Toward a New Theory of Spherical Nuclei. I*†

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(Received 9 November 1966)

A tentative theory of the quadrupole moment Q_{2^+} of the first 2^+ excited state of spherical nuclei is developed as a logical extension of the methods used in the usual theory based on pairing plus quadrupole-quadrupole interaction. In the latter, in which the quasiparticle occupation amplitudes in the ground state are taken to be of zero order and the off-diagonal amplitudes connecting the ground state with the 2^+ state to be of first order (and described by the random-phase approximation), Q_{2^+} is nominally of second order. We find that, in a consistent calculation, the quadrupole deformation is driven by the off-diagonal amplitudes, but that there is also the possibility of a self-sustained deformation for sufficiently large quadrupole coupling constants. Numerically, one finds ranges of the latter, all in accord with the excitation energy and the $E2$ transition probability, for which Q_{2^+} shows extremely rapid variations, and in particular, also assumes values sufficiently large to contradict the whole basis of the calculation. The need for a self-consistent intermediate coupling calculation is indicated.

I. INTRODUCTION

RECENT measurements of the quadrupole moment of the first excited 2^+ state[1] of Cd^{114} and of other so-called spherical nuclei[2] have yielded rather large values compared to *a priori* expectations. Qualitatively, the results can be summarized by the statement that the static quadrupole moment is comparable to the transition quadrupole moment to the ground state. This is certainly incompatible with the traditional semiclassical picture[3] of the 2^+ state as a quadrupole surface vibration about an equilibrium spherical shape. It appears also to be inexplicable by existing microscopic theories,[4] in particular by the usual mixture of BCS theory plus random-phase approximation (RPA).[5] In this and in the following paper, we begin the quest for a revised theory. The aim of the present work is modest: It is to show that with appropriate methods of investigation,[6] one need go only one step beyond existing ideas to encounter mechanisms capable of producing a permanent deformation of the 2^+ states.

The mechanism studied in this paper may be referred to loosely as the self-consistent blocking effect. Here we mean by blocking any physical effect, which will serve to distinguish two states whose essential properties are otherwise identified in some well-defined zero-order approximation. Thus, the usual theory assumes that in zero order the 2^+ state can be characterized by the same average (spherical) density and the same energy gap as the ground state. In first order, we encounter the off-diagonal density matrix and gap function, which are determined by the RPA in an approximation which explicitly incorporates the zero-order assumptions. Our purpose is to investigate the leading nonvanishing corrections to the density matrix of the 2^+ state which distinguish it from the ground state; these first occur in second order.

The core of this work is Sec. V, where this investigation is carried out. In fact, we compute the reduced quadrupole moment directly. It is ostensibly of second order in that it is driven by terms quadratic in the (first-order) off-diagonal density. However, because we are studying perturbations of basically nonlinear equations, we find that the quadrupole moment is in part induced self-consistently and this accounts largely for the rather striking results reported in Sec. VI.

In the next section, we essentially continue and complete the Introduction, though in a more technical vein. The following two sections contain brief treatments of the ground state and of the RPA, before we turn to the subject of primary interest.

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* Work supported in part by the U. S. Atomic Energy Commission.

† A brief account of the results of this and of the following paper has already appeared: G. Do Dang, R. Dreizler, A. Klein, and C. S. Wu, Phys. Rev. Letters **17**, 709 (1966).

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[1] J. DeBoer, R. G. Stokstad, G. D. Symons, and A. Winther, Phys. Rev. Letters **14**, 564 (1965); P. H. Stelson, W. T. Milner, J. L. C. Ford, Jr., F. K. McGowan, and R. L. Robinson, Bull. Am. Phys. Soc. **10**, 427 (1965).

[2] Coulomb Excitation Conference, Copenhagen, 1966 (unpublished). The authors are indebted to R. A. Sorensen for informing them of the results of this conference and to P. H. Stelson for a private communication of unpublished results.

[3] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selbskab, Mat. Fys. Medd. **27**, No. 16 (1953); G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1965).

[4] For a survey of the results of these see T. Tamura and T. Udagawa, Phys. Rev. Letters **15**, 765 (1965).

[5] L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35**, 853 (1963). Referred to as K.S.

[6] A. K. Kerman and A. Klein, Phys. Rev. **132**, 1326 (1963); G. Do Dang and A. Klein, *ibid.* **147**, 689 (1966).

II. FUNDAMENTALS AND OUTLINE OF PROCEDURE

We study the Hamiltonian of K.S.,[5]

$$H=\sum_{\alpha i} h_{ai}a_{\alpha i}{}^\dagger a_{\alpha i}-\tfrac{1}{4}\sum_{i\alpha\beta} G_i(s_\alpha a_{\alpha i}{}^\dagger a_{-\alpha i}{}^\dagger)(s_\beta a_{-\beta i}a_{\beta i})$$
$$-\tfrac{1}{2}\sum_{qi} X_i Q_q{}^i Q_q{}^{i\dagger}-\tfrac{1}{2}X_{np}\sum_q (Q_q{}^p Q_q{}^{n\dagger}+Q_q{}^n Q_q{}^{p\dagger}),\quad (2.1)$$

where $i=p$, n for proton and neutron, respectively, $\alpha=(nlj_am_\alpha)$, $s_\alpha=(-1)^{j_a-m_\alpha}$, $\bar{\alpha}=(nl,j_a,\bar{m}_a=-m_\alpha)$, and the quadrupole operator

$$Q_q{}^{i\dagger}=\sum_{\alpha\beta} F_i(ab)s_\beta(abm_\alpha\bar{m}_\beta|2q)a_{\alpha i}{}^\dagger a_{\beta i}.\quad (2.2)$$

For typographical reasons, subscripts $\bar{\alpha}$, $\bar{\beta}$, $\cdots$ are printed as $-\alpha$, $-\beta$, $\cdots$.

