PHYSICAL REVIEW VOLUME 156, NUMBER 4 20 APRIL 1967

Shell-Model Spin-Orbit Force Radius Anomaly*

DONALD W. L. SPRUNG

AND

P. C. BHARGAVA†

Physics Department, McMaster University, Hamilton, Ontario, Canada

(Received 14 October 1966)

It has recently been shown that the spin-orbit part of the optical-model potential, taken to be a surface-gradient force, is displaced about $0.1A^{1/3}$ F inside the central well. For the shell-model potential this feature arises very naturally from the Brueckner many-body theory, the spin-orbit force being strongly density-dependent while the central force saturates at nuclear densities.

RECENTLY, improved experiments on the polarization in proton-nucleus scattering have allowed refinements in the fitting of optical-model potentials. Goldfarb, Greenlees, and Hooper,[1] in particular, have made careful fits employing wells of the usual type with Woods-Saxon central and surface-derivative $(1/r)df/dr$ spin-orbit potentials. Paying particular attention to forward angles, where the differential cross section is well understood, they show that a good fit to the polarization angular pattern can be obtained only by allowing the radius R_{LS} of the shape factor $f(r)$ for the spin-orbit force to be smaller (about $1.1A^{1/3}$ F) than the corresponding radius of the central well (about $1.2A^{1/2}$ F). A number of similar studies[2–4] have obtained values of R_{LS} between 1.06 and $1.1A^{1/3}$ F. The significant fact seems to be that this radius parameter is less than or just equal to the half-density radius $c\simeq 1.10$ F of the nuclear-density distribution as determined by electron-scattering experiments.[5] On the other hand, it is well known that the central well extends beyond the nuclear-density distribution, a fact usually ascribed to the finite range of the two-nucleon force but never calculated in a convincing manner.[6] For the related case of the shell-model potential, these opposite tendencies arise very naturally from the Brueckner theory.

We use the local-density approximation (LDA) of Brueckner, Gammel, and Weitzner,[7] according to which the effective two-body interaction inside a nucleus is the G matrix taken from nuclear matter at the local density. This approximation may involve about a 10% inaccuracy, but is believed adequate for the qualitative point at issue. Wong[8] has recently shown that the LDA can be made very accurate by using an effective density extending slightly beyond the actual density. We used the G matrix from our recent nuclear-matter calculations[9] for the Hamada-Johnston potential (which gave 11 MeV/A binding at a density $k_F=1.35$ F^{-1}) to construct the (nonlocal) spin-orbit amplitude $\langle r'/G_{LS}/r\rangle$. A method can be found in Ref. 7. In the $T=1$ state, we find that G_{LS} is very nearly equal to the free-nucleon potential $V_{LS}(r)$, except right at the edge of the hard core. This agrees with the procedure of Brueckner and Gammel.[10] The $T=1$ tensor force contributes negligibly to the spin-orbit interaction.

In our nuclear-matter calculation, we treated the states with $J\leqslant 2$ and the ${}^3D_3$-${}^3G_3$ system exactly, and used the Born approximation to the one-pion-exchange potential (OPEP) for the remainder. The spin-orbit amplitude then arises from the ${}^3P(T=1)$ and ${}^3D(T=0)$ waves. It would be somewhat arbitrary to assume that the higher waves contribute precisely zero, so for simplicity, we assumed separately for $T=0$ and $T=1$ that each multipole of the force is equal. This replaces the angular dependence P_L of the nonlocality by $\delta(\Omega-\Omega')\pm\delta(\Omega+\Omega')$, making the force "local in angle" but still nonlocal radially.

To deduce the one-body or shell-model spin-orbit force, we consider a closed-shell ± 1 nucleus. The method of Blin-Stoyle[11] was extended slightly to cover the case of a nonlocal two-body interaction, with the result

$$V_{SL}\boldsymbol{\sigma}\cdot\mathbf{l}=-K\frac{1}{r}\frac{d\rho}{dr}\boldsymbol{\sigma}\cdot\mathbf{l},$$

$$K=\tfrac{3}{4}K_1+\tfrac{1}{4}K_0+\left(\frac{N-Z}{4A}\right)(K_1-K_0), \qquad (1)$$

$$K_{T=1}=\frac{2\pi}{3}\int drdr'\,(rr')^3G_{LS}(r,r',T=1).$$

* Research supported by the National Research Council of Canada under Grant No. A3198.

