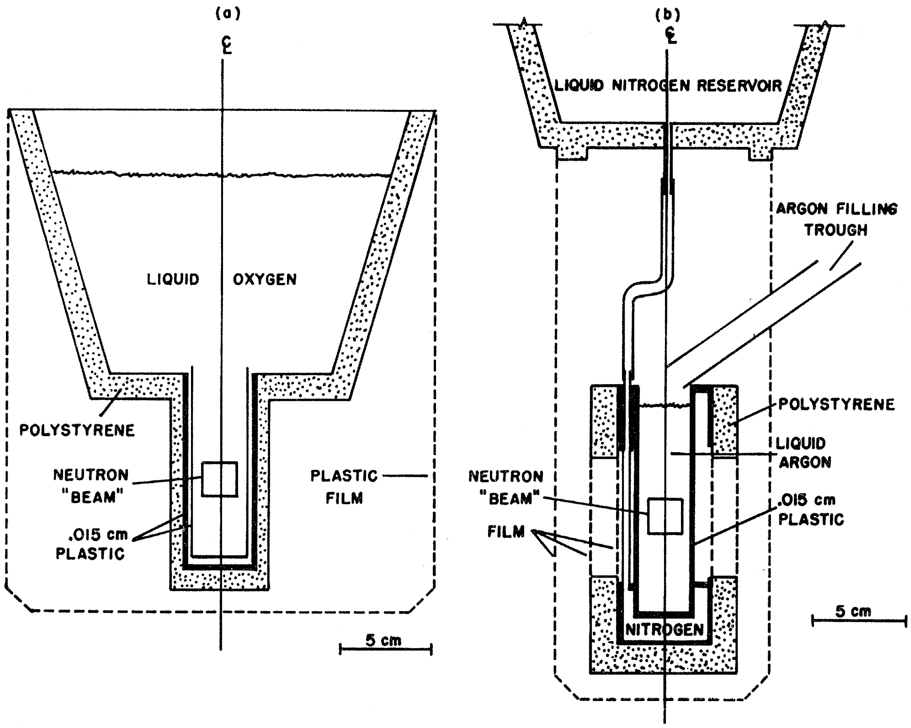

FIG. 2. Scattering-sample Dewars.

cylinder through a copper condensing coil which was maintained at liquid-nitrogen temperature.

The sensitive volume of the scattering material is given by the intersection of the sample Dewar and the solid angle of the neutrons which are emitted from the source in coincidence with a detected α particle. The maximum dimension of the sensitive volume was 3.14 cm, corresponding to a sample thickness of 0.11 mean free paths for oxygen and 0.14 mean free paths for argon.

Because the first excited state of O^{16} is higher in energy than the ground state by 6.05 MeV, energy resolution was not a serious problem in this case. The neutron spectrometer was modified to give better efficiency, albeit poorer resolution, by using a shorter flight path (1.24 m) and a thicker neutron scintillator (10.15 cm) than were quoted in Ref. 1. The actual time resolution was approximately 1.5 nsec, which was quite adequate to resolve neutrons scattered inelastically from oxygen. For the argon measurements, the flight path was increased to 1.80 m. This was the shortest flight path at which neutrons scattered inelastically from the 1.46-MeV state in Ar^{40} could be resolved from the elastic-scattering events.

A typical time-of-flight spectrum for neutrons scattered elastically from oxygen is shown in Fig. 3. A background spectrum corresponding to scattering from the empty sample Dewar is also shown. The Dewar background was not energy independent; in fact, small peaks due to scattering from carbon and hydrogen in the Dewar walls were discerned. Thus, it was necessary to obtain and subtract a Dewar-in, sample-out spectrum for each sample-in run.

The absolute efficiency of the spectrometer was determined by measuring the elastic scattering of 14-MeV neutrons from hydrogen and carbon. Polyethylene and carbon samples with the same dimensions as those of the liquid samples were used. The spectrum of neutrons scattered from hydrogen was obtained by subtracting the carbon contribution from the polyethylene spectrum at each of several forward angles. For the known n-p cross sections, the phase-shift analysis (YLAN4M, 1965) by Breit[2] was used. A separate efficiency determination was made for each angular distribution measured. In this way, changes in flight path, discriminator bias levels, etc., were experimentally accounted for.