For harmonic-oscillator shell-model wave functions, $F_i(ab)$ is defined by the expressions

$$(\alpha i|r^2Y_{2q}|\beta i)\equiv s_\beta(abm_\alpha\bar{m}_\beta|2q)(ai\|r^2Y_2|bi),$$
$$(ai\|r^2Y_2\|bi)\equiv \nu_i(5/4\pi)^{1/2}F_i(ab),\quad (2.3)$$
$$\nu_i=[N_i+\tfrac{3}{2}](\hbar/M\omega_0),$$

where ω_0 is the harmonic-oscillator frequency, $\hbar\omega_0\cong 41/A^{1/3}$, M is the nucleon mass, and N is the principal quantum number of the shell considered.

The following equations of motion are easily obtained:

$$[a_{\alpha i},H]=(h_{ai}+F_{ai})a_{\alpha i}-\tfrac{1}{2}G_i s_\alpha a_{-\alpha i}{}^\dagger\sum_\beta s_\beta a_{-\beta i}a_{\beta i}$$
$$-X_i\sum_{q\beta}F_i(ab)s_\beta(abm_\alpha\bar{m}_\beta|2q)a_{\beta i}\Lambda_q{}^i,\quad (2.4)$$

$$[a_{-\alpha i}{}^\dagger,H]=-(h_{ai}-F_{ai}-G_i)a_{-\alpha i}{}^\dagger-\tfrac{1}{2}G_i s_\alpha a_{\alpha i}\sum_\beta a_{\beta i}{}^\dagger a_{-\beta i}{}^\dagger$$
$$+X_i\sum_{q\beta}F_i(ba)(-)^q s_\beta(abm_\alpha\bar{m}_\beta|2q)a_{-\beta i}{}^\dagger\Lambda_q{}^i,\quad (2.5)$$

where

$$F_{ai}\equiv\tfrac{1}{2}X_i\sum_\beta 5F_i{}^2(ab)/2\Omega_a,\quad (2.6)$$
$$2\Omega_a=2j_a+1,$$

$$\Lambda_q{}^i\equiv Q_q{}^i+X_{np}{}^iQ_q{}^{i'},\quad i\neq i',\quad (2.7)$$
$$X_{np}{}^i=X_{np}/X_i.$$

Our major endeavor is to extract information from selected matrix elements of Eqs. (2.4) and (2.5) and from associated normalization and consistency conditions. These matrix elements, of which there will eventually be a fair number, are all associated in some well-defined manner either with the ground state $|0\rangle$ of a given even nucleus, or with the first 2^+ excited state $|2,q\rangle$, where q is the magnetic quantum number. It may clarify matters if by way of further introduction we collect and define the relevant amplitudes used in this paper, as well as state briefly their role in the calculation. A more general discussion is to be found in the following paper.

In beginning our study of the ground state, we suppose as in the usual theory that the dominant single-particle amplitudes are those which carry us to the single quasiparticle states $|\alpha_i\rangle$,

$$\langle\bar{\alpha}i|a_{\alpha i}|0\rangle=s_\alpha v_{ai},\qquad \langle\bar{\alpha}i|a_{-\alpha i}{}^\dagger|0\rangle=u_{ai}.\quad (2.8)$$

By failing to distinguish between two adjacent even nuclei in (2.8), we are of course, making the BCS approximation. In what follows, it is natural to consider these quantities to be of zero order. The subsequent examination of corrections to the lowest-order equations brings in two additional sets of amplitudes. Explicitly we encounter the quantities $[\theta(abc\cdots)=(-1)^{j_a+j_b+j_c+\cdots}]$

$$\langle\bar{\alpha}i|a_{\beta i}|I\bar{q}\rangle=\theta(ab)(abm_\alpha\bar{m}_\beta|2q)v_{ab}{}^i(I),$$
$$\langle\bar{\alpha}i|a_{-\beta i}{}^\dagger|I\bar{q}\rangle=\theta(ab)s_\beta(abm_\alpha\bar{m}_\beta|2q)u_{ab},(I).\quad (2.9)$$

The first amplitude, for instance, describes the possibility that if we start with an excited 2^+ state (I refers henceforth to any one of these, not necessarily to the lowest one), we thereby deexcite the core and end in one of the quasiparticle (hole) states. With these as with subsequent amplitudes, the rules of angular-momentum coupling must be enforced.

Relative to the amplitudes of (2.8), we suppose throughout this paper that the quantities (2.9) are of first order. Equally of first order are the amplitudes

$$\langle(Ib)\bar{\alpha}_i|a_{\alpha i}|0\rangle=s_\alpha(5/2\Omega_a)^{1/2}v_a{}^i(Ib),$$
$$\langle(Ib)\bar{\alpha}_i|a_{-\alpha i}{}^\dagger|0\rangle=(5/2\Omega_a)^{1/2}u_a{}^i(Ib).\quad (2.10)$$

In this case we start with the ground state and end up with a state $|(Ib)\bar{\alpha}_i\rangle$ which we interpret as arising to a good approximation from core-hole or core-particle coupling. When we study the equations for the two sets (2.9) and (2.10), we discover that in a linearized approximation they yield a version of the RPA. We subsequently find that the quadrupole transition amplitude is linear in these quantities, whereas corrections to the BCS amplitudes (2.8) are quadratic.