† Present address: University of British Columbia, Vancouver, British Columbia, Canada.

[1] L. B. J. Goldfarb, G. W. Greenlees, and M. B. Hooper, Phys. Rev. **144**, 829 (1966).

[2] J. A. R. Griffith and S. Roman, Phys. Letters **19**, 410 (1965).

[3] D. A. Lind, D. E. Heagerty, and J. G. Kelly, Bull. Am. Phys. Soc. **10**, 104 (1965).

[4] L. N. Blumberg, R. H. Bassel, E. E. Gross, V. Van der Woude, and A. Zucker, Bull. Am. Phys. Soc. **10**, 103 (1965).

[5] L. R. B. Elton, *Nuclear Sizes* (Oxford University Press, New York, 1961), p. 31.

[6] R. E. Schentner and B. W. Downs, Phys. Rev. **129**, 2292 (1963).

[7] K. A. Brueckner, J. L. Gammel, and H. Weitzner, Phys. Rev. **110**, 431 (1958).

[8] C. W. Wong Nucl. Phys. **A91**, 399 (1967).

[9] D. W. L. Sprung, P. C. Bhargava, and T. Dalhblom, Phys. Letters **21**, 538 (1966); **23**, 438(E) (1966).

[10] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958).

[11] R. J. Blin-Stoyle, Phil. Mag. **46**, 973 (1955).

TABLE I. The coefficient K_T of $(1/r)d\rho/dr$ in the shell-model spin-orbit force, as a function of density. ρ is measured in F^{-3} and K_T is tabulated in units of F^{+3}. Multiplication by 41.469 gives K in MeV F^5.

k_F (F^{-1})	(ρ/ρ_0)	$K_{T=1}$	$K_{T=0}$	$\frac{3}{4}K_1+\frac{1}{4}K_0$	$\frac{1}{4}(K_1-K_0)$
1.43	1.16	4.131	−1.111	2.820	1.310
1.36	1.0	4.114	−1.292	2.762	1.352
1.23	0.74	4.093	−1.896	2.595	1.497
1.10	0.53	4.056	−2.600	2.391	1.664
0.90	0.29	3.962	−4.230	1.913	2.048
0.80	0.20	3.920	−5.454	1.576	2,344
0.70	0.14	3.871	−7.165	1.112	2.759

The angular locality of G_{LS} has been used. For a radially local two-body interaction $[\delta(r-r')/rr']$, we have twice his[11] result. In the actual work, the r and r' integrations were done analytically, and only one integration over a relative momentum k was done numerically, greatly improving the accuracy. The $T=1$ and $T=0$ coefficients are given as functions of density in Table I. The $T=0$ contribution arises largely from a two-body tensor force. It is seen to have the wrong sign and to be highly density-dependent, being small near normal nuclear density. This reflects the saturation property of the tensor force. We have remarked that K_1 arises almost entirely from the two-body spin-orbit potential, and is quite independent of density. The resulting K increases with density. Thus, in the local-density approximation, our one-body spin-orbit force is very roughly $-C\rho(1/r)d\rho/dr$, and can be seen in Fig. 1 to look very like the usual force but with a shift *inside* the density distribution $\rho(r)$. The strength is equivalent to about 20 times the "Thomas term." Using oscillator single-particle wave functions with oscillator parameters close to $\hbar\omega=41A^{-1/3}$ MeV, experimental values of spin-orbit splittings are well fitted[12] from O^{16} to Pb^{208}. This agrees with the more exact calculation of Kuo and Brown[13] for O^{16}, but our method, being simpler, can be applied to more cases.

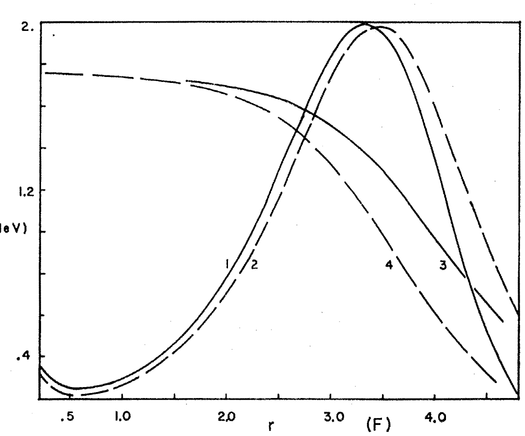

FIG. 1. The one-body (shell-model) spin-orbit force calculated as described in the text is the solid line marked 1. For comparison, the dashed line 2 is a constant times $(1/r)d\rho/dr$. Line 4 is the Fermi density distribution ρ appropriate to Ca^{40}, while 3 is the local value of the central potential taken from nuclear matter at the local density. Note the shift in the opposite direction to the spin-orbit force. The vertical scale refers only to lines 1 and 2.