Data were taken at 22 angles between 10 and 160 deg in the laboratory. Background spectra were taken at each angle. Because the neutron intensity is a sensitive function of angle, particularly in the forward direction, the location of the unscattered neutron beam was frequently checked so that the absolute angle was known to better than 0.4 deg at all times.

III. RESULTS

Experimental results are given numerically in Table I and shown graphically in Figs. 4 and 5. The errors quoted are due to counting statistics only. A second source of error would be the uncertainties in those cross

[2] G. Breit (private communication).

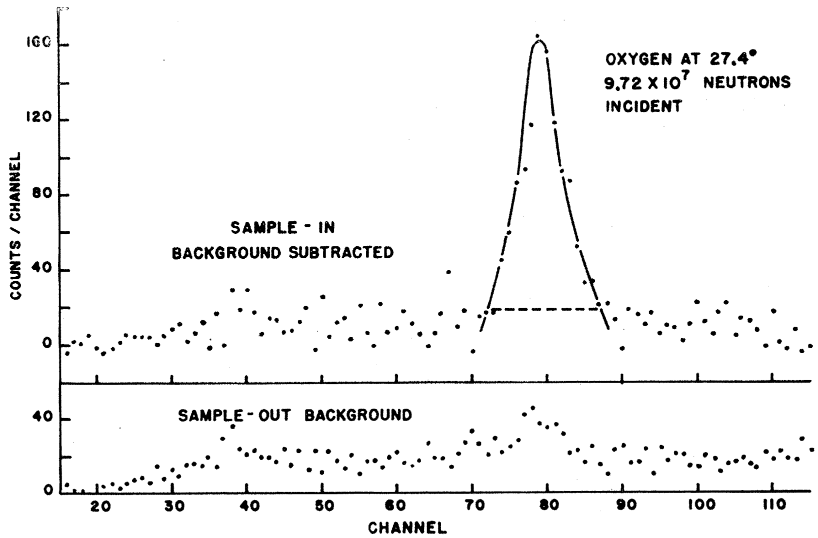

FIG. 3. Typical time spectrum of neutrons scattered from oxygen and the associated background spectrum.

sections which were taken as standards (4%). Finally, counting errors in the normalization measurements themselves amounted to 2 or 3%.

The data were corrected for the effects of finite angular resolution of the spectrometer, but this seldom exceeded 1% and can be considered insignificant within the stated errors of the experiment. By the same reasoning, multiple-scattering corrections were not attempted.

TABLE I. Measured differential cross sections for the scattering of 14-MeV neutrons from oxygen and argon.

	Oxygen			Argon	
$\theta_{c.m.}$ (deg)	$(d\sigma/d\Omega)_{c.m.}$ (mb/sr)	$\Delta(d\sigma/d\Omega)^a$ (percent)	$\theta_{c.m.}$ (deg)	$(d\sigma/d\Omega)_{c.m.}$ (mb/sr)	$(d\sigma/d\Omega)^a$ (percent)
13.17	663.6	5.75	10.25	1377.5	4.76
18.47	507.0	6.88	15.37	987.0	7.31
23.76	428.6	5.53	20.49	644.9	3.75
29.05	297.6	4.70	25.60	352.5	3.97
34.32	190.5	5.43	30.72	179.9	3.83
39.57	104.5	5.56	35.82	80.1	4.87
44.81	74.0	5.64	40.92	46.4	5.45
50.04	41.2	6.57	46.01	69.3	4.90
55.24	38.7	7.42	51.10	50.8	4.90
60.42	38.4	8.29	56.18	76.9	4.72
65.57	48.9	5.36	61.24	61.2	4.03
70.71	63.2	6.00	66.30	61.0	5.59
75.81	74.7	4.37	71.35	33.6	5.53
80.90	74.0	4.65	76.38	21.3	7.65
85.95	71.4	4.58	81.41	19.1	10.00
95.98	53.5	5.12	91.43	21.5	6.71
105.90	39.9	6.39	101.41	29.9	10.00
115.71	21.8	10.00	111.35	24.3	6.30
125.42	13.7	12.81	121.24	12.0	9.12
138.87	17.3	8.61	131.10	9.42	11.79
148.95	31.5	5.87	140.92	11.2	9.76
153.30	32.5	7.15	150.72	12.0	11.22
			159.12	13.4	13.50

[a] Includes the errors due to counting statistics only.