Finally, to investigate the average quadrupole moment in one of the states $|Iq\rangle$ (the major aim of this work), we study the equations for the "diagonal" matrix elements,

$$\langle(Ia)\beta_i|a_{\alpha i}|Iq\rangle\equiv(2aq\bar{m}_\alpha|bm_\beta)s_\alpha v_{aI}{}^i(b),$$
$$\langle(Ia)\beta_i|a_{-\alpha i}{}^\dagger|Iq\rangle\equiv(2aq\bar{m}_\alpha|bm_\beta)u_{aI}{}^i(b).\quad (2.11)$$

These should be the dominant core-hole, core-particle matrix elements according to the usual theory. Indeed, if we temporarily discount the existence of static quadrupole deformations in the states $|Iq\rangle$, then in zero order $v_{aI}{}^i$, $u_{aI},(b)$ are independent of b and equal to the ground-state amplitudes v_{ai}, u_{ai} of Eq. (2.8), respectively. As in the case of the latter, the corrections are ostensibly of second order in the small amplitudes (2.9) and (2.10). Though many of the corrections are the same as those operative for the ground state, there remain terms for which the different angular-momentum coupling and different transitions available from the 2^+ states make their weight felt and a nonvanishing static quadrupole moment can be induced, not only, it appears, as the direct second-order effect of the fluctuating quadrupole moment associated with the RPA, but also as the result of a self-consistency requirement which makes the induced quadrupole moment, in fact proportional to itself.

A closer inspection of the expansion for the quadrupole moment reveals, moreover, the intervention of one further set of amplitudes, namely,

$$\begin{aligned}\langle (Ib)\bar{\gamma}_i|a_{\alpha i}|Iq\rangle &\equiv s_\alpha(2aq\bar{m}_\alpha|c\bar{m}_\gamma)u_a{}^i(Ib,c)\,,\\ \langle (Ib)\bar{\gamma}_i|a_{-\alpha i}{}^\dagger|Iq\rangle &\equiv (2aq\bar{m}_\alpha|c\bar{m}_\gamma)u_a{}^i(Ib,c)\,,\end{aligned}\tag{2.12}$$

where $a\neq b$. Thus we recognize the possibility that a given state $|(Ib)\bar{\gamma}_i\rangle$, though mainly of a character indicated by the notation, may nevertheless be partly composed by coupling to the core state $|I\rangle$ a particle in a subshell other than b. The equations of motion for the amplitudes (2.12) indicate that these are of second order and that they, too, are in fact driven by the quadrupole amplitudes themselves. Altogether, we obtain finally a pair of linear equations for the neutron and proton reduced quadrupole matrix elements. This is a direct consequence of the fact that our starting equations were nonlinear. As we shall see, this central formula of our paper yields numerical results strikingly different from that associated with the RPA itself.

III. THE GROUND-STATE PROBLEM (PAIRING)

Taking a matrix element of (2.4), we have (the W are total energies)

$$\begin{aligned}&[W_0(A)-W_{ai}(A-1)-h_{ai}-F_{ai}]\langle\bar{\alpha}i|a_{\alpha i}|0\rangle\\ &\quad=-\tfrac{1}{2}G_i\sum_\beta s_\alpha s_\beta\langle\bar{\alpha}i|a_{-\alpha i}{}^\dagger a_{-\beta i}a_{\beta i}|0\rangle\\ &\qquad-X_i\sum_{q\beta}F_i(ab)s_\beta(abm_\alpha\bar{m}_\beta|2q)\langle\bar{\alpha}i|a_{\beta i}\Lambda_q{}^i|0\rangle.\end{aligned}\tag{3.1}$$

To evaluate the first term on the right-hand side, we note that the operator $\hat{\Delta}_i=\sum_\alpha s_\alpha a_{-\alpha i}a_{\alpha i}$ carries zero angular momentum and thus connects only states with the same spin. Neglecting seniority-zero excited states, we therefore get

$$\begin{aligned}\langle\bar{\alpha}i|a_{-\alpha i}{}^\dagger\hat{\Delta}_i|0\rangle&\cong\langle\alpha_i|a_{-\alpha i}{}^\dagger|0\rangle\langle 0|\hat{\Delta}_i|0\rangle\langle|\hat{\Lambda}_i|0\rangle\\ &=\Delta_i\langle\bar{\alpha}i|a_{-\alpha i}{}^\dagger|0\rangle.\end{aligned}\tag{3.2}$$