Also shown in Fig. 1 are the Fermi density distribution for Ca^{40} and the central potential in the LDA. The latter extends well beyond the density, because of the saturation property of the central field (making it vary slowly with density). Such a simple approximation is not reasonable for the central field, but it does show that the tendency is just the reverse to the spin-orbit case.

Greenlees, Pyle, and Tang[14] proposed a different explanation of the effect, based on the premise that the range of the two-body spin-orbit potential is much less than the range of the two-body central force. However, the intermediate-range part of the central force (the part effective in smearing the optical potential beyond the nuclear density) is actually of range about $\frac{1}{3}\hbar/m_\pi C$, which is the same as the range of the spin-orbit potential in the Hamada-Johnston force used here or of other modern potentials. The new effect suggested here operates independently of these force ranges. Furthermore, it seems doubtful that the high-energy approximation referred to by the above authors can be relied upon down to energies as low as 10 MeV.

In conclusion, the one-body spin-orbit force of the shell model can be represented to good approximation as a surface-gradient force, at or just inside the nuclear-*density* distribution. The shift inside by about $0.05A^{1/3}$ F is due to the strong density dependence of the $T=0$ contribution. For the central force, the tendency is to shift the potential beyond the density distribution because of the saturating character of the central field. The one-body spin-orbit force calculated here gives good agreement with experimental single-particle level splittings over a wide range of nuclei.

The density distribution in question might possibly be the "effective" local density of Wong,[8] which extends slightly beyond the nuclear charge density by about $0.09A^{1/3}$. In either case, however, our work provides a mechanism for the spin-orbit radius R_{LS} to be smaller than the central-well radius, since they shift in opposite directions from the nuclear density. From Fig. 1 it is seen that the diffuseness parameter a_{LS} is about the same as for the density, while the local-density central well has greater diffuseness. These points are in qualitative agreement with a recent optical-potential fit to 30.3-MeV proton scattering and polarization.[15]

[12] P. C. Bhargava, thesis, McMaster University, 1966 (unpublished).
[13] T. T. S. Kuo and G. E. Brown, Nucl. Phys. **85**, 40 (1966).
[14] G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. Letters **17**, 33 (1966).
[15] G. R. Satchler, Nucl. Phys. (to be published).

Note added in proof. The single particle level splittings referred to above[12] were inadvertently multiplied by two before comparison with experiment. This disagreement is incidental to the main point of the paper, but it does weaken the conclusion pending a satisfactory calculation of the spin-orbit force. We thank H. A. Bethe and C. W. Wong for correspondence concerning this difficulty.

PHYSICAL REVIEW VOLUME 156, NUMBER 4 20 APRIL 1967

$^{13}C(\alpha,n)^{16}O$ Reaction Cross Section between 1.95 and 5.57 MeV

K. K. SEKHARAN, A. S. DIVATIA, M. K. MEHTA, S. S. KEREKATTE, AND K. B. NAMBIAR
Van de Graaff Laboratory, Atomic Energy Establishment, Trombay, Bombay, India
(Received 14 November 1966; revised manuscript received 28 December 1966)

The $^{13}C(\alpha,n)^{16}O$ total cross section has been determined for the incident α-particle energy range 1.95 to 5.57 MeV using a 4π neutron detector. The total cross section of the inverse reaction $^{16}O(n,\alpha)^{13}C$ has been calculated by applying the reciprocity theorem. The total level widths for 16 levels, and the partial widths Γ_α and Γ_n along with the reduced widths γ_α^2 and γ_n^2 for the levels corresponding to the 2.68, 2.81, 3.72, and 4.62-MeV resonances, have been determined.