Elastic scattering from oxygen has been previously reported by Bauer *et al.*[3] and by McDonald *et al.*[4] There is a complete agreement among all three sets of data in the forward lobe of the angular distribution, with the present results being somewhat larger than the other two at angles greater than 50 deg.

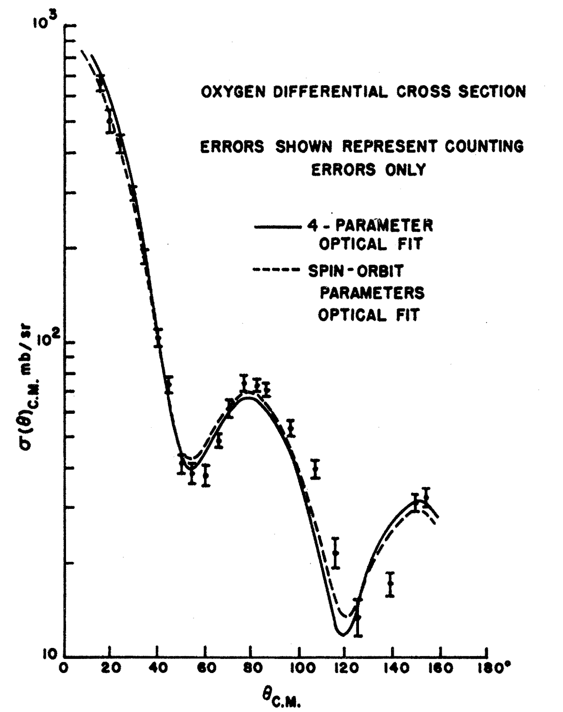

FIG. 4. Differential cross section for elastic scattering from oxygen. The solid curve is the four-parameter optical-model prediction and the dashed curve is the prediction obtained when the spin-orbit parameters were allowed to vary.

[3] R. W. Bauer, J. D. Anderson, and L. J. Christensen, Nucl. Phys. **47**, 241 (1963).

[4] W. J. McDonald, J. H. Robson, and R. Malcolm, Nucl. Phys. **75**, 353 (1966).

TABLE II. Parameters and cross sections obtained in the four-parameter optical-model analysis of the elastic scattering of 14-MeV neutrons from oxygen and argon.

Nucleus	V_C (MeV)	W_D (MeV)	r_D (F)	a_C (F)	χ^2	σ_{el} (mb)	σ_r (mb)	σ_{tot} (mb)
Oxygen	46.4	4.36	1.15	0.655	151	1003	525	1528
Argon	47.3	6.49	1.28	0.497	410	837	1007	1844

The differential cross section at zero deg was determined by making a reasonable extrapolation of a plot of $\ln\sigma(\theta)_{\text{lab}}$ versus $\cos\theta_{\text{lab}}$ for the forward lobe of the distribution.[3] The value obtained for oxygen by this method was 0.98 ± 0.10 b/sr, which may be compared with the value of 1.11 ± 0.10 b/sr quoted by Bauer. Wick's limit[5] may be calculated for this case using a known value of the total cross section for oxygen, $\sigma_{\text{tot}}=1.61\pm0.04$ b,[6] with the result $\sigma_{\text{el}}(0)=1.10\pm0.04$ b/sr. All three results are in very good agreement.

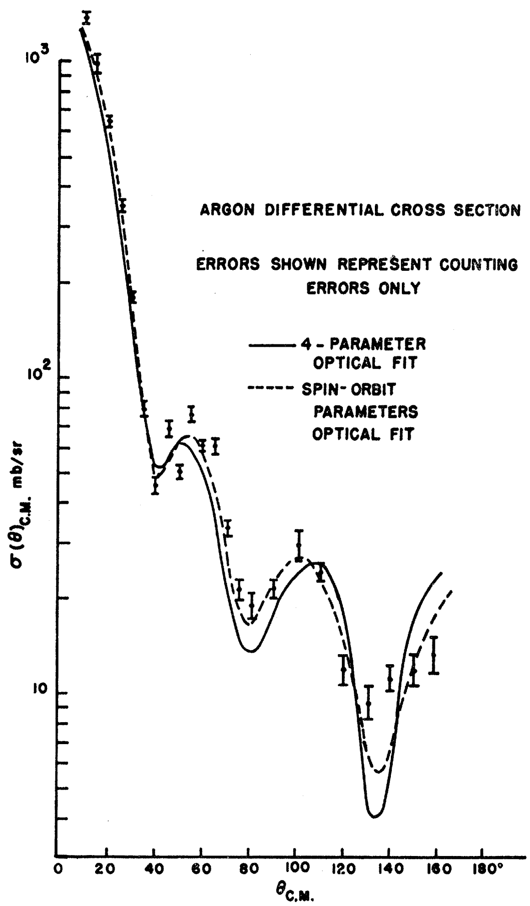

FIG. 5. Differential cross section for elastic scattering from argon. The solid curve is the four-parameter optical-model prediction and the dashed curve is the prediction obtained when the spin-orbit parameters were allowed to vary.