On the other hand, since $\Lambda_q{}^i$ carries angular momentum 2, we find for the second term

$$\langle\bar{\alpha}_i|a_{\beta i}\Lambda_q{}^i|0\rangle=\sum_I\langle\bar{\alpha}_i|\alpha_{\beta i}|I\bar{q}\rangle\Lambda_I{}^i(-1)^q\,,\tag{3.3}$$

where the sum is over states of angular momentum 2 and

$$\Lambda_I{}^i=\langle Iq|\Lambda_q{}^{i\dagger}|0\rangle.\tag{3.4}$$

Utilizing the definitions (2.8) and (2.9), we consequently find for (3.1), and by a similar procedure for the matrix element of (2.5), the following equations:

$$\begin{aligned}[E_{ai}+\epsilon_{ai}]v_{ai}=\Delta_i u_{ai}-F_{ai}v_{ai}&\\ -X_i\sum_{Ib}[5F_i(ab)/2\Omega_a]v_{ab}{}^i(I)\Lambda_I{}^i,&\end{aligned}\tag{3.5}$$

$$\begin{aligned}[E_{ai}-\epsilon_{ai}]u_{ai}=\Delta_i v_{ai}-F_{ai}u_{ai}&\\ +X_i\sum_{Ib}[5F_i(ab)/2\Omega_a]u_{ab}{}^i(I)\Lambda_I{}^i,&\end{aligned}\tag{3.6}$$

wherein E_{ai} and ϵ_{ai} are defined as usual,

$$\begin{aligned}E_{ai}=W_{ai}(A-1)-\tfrac{1}{2}[W_0(A)+W_0(A-2)]+\tfrac{1}{2}G,&\\ \epsilon_{ai}=h_{ai}-\lambda_1,\quad 2\lambda_1=2\lambda+G,&\\ 2\lambda=W_0(A)-W_0(A-2).&\end{aligned}\tag{3.7}$$

In what follows, we treat the quadrupole coupling as a perturbation. For $X_i=0$, (3.5) and (3.6) reduce to the usual BCS equations,

$$\begin{aligned}[E_{ai}{}^{(0)}+\epsilon_{ai}{}^{(0)}]v_{ai}{}^{(0)}&=\Delta_i{}^{(0)}u_{ai}{}^{(0)}\,,\\ [E_{ai}{}^{(0)}-\epsilon_{ai}{}^{(0)}]u_{ai}{}^{(0)}&=\Delta_i{}^{(0)}v_{ai}{}^{(0)}.\end{aligned}\tag{3.8}$$

Subject to the normalization condition,

$$u_{ai}{}^{(0)2}+v_{ai}{}^{(0)2}=1\,,\tag{3.9}$$

we can deduce the well-known consequences.[5] What is essential for the further development is that if the physical solution of (3.8) (positive E_{ai}) is written

$$\psi_{ai}{}^{(0)}=\begin{pmatrix}v_{ai}{}^{(0)}\\ u_{ai}{}^{(0)}\end{pmatrix},\tag{3.10}$$

then the other solution has energy $-E_{ai}$ and a wave function

$$i\tau_2\psi_{ai}{}^{(0)}=\begin{pmatrix}u_{ai}{}^{(0)}\\ -v_{ai}{}^{(0)}\end{pmatrix}\tag{3.11}$$

orthogonal to (3.10).

As the prototype of the somewhat more elaborate considerations necessary for the excited states, we shall study briefly the leading corrections to (3.8)–(3.10). To the required order, we write

$$\psi_{ai}=\begin{pmatrix}v_{ai}\\ u_{ai}\end{pmatrix}=(1+\tfrac{1}{2}\eta_{ai})\psi_{ai}{}^{(0)}+c_{ai}(i\tau_2\psi_{ai}{}^{(0)}).\tag{3.12}$$

The normalization constant η_{ai} is calculated from the anticommutator

$$\langle 0|[a_{\alpha i},a_{\alpha i}{}^\dagger]_+|0\rangle=1.\tag{3.13}$$

For example,

$$\langle 0|a_{\alpha i}{}^{\dagger}a_{\alpha i}|0\rangle = \langle 0|a_{\alpha i}{}^{\dagger}|\bar{\alpha}_i\rangle\langle\bar{\alpha}_i|a_{\alpha i}|0\rangle + \sum_{Ib}\langle 0|a_{\alpha i}{}^{\dagger}|(Ib)\bar{\alpha}_i\rangle\langle(Ib)\bar{\alpha}_i|a_{\alpha i}|0\rangle. \quad (3.14)$$

Recognizing the amplitudes (2.10), we find from (3.14) and from the commuted product that (3.13) becomes [with an obvious definition—compare (3.10)]

$$\tilde{\psi}_{ai}\psi_{ai}+\frac{5}{2\Omega_a}\sum_{Ib}\tilde{\psi}_a{}^i(Ib)\psi_a{}^i(Ib)=1. \quad (3.15)$$

Substituting (3.12) and keeping only linear terms in η_{ai} or c_{ai} (there are none of the latter here), we derive

$$\eta_{ai}=-\sum_{Ib}\left(\frac{5}{2\Omega_a}\right)\tilde{\psi}_a{}^i(Ib)\psi_a{}^i(Ib). \quad (3.16)$$