I. INTRODUCTION

THE measurement of the $^{13}C(\alpha,n)^{16}O$ reaction cross section is important for studying the level structure in the compound nucleus ^{17}O. The cross section in the range $E_\alpha=1.95$ to 5.57 MeV gives information about the levels in the excitation energy range of about 7.8 to 10.6 MeV. The elastic sacttering of α particles by ^{13}C, neutrons by ^{16}O, and the radiative capture of α particles by ^{13}C are other possible reactions leading to this range of excitation energies in ^{17}O. No direct measurements of the total $^{13}C(\alpha,n)^{16}O$ reaction cross sections are available, whereas there are many previous measurements of the differential cross sections of this reaction. Becker and Barschall[1] and Walton *et al.*[2] have measured the differential cross sections for the incident α-particle energy range 2–3.5 MeV and Bonner *et al.*[3] for the energy range 2–5 MeV. Barnes *et al.*[4] have measured the $^{13}C(\alpha,\alpha)^{13}C$ differential cross section from $E_\alpha=2$ to 3.5 MeV. A useful aspect of the total-cross-section measurement of $^{13}C(\alpha,n)^{16}O$ reaction is the calculation of the total cross section for the inverse reaction, $^{16}O(n,\alpha)^{13}C$, using reciprocity. The $^{16}O(n,\alpha)^{13}C$ reaction cross sections are useful for reactor calculations. Direct measurements of this cross section have been made by Seitz and Huber,[5] Davis *et al.*,[6] and Lister and Sayres.[7] From the $^{13}C(\alpha,n)^{16}O$ differential cross-section values, Walton *et al.*[2] have obtained the total (α,n) cross section for this reaction up to 3.5 MeV. A direct measurement of this reaction cross section has been done at Trombay for the incident α-particle energy range 1.95–5.57 MeV using a 4π geometry neutron detector.

II. EXPERIMENTAL METHOD

Singly ionized helium ions from the 5.5-MeV HVEC Van de Graaff Accelerator at Trombay[8] were used to bombard an electromagnetically enriched (enrichment $\approx 30\%$) ^{13}C target[9] deposited on a 0.25-mm-thick tantalum backing. The neutrons were detected by a calibrated 4π detector built according to the design of Marion *et al.*,[10] the ^{13}C target being mounted at the center of the counter. The counter consists of an inner and outer set of BF_3 counters embedded in a block of paraffin. The efficiency of this counter, as a function of the neutron energy, was obtained from the measurement of the neutron yields from the $^7Li(p,n)^7Be$ reaction.[11] The cross section of this reaction has been determined by Gibbons and Macklin.[12] The efficiency curve was extended up to 5-MeV neutron energy by using a 50-millicurie Ra-α-Be neutron source. The efficiency curves are shown in Fig. 1.

The number of ^{13}C nuclei per square centimeter present in the carbon target was determined by measuring the neutron yield at 3.5 and 4.1 MeV from the $^{13}C(p,n)^{13}N$ reaction using the same neutron counter and comparing the results with the previous cross-section measurements of Gibbons and Macklin.[12] From the ex-

[1] R. L. Becker and H. H. Barschall, Phys. Rev. **102**, 1384 (1956).
[2] R. B. Walton, J. D. Clement, and F. Boreli, Phys. Rev. **107**, 1065 (1957).
[3] T. W. Bonner, Alfred A. Kraus, Jr., J. B. Marion, and J. P. Schiffer, Phys. Rev. **102**, 1348 (1956).
[4] B. K. Barnes, T. A. Belote, and J. R. Risser, Phys. Rev. **140**, B616 (1965).
[5] J. Seitz and P. Huber, Helv. Phys. Acta **28**, 227 (1955).
[6] E. A. Davis, T. W. Bonner, D. W. Worley, Jr., and R. Base, Nucl. Phys., **48**, 169 (1963).
[7] D. Lister and A. Sayres, Phys. Rev. **143**, 745 (1966).
[8] A. S. Divatia *et al.*, Atomic Energy Establishment Trombay Report No. AEET/NP/5, 1962 (unpublished).
[9] Supplied by the Electromagnetic Separation Group, Atomic Energy Research Establishment, Harwell, England.
[10] J. B. Marion, R. J. A. Levesque, C. A. Ludemann, and R. W. Detenbeck, Nucl. Instr. Methods **8**, 297 (1960).
[11] K. K. Sekharan, M.Sc. thesis, University of Bombay, 1965 (unpublished).
[12] J. H. Gibbons and R. L. Macklin, Phys. Rev. **114**, 571 (1959).