[5] G. C. Wick, Atti Reale Accad. Italia. Mem. Classe Sci. Fis. Mat. e Nat. **13**, 1203 (1943).

[6] J. H. Coon, E. R. Graves, and H. H. Barschall, Phys. Rev. **88**, 562 (1952).

The integrated elastic cross section for oxygen was found to be 1.05 ± 0.08 b (see Table II). Bauer obtained 0.88 ± 0.09 b, and McDonald originally quoted 1.07 ± 0.25 b and subsequently renormalized to 0.87 b. The present work is in better agreement with the earlier result of McDonald.

The differential cross section for the scattering of 14-MeV neutrons from argon has not been previously measured. However, a comparison of the cross section at zero deg can be made with the values for Wick's limit. Using a value of 2.01 ± 0.10 b for the total cross section of argon at 14-MeV,[7] Wick's limit is $\sigma_{\text{el}}(0)=1.74\pm0.17$ b/sr, which compares favorably with the value of $\sigma_{\text{el}}(0)=1.88\pm0.10$ b/sr extracted from the present data.

Finally, the integrated elastic scattering cross section for argon can be quoted as $\sigma_{\text{el}}=1.00\pm0.07$ b.

IV. DISCUSSION

The results obtained in the present experiment have been compared with optical-model calculations. The optical-model program CLOUDY[8] was used to perform the calculations. The complex potential used in the calculations contained a surface derivative form for the imaginary part and is given by

$$V(r)=-V_C f(r,r_C,a_C)+i4a_D W_D d[f(r,r_D,a_D)]/dr + (V_{\text{SO}}/r)(\hbar/m_\pi c)^2 d[f(r,r_{\text{SO}},a_{\text{SO}})]/dr\,\boldsymbol{\sigma}\cdot\mathbf{l}, \quad (1)$$

where

$$f(r,r_i,a_i)=\{1+\exp[(r-r_iA^{1/3})/a_i]\}^{-1}. \quad (2)$$

Here m_π is the mass of the pion and A is the mass number of the target nucleus.

In order to compare the optical calculations with the experimental results, the automatic search routine of the program CLOUDY was used. This routine minimizes the quantity χ^2 for each angular distribution. χ^2 is given by

$$\chi^2=\sum_{i=1}^{N}\left(\frac{\sigma^{\text{th}}(\theta_i)-\sigma^{\text{expt}}(\theta_i)}{\Delta\sigma^{\text{expt}}(\theta_i)}\right)^2, \quad (3)$$

where N is the number of experimental points, $\sigma^{\text{th}}(\theta_i)$ is the optical-model value of the differential cross section at the angle θ_i, $\sigma^{\text{expt}}(\theta_i)$ is the corresponding experimental value, and $\Delta\sigma^{\text{expt}}(\theta_i)$ is the experimental error at θ_i.

[7] F. J. Vaughn, W. L. Imhof, R. G. Johnson, and M. Walt, Phys. Rev. **118**, 683 (1960).

[8] For a description of the program CLOUDY, see R. D. Koshel and R. L. Cassola, Ohio University Report No. GP-1848, 1965, p. 27 (unpublished).

The potential given in Eq. (1) contains nine parameters. These are the three potential depths, V_C, W_D, and V_{SO}, the three radial parameters, r_C, r_D, and r_{SO}, and the three diffuseness parameters, a_C, a_D, and a_{SO}. It is usually possible to fit the data very well by allowing all these parameters to vary; however, one usually sacrifices quality in the fit by fixing some of the parameters and looks just at the behavior of the remaining free parameters. In this way one usually finds a much smoother trend in the behavior of the free parameters.[9]

Since the elastic-scattering distribution is usually not so sensitive to changes in the spin-orbit potential, the three parameters appearing in the spin-orbit part of the potential were initially held fixed at the values suggested by Perey.[9] These are $V_{SO}=7.5$ MeV, $r_{SO}=1.25$ F, and $a_{SO}=0.65$ F.