To find c_{ai} and, incidentally, δE_{ai}, where

$$\delta E_{ai}=E_{ai}-E_{ai}{}^{(0)}, \quad (3.17)$$

we write (3.5) and (3.6) in the condensed form, using Pauli matrices,

$$E_{ai}\psi_{ai}=(-\epsilon_{ai}{}^{(0)}\tau_3+\Delta_i{}^{(0)}\tau_1)\psi_{ai}-\chi_{ai}, \quad (3.18)$$

where

$$\chi_{ai}=\begin{bmatrix} -\delta\Delta_i u_{ai}{}^{(0)}+F_{ai}v_{ai}{}^{(0)}+\sum_{Ib}\dfrac{5X_iF_i(ab)}{2\Omega_a}v_{ab}{}^i(I)\Lambda_I{}^i \\ -\delta\Delta_i v_{ai}{}^{(0)}+F_{ai}u_{ai}{}^{(0)}-\sum_{Ib}\dfrac{5X_iF_i(ab)}{2\Omega_a}u_{ab}{}^i(I)\Lambda_I{}^i \end{bmatrix} \quad (3.19)$$

is the perturbation. Here we have recognized that the self-consistent potential Δ_i must also change,

$$\Delta_i=\Delta_i{}^{(0)}+\delta\Delta_i. \quad (3.20)$$

With the help of (3.12) we find to first order

$$2E_{ai}{}^{(0)}c_{ai}=\tilde{\psi}_{ai}{}^{(0)}(i\tau_2\chi_{ai}), \quad (3.21)$$

$$\delta E_{ai}=-\tilde{\psi}_{ai}{}^{(0)}\chi_{ai}, \quad (3.22)$$

and, after a short calculation, utilizing the definition (3.2),

$$\delta\Delta_i=\tfrac{1}{2}G_i\sum_a\{\tfrac{5}{2}\sum_{Ib}\tilde{\psi}_a{}^i(Ib)\tau_1\psi_a{}^i(Ib) +\Omega_a[\eta_{ai}\tilde{\psi}_{ai}{}^{(0)}\tau_1\psi_{ai}{}^{(0)}-2c_{ai}\tilde{\psi}_{ai}{}^{(0)}\tau_3\psi_{ai}{}^{(0)}]\}. \quad (3.23)$$

Comparing (3.19), (3.21), and (3.23), we see that in virtue of the change in the self-consistent potential Δ_i Eq. (3.21) is not yet an explicit solution for c_{ai}. The latter is, however, easily found. We omit the result, since it is not required in any of our calculations.

IV. THE RANDOM-PHASE AMPLITUDES

We first study the equations of motion for the amplitudes (2.9). With the definition

$$\omega_I=W_I-W_0, \quad (4.1)$$

Eq. (2.4) yields

$$[\omega_I-E_{ai}-\epsilon_{bi}-F_{bi}]\langle\bar{\alpha}_i|a_{\beta i}|Iq\rangle = -\tfrac{1}{2}G_i\sum_\gamma s_\beta s_\gamma\langle\bar{\alpha}_i|a_{-\beta i}{}^{\dagger}a_{-\gamma i}a_{\gamma i}|Iq\rangle - X_i\sum_{q'\gamma}F_i(bc)s_\gamma \times\langle bcm_\beta\bar{m}_\gamma|2q'\rangle\langle\bar{\alpha}_i|a_{\gamma i}\Lambda_{q'}{}^i|Iq\rangle. \quad (4.2)$$

We shall in fact limit ourselves to a linearized version of this equation. In the approximation to be considered, we set

$$\Delta_i(II)=\tfrac{1}{2}G_i\sum_\alpha s_\alpha\langle Iq|a_{-\alpha i}a_{\alpha i}|Iq\rangle\cong\Delta_i{}^{(0)}, \quad (4.3)$$

and

$$\langle\bar{\alpha}_i|a_{\gamma i}\Lambda_{q'}{}^i|Iq\rangle=\sum_{I'}\langle\bar{\alpha}_i|a_{\gamma i}|I'q''\rangle\langle I'q''|\Lambda_{q'}{}^i|Iq\rangle \simeq\langle\bar{\alpha}_i|a_{\gamma i}|0\rangle\langle 0|\Lambda_{q'}{}^i|Iq\rangle\simeq\delta(\alpha\gamma)\delta(qq')s_\alpha v_{ai}{}^{(0)}\Lambda_I{}^i. \quad (4.4)$$

Counting the amplitudes (2.9) and (2.10) as first order, it is not difficult to ascertain that the corrections to (4.3) and (4.4) are two orders smaller than the terms retained. Equation (4.3), in particular, emphasizes that our initial treatment neglects the blocking effect.

By means of (4.3) and (4.4) and of similar approximations applied to Eq. (2.5), we obtain the pair of equations:

$$[\omega_I-E_{ai}{}^{(0)}-\epsilon_{ai}{}^{(0)}]v_{ab}{}^i(I)=-\Delta_i{}^{(0)}u_{ab}{}^i(I) +X_iF_i(ab)v_{ai}{}^{(0)}\Lambda_I{}^i, \quad (4.5)$$

$$[\omega_I-E_{ai}{}^{(0)}+\epsilon_{ai}{}^{(0)}]u_{ab}{}^i(I)=-\Delta_i{}^{(0)}v_{ab}{}^i(I) -X_iF_i(ab)u_{ai}{}^{(0)}\Lambda_I{}^i. \quad (4.6)$$

For the amplitudes (2.10), we obtain similar equations,

$$[\omega_I+E_{bi}{}^{(0)}+\epsilon_{ai}{}^{(0)}]v_a{}^i(Ib)=\Delta_i{}^{(0)}u_a{}^i(Ib) +X_iF_i(ba)v_{bi}{}^{(0)}\Lambda_I{}^i, \quad (4.7)$$

$$[\omega_I+E_{bi}{}^{(0)}-\epsilon_{ai}{}^{(0)}]u_a{}^i(Ib)=\Delta_i{}^{(0)}v_a{}^i(Ib) -X_iF_i(ba)u_{bi}{}^{(0)}\Lambda_I{}^i, \quad (4.8)$$

if we suppose, from the definition of the state $|(Ib)\alpha i\rangle$, that its energy is given, consistent with the current approximation, by

$$W_a(Ib)\simeq W_0+E_b+\omega_I. \quad (4.9)$$

From (4.5)–(4.8) it is not difficult to deduce the well-known phonon dispersion relation of the RPA, if we notice from (2.7) and (3.3), that $\Lambda_I{}^i$ is a linear combination of the quantities

$$Q_I{}^i\equiv\langle Iq|Q_q{}^{i\dagger}|0\rangle\cong\sum_{ab}F_i(ab) \times[\theta(ab)v_{bi}{}^{(0)}v_{ba}{}^i(I)+v_{ai}{}^{(0)}v_b{}^i(Ia)]. \quad (4.10)$$

We find after some algebra, the dispersion equation[5]:

$$\mathfrak{F}(\omega^2)=[1-X_pS_p(\omega)][1-X_nS_n(\omega)]-X_{np}^2S_p(\omega)S_n(\omega)=0\,, \quad (4.11)$$

where

$$S_i(\omega)=\sum_{ab} F_i^2(ab)E_i(ab)(u_{ai}^{(0)}v_{bi}^{(0)}+u_{bi}^{(0)}v_{ai}^{(0)})^2[E_i^2(ab)-\omega^2]^{-1}\,, \quad (4.12)$$

$$E_i(ab)=E_{ai}^{(0)}+E_{bi}^{(0)}\,.$$

The excitation energies are thus determined from (4.11). To normalize our amplitudes, we utilize the energy self-consistency condition

$$\omega_I=W_I-W_0. \quad (4.13)$$

Several examples of this kind of calculation have been carried out in our previous work[6] and we shall therefore omit the details. The result is

$$\omega_I=\tfrac{1}{2}\sum_{ab,i}\{\psi_{ba}{}^i(I)[E_{ai}^{(0)}+\epsilon_{ai}^{(0)}\tau_3-\Delta_i^{(0)}\tau_1]\psi_{ba}{}^i(I)+\tilde{\psi}_a{}^i(Ib)[E_{ai}^{(0)}+\epsilon_{ai}^{(0)}\tau_3-\Delta_i^{(0)}\tau_1]\psi_a{}^i(Ib)\}-\sum_i X_iQ_I{}^i\Lambda_I{}^i. \quad (4.14)$$

Since part of the proof of Eq. (4.14) involves the demonstration that

$$Q_I{}^i=X_iS_i(\omega_I)\Lambda_I{}^i\,, \quad (4.15)$$

we see from (4.9)–(4.14) that ω_I is a known quadratic function of $\Lambda_I{}^i$, which is then determined up to a sign. Returning to (4.9)–(4.14), the random-phase amplitudes are similarly determined. It can be shown that the condition (4.14) is equivalent to the usual RPA normalization condition.[7]

Finally, we add that from the results of this section, in particular from the $Q_I{}^i$, Eq. (4.10), we can calculate the $E2$ transition probability from a one-phonon state I to the ground state

$$B(E2, I\rightarrow 0)=\frac{5}{4\pi}|\sum_i e_i\nu_iQ_I{}^i|^2\,, \quad (4.16)$$

where e_i is the effective charge of nucleons of type i, namely,

$$e_p=(1+e_{\text{eff}})(e)\,,\qquad e_n=e_{\text{eff}}(e)\,, \quad (4.17)$$

and e_{eff} is chosen to fit the experimental value. We shall then utilize the same value to study the static quadrupole moments of the one-phonon states.

V. STATIC QUADRUPOLE MOMENT OF THE ONE-PHONON STATE

We come now to the essentially new part of our work. We shall first study the relevant amplitudes (2.11), which we unite with the definition

$$\psi_{aI}{}^i(b)=\begin{pmatrix} v_{aI}{}^i(b)\\ u_{aI}{}^i(b)\end{pmatrix}. \quad (5.1)$$

In order to maintain a firm hold on the chain of argument we restrict ourselves in this section mainly to results. If we write the energy of the state $|(Ia)\beta i\rangle$ as

$$W_{aI}{}^i(b)=W_0+E_{ai}+\omega_I+\delta E_{aI}{}^i(b)\,, \quad (5.2)$$

we ultimately obtain by the methods exposed in the previous sections for the quantity (5.1) the following equation:

$$[E_{ai}+\delta E_{aI}{}^i(b)]\psi_{aI}{}^i(b)=[-\epsilon_{ai}^{(0)}\tau_3+\Delta_i^{(0)}\tau_1]\psi_{aI}{}^i(b)-\chi_{ai}-\chi_{aI}{}^i(b)\,, \quad (5.3)$$

where χ_{ai} is given by Eq. (3.19) and

$$\chi_{aI}{}^i(b)=-\delta\Delta_i(I)(\tau_1\psi_{ai}^{(0)})-(5/2\Omega_b)X_iF_i(ab)\Lambda_I{}^i\tau_3\psi_{b}{}^i(Ia)-X_i\sum_{b'}5\theta(bb')W(2bb'2,aa)F_i(ab')\Lambda_I{}^i\tau_3\psi_{ab'}{}^i(I)+X_i5\theta(ab)F_i(aa)W(2a2a,b2)\langle I\|\Lambda^i\|I\rangle\tau_3\psi_{ai}^{(0)}. \quad (5.4)$$

In Eq. (5.4)

$$\delta\Delta_i(I)=\Delta_i(II)-\Delta_i^{(0)}-\delta\Delta_i\,, \quad (5.5)$$

$$\langle Iq''|\Lambda_{q'}{}^{i\dagger}|Iq\rangle=(22qq'|Iq'')\langle I\|\Lambda^i\|I\rangle. \quad (5.6)$$

Both of these quantities, so far unknown, will be computed below.