It is known that ambiguities of the form $V_C r_C^n=$ constant and $W_D a_D=$ constant[9] exist in some cases, so r_C and a_D were held fixed in the initial calculations at the values suggested by Perey. These are $r_C=1.25$ F and $a_D=0.47$ F.

Thus, in the first calculations the depth of the real central potential V_C, the depth of the imaginary part of the potential W_D, the diffuseness of the real central potential a_C, and the radius of the imaginary central potential r_D, were allowed to vary. The results of these initial calculations together with the experimental points are shown in Figs. 4 and 5. The solid line in each case is the minimum χ^2 fit of the four-parameter search to the experimental data. The parameters obtained along with the predicted elastic, reaction, and total cross sections are presented in Table II. The experimental values of the elastic, reaction, and total cross sections are given in Table III.

The fits found are good, the oxygen fit being the better of the two. The parameters found are reasonable and the agreement between the optical-model predictions for the elastic, reaction, and total cross sections given in Table II with the experimental values shown in Table III is quite good.

In order to see if the ambiguities in the products of the potential depths and the radial and diffuseness parameters were present, further calculations were made with the optical model. In addition to the four parameters which were previously varied, r_C and a_D were also varied. The parameters which gave the best fit in the χ^2 sense for this six-parameter search are shown in Table IV. The values found for the elastic, reaction, and total cross section are also shown in the same table. Graphs of the optical-model predictions for the six-parameter analysis are not given, because they are indistinguishable from those found in the four-parameter search.

TABLE III. Experimental elastic and total cross sections for scattering of 14-MeV neutrons from oxygen and argon.

Nucleus	σ_{el}^{expt}(b)	σ_{tot}^{expt}(b)
	1.05 ± 0.08[a]	1.61 ± 0.04[d]
Oxygen	0.88 ± 0.09[b]	1.56 ± 0.04[d]
	1.07 ± 0.25[c]	1.64 ± 0.04[e]
Argon	1.00 ± 0.07[a]	2.01 ± 0.10[f]

[a] Present work.
[b] Reference 3.
[c] Reference 4.
[d] Reference 6.
[e] H. L. Poss, E. O. Salant, G. A. Snow, and L. C. L. Yuan, Phys. Rev. 87, 11 (1952).
[f] Reference 4.

On comparing the parameters obtained for oxygen in the two cases, it is seen that V_C and a_C did not change significantly. The radial parameter r_C, found in the second search, is now 1.24 F instead of the fixed value 1.25 F. This is not a significant change. However, there were changes in the depth and the diffuseness of the imaginary potential. If the products of W_D and a_D are formed for the two cases, the following results are found. For the four-parameter search $W_D a_D=2.05$ MeV F. For the six-parameter search, $W_D a_D=1.89$ MeV F. While the two results are not exactly equal, they are similar enough to warrant the conclusion that the $W_D a_D$ ambiguity exists for oxygen. This change in W_D and a_D gave rise to a change in the elastic, reaction, and total cross sections. As can be seen on comparing the optical-model results for these cross sections found in the six-parameter search (Table IV) and the four-parameter search (Table II) with those found experimentally (Table III), the six-parameter results give a slightly poorer comparison with the experimental results.

For argon, the final values found in the six-parameter analysis for r_C and a_D, shown in columns 3 and 6, respectively, of Table IV, agree quite well with fixed values of the four-parameter analysis. The remaining four parameters obtained in the six-parameter analysis agreed very well with those found in the four-parameter analysis. The changes in χ^2, the elastic, reaction, and total cross sections were small. Thus, it would seem that the four-parameter model works well in argon.

For both oxygen and argon, the final value found for r_C in the six-parameter analysis was close to 1.25 F,

TABLE IV. Parameters and cross sections obtained in the six-parameter optical-model analysis of the elastic scattering of 14-MeV neutrons from oxygen and argon.

Nucleus	V_C (MeV)	W_D (MeV)	r_C (F)	r_D (F)	a_C (F)	a_D (F)	χ^2	σ_{el} (mb)	σ_r (mb)	σ_{tot} (mb)
Oxygen	46.8	5.50	1.24	1.17	0.651	0.344	125	992	492	1484
Argon	46.9	6.50	1.26	1.27	0.493	0.473	407	838	1008	1846

[9] F. G. Perey, Phys. Rev. 131, 745 (1963).