The comparison of (5.3) with (3.18) and (3.12) suggests that a solution to the first may be written as

$$\psi_{aI}{}^i(b)=\psi_{ai}+\tfrac{1}{2}\eta_{aI}{}^i(b)\psi_{ai}^{(0)}+c_{aI}{}^i(b)i\tau_2\psi_{ai}^{(0)}\,, \quad (5.7)$$

where ψ_{ai} is given by (3.12). The extra normalization constant $\eta_{aI}{}^i(b)$ is determined from the anticommutator condition

$$\langle Iq|[a_{\alpha i},a_{\alpha i}{}^\dagger]_+|Iq\rangle=1\,. \quad (5.8)$$

This leads, after some calculation to the following result:

$$\eta_{aI}{}^i(b)=-\frac{5}{2\Omega_b}\tilde{\psi}_{ba}{}^i(I)\psi_{ba}{}^i(I)+\sum_c 5\theta(bc)\times W(2bc2,aa)\tilde{\psi}_a{}^i(Ic)\psi_a{}^i(Ic)\,, \quad (5.9)$$

where W is the original Racah coefficient.[8] In reaching this result and others to follow, we make use of the relations[9]

$$\sum_{J(\text{even}),\,J(\text{odd})} 2(2a+1)(2J+1)W(22bc,Ja')W(22bc,Ja)=\delta(aa')\pm(2a+1)\theta(bcaa')W(2bc2,a'a)\,, \quad (5.10)$$

[7] The proof is similar to demonstrations carried out in both papers cited under Ref. 6.

[8] See, for instance, A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957).

[9] A version of this formula has been given by Rottenberg *et al.*, in *The 3j and 6j Symbols* (Technology Press, Cambridge, Massachusetts, 1959).

where the upper sign goes with J(even) and the lower with J(odd).

On the other hand, $c_{aI}{}^i(b)$ and $\delta E_{aI}{}^i(b)$ are determined from Eq. (5.3) as follows:

$$2E_{ai}{}^{(0)}c_{aI}{}^i(b)=\tilde{\psi}_{ai}{}^{(0)}[i\tau_2\chi_{aI}{}^i(b)],\qquad(5.11)$$

$$E_{aI}{}^i(b)=-\tilde{\psi}_{ai}{}^{(0)}\chi_{aI}{}^i(b).\qquad(5.12)$$

In order to evaluate the right-hand sides of Eqs. (5.11) and (5.12), we must, according to (5.4), evaluate the quantities defined by (5.5) and (5.6). For the former we find

$$\delta\Delta_i(I)=\tfrac{1}{4}G_i\sum_{ab}\Big\{\tilde{\psi}_{ba}{}^i(I)\tau_1\psi_{ba}{}^i(I)+\tilde{\psi}_a{}^i(Ib)\tau_1\psi_a{}^i(Ib)+\frac{2\Omega_b}{5}[\eta_I{}^i(b)\tilde{\psi}_{ai}{}^{(0)}\tau_1\psi_{ai}{}^{(0)}-2c_I{}^i(b)\tilde{\psi}_{ai}{}^{(0)}\tau_3\psi_{ai}{}^{(0)}]\Big\}.\qquad(5.13)$$

As to the quantity $\langle I\|\Lambda^i\|I\rangle$, we shall study it in a little more detail, since it is the core of this work, and because the analogy with previous calculations is not complete.

We begin by calculating the quantity

$$\langle iq''|Q_{q'}{}^{i\dagger}|Iq\rangle\equiv\sum_{\alpha\beta}F_i(ab)s_\beta(abm_\alpha\bar{m}_\beta|2q')\times\langle Iq''|a_{\alpha i}{}^\dagger a_{\beta i}|Iq\rangle,\qquad(5.14)$$

which is linearly related to the quantity of interest. For the contribution from zero- and two-phonon intermediate states, we get easily

$$-\tfrac{1}{2}\sum_{abc}5\theta(ac)F_i(ab)W(2a2b,c2)(22qq'|2q'')\times[\psi_{ca}{}^i(I)\tau_3\psi_{cb}{}^i(I)+\tilde{\psi}_a{}^i(Ic)\tau_3\psi_b{}^i(Ic)].\qquad(5.15)$$

The calculation of the contributions from the one-phonon states, however, requires special care. For this part, we have for $a=b$,

$$\langle Iq''|a_{\alpha i}{}^\dagger a_{\beta i}|Iq\rangle\to\langle Iq''|a_{\alpha i}{}^\dagger|(Ia)\bar{\gamma}_i\rangle\times\langle(Ia)\bar{\gamma}_i|a_{\beta i}|Iq\rangle,\qquad(5.16)$$

which results in the following contribution to (5.14):

$$-\tfrac{1}{2}\sum_{ac}F_i(aa)(2\Omega_c)\theta(ac)W(2a2a,c2)(22qq'|2q'')\times[\eta_{aI}{}^i(c)\tilde{\psi}_{ai}{}^{(0)}\tau_3\psi_{ai}{}^{(0)}+2c_{aI}{}^i(c)\tilde{\psi}_{ai}{}^{(0)}\tau_1\psi_{ai}{}^{(0)}].\qquad(5.17)$$