TABLE V. Parameters and cross sections obtained when only the spin-orbit parameters were varied. The remaining optical parameters were held fixed at the values given in Table IV.

Nucleus	V_{SO} (MeV)	r_{SO} (F)	a_{SO} (F)	χ^2	σ_{el} (mb)	σ_r (mb)	σ_{tot} (mb)
Oxygen	9.74	1.05	0.866	119	985	496	1481
Argon	12.2	0.636	2.03	166	919	957	1876

the value given by Perey. However, the value for a_D was found to be 0.473 F for argon and 0.344 F for oxygen. The value for argon agrees with 0.47 F, the Perey value. The value for oxygen differs greatly from the Perey value; however, since a W_Da_D ambiguity was discovered in the analysis, the value of 0.47 F is probably a very good one to use.

It was then decided to investigate the effect of the three parameters appearing in the spin-orbit potential on the calculated angular distributions. The six parameters appearing in the central and imaginary central potentials were fixed at the values found in the six-parameter analysis. The results are shown in Figs. 4 and 5. The dashed line corresponds to the minimum χ^2 fit to the experimental data. The spin-orbit parameters corresponding to the minimum χ^2 are shown in Table V. The optical-model predictions for the elastic, reaction, and total cross sections are also given in this table.

On comparing the two optical-model curves in Fig. 4, it can be seen that there is very little difference in the angular distributions. The χ^2 given in column four, Tables IV and V, are near each other. There was, however, a significant change in the spin-orbit parameters. No special significance should be placed in this result as it was found that the χ^2 was relatively flat for changes in the spin-orbit parameters. This would indicate that the spin-orbit potential is relatively unimportant for oxygen.

For argon the situation was not the same. On comparing the two curves given in Fig. 5, it can be seen that there is a noticeable difference in the two angular distributions, particularly at the minima. χ^2 was dramatically reduced, as can be seen on comparing the value found in column 4 of Table V with that found in column 7 of Table IV. The spin-orbit parameters differ greatly from those usually encountered in optical-model analysis. It is believed that the high initial value of χ^2 for the spin-orbit search, which was found in the six-parameter analysis, forced the search program to find a minimum in a region of the parameter space which is considered unphysical. Thus, not too much reliance should be placed in the values found for the spin-orbit parameters. It was also observed that the value of χ^2 did not appreciably change until the parameters took on "unreasonable" values. It may be then said that even though the value of χ^2 was greatly reduced, not too much faith can be placed in the parameters and it would be better to use the set of parameters obtained when the spin-orbit parameters were held at fixed at the values suggested by Perey.

The parameters found in the four-parameter analysis agree reasonably well with the average parameters found by Perey and Buck[10] and Rosen,[11] with the exception of the diffuseness parameter a_C of argon. The value found in this analysis is smaller. However, since the parameters with which the comparison is being made are average parameters, it is felt that the low value found for this parameter does not present an objection to the results of the analysis.

V. CONCLUSION

It has been found that a four-parameter optical model using a derivative form for the imaginary central potential very well describes the angular distributions of 14-MeV neutrons elastically scattered from oxygen and argon. The four parameters that were allowed to vary were the depths of the real and imaginary central potentials, the diffuseness of the real central potential, and the radius of the imaginary central potential. In addition, the model used gave very good values for the total cross sections for these two elements.

When an analysis was performed using a six-parameter optical model, it was found that there was little change in the parameters and fits obtained for argon; however, in oxygen there was a change in the depth and diffuseness of the imaginary central potential. This change was consistent with a W_Da_D ambiguity. The changes in the theoretical angular distributions were small, and the six-parameter model gave slightly poorer results for the total cross section. It was also found that changes in the spin-orbit parameters from those values usually used in optical-model analyses had little effect on the angular distributions unless one would allow unreasonable values for these parameters. Thus, it would seem that the four-parameter model used in this analysis could be used as a general model for the investigation of elastic scattering of neutrons; however, before this statement can be accepted, further investigations of different elements and at different energies must be performed.

[10] F. G. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962).

[11] L. Rosen, in *Proceedings of the Second International Symposium on Polarization Phenomena of Nucleons*, edited by P. Huber and H. Schopper (Birkhäuser Verlag, Basel, Germany, 1966), p. 253.