For $a\neq b$, a new type of particle nondiagonal matrix element is involved, namely, that defined in Eq. (2.12). We have, for example,

$$\langle Iq''|a_{\alpha i}{}^\dagger a_{\beta i}|Iq\rangle=\langle Iq''|a_{\alpha i}{}^\dagger|(Ia)\bar{\gamma}i\rangle\langle(Ia)\bar{\gamma}i|a_{\beta i}|Iq\rangle+\langle Iq''|a_{\alpha i}{}^\dagger|(Ib)\bar{\gamma}_i\rangle\langle(Ib)\bar{\gamma}_i|a_{\beta i}|Iq\rangle.\qquad(5.18)$$

Utilizing Eq. (2.12), terms of the form (5.18) can be shown to contribute to (5.14) the amount

$$-\sum_{a\neq b}F_i(ab)[v_{bi}r_a{}^i(Ib)-u_{bi}s_a{}^i(Ib)](22qq'|2q''),\qquad(5.19)$$

where

$$r_a{}^i(Ib)\equiv\sum_c\theta(bc)2\Omega_cW(2a2b,c2)v_a{}^i(Ib,c),$$

$$s_a{}^i(Ib)\equiv\sum_c\theta(bc)2\Omega_cW(2a2b,c2)u_a{}^i(Ib,c).\qquad(5.20)$$

By studying the equations of motion for these quantities we shall ascertain that they are of second order. The equations sought can be obtained by now standard methods, and we find in lowest approximation

$$(E_{bi}+\epsilon_{ai})r_a{}^i(Ib)=\Delta_is_a{}^i(Ib)-X_iF_i(ab)v_{bi}\langle I\|\Lambda^i\|I\rangle+X_i\sum_c5F_i(ac)\theta(bc)W(2a2b,c2)\Lambda_I{}^i[v_c{}^i(Ib)-v_{bc}{}^i(I)],$$

$$(E_{bi}-\epsilon_{ai})s_a{}^i(Ib)=\Delta_ir_a{}^i(Ib)+X_iF_i(ab)u_{bi}\langle I\|\Lambda^i\|I\rangle-X_i\sum_c5F_i(ac)\theta(bc)W(2a2b,c2)\Lambda_I{}^i[u_c{}^i(Ib)-u_{bc}{}^i(I)].\qquad(5.21)$$

We see indeed that the "driving terms" of (5.21) are, according to our assumptions, second order. The total contribution (5.19) is then of the same order as the previously computed contributions (5.15) and (5.12). Altogether we find

$$\langle I\|Q^i\|I\rangle=-\tfrac{1}{2}\sum_{abc}5\theta(ac)F_i(ab)W(2a2b,c2)[\tilde{\psi}_{ca}{}^i(I)\tau_3\psi_{cb}{}^i(I)+\tilde{\psi}_a{}^i(Ic)\tau_3\psi_b{}^i(Ic)]-\tfrac{1}{2}\sum_{ab}F_i(aa)2\Omega_a\theta(ab)W(2a2a,b2)\times[\eta_{aI}{}^i(b)\tilde{\psi}_{ai}{}^{(0)}\tau_3\psi_{ai}{}^{(0)}+2c_{aI}{}^i(b)\tilde{\psi}_{ai}{}^{(0)}\tau_1\psi_{ai}{}^{(0)}-\sum_{a\neq b}F_i(ab)[v_{bi}r_a{}^i(Ib)-u_{bi}s_a{}^i(Ib)].\qquad(5.22)$$

From this expression we obtain $\langle I\|\Lambda,\|I\rangle$ by means of the equation

$$\langle I\|\Lambda^i\|I\rangle=\langle I\|Q^i\|I\rangle+X_{np}{}^i\langle I\|Q^{i'}\|I\rangle,\quad i'\neq i.\qquad(5.23)$$

The most important observation of this section is that we can solve (5.22) explicitly for $\langle I\|Q\|I\rangle$. First of all, from (5.9) we find

$$\sum_b\theta(ab)(2b+1)W(2a2a,b2)\eta_{aI}{}^i(b)=-5\sum_b\theta(ab)W(2a2a,b2)[\tilde{\psi}_{ba}{}^i(I)\psi_{ba}{}^i(I)+\tilde{\psi}_a{}^i(Ib)\psi_a{}^i(Ib)].\qquad(5.24)$$

From (5.4) and (5.11) we find in turn[10]

$$\sum_b \theta(ab)(2b+1)W(2a2a,b2)2c_{aI}{}^i(b)=\sum_b \theta(ab)(2b+1)W(2a2a,b2)\tilde{\psi}_{ai}{}^{(0)}[i\tau_2\chi_{aI}{}^i(b)]/E_{ai}{}^{(0)}$$
$$=(E_{ai})^{-1}\tilde{\psi}_{ai}{}^{(0)}i\tau_2[K_1{}^i(a)+K_2{}^i(a)+K_2{}^i(a)\langle I\|\Lambda^i\|I\rangle], \quad (5.25)$$

where the two-column matrices K_1 and K_2 are given by

$$K_1{}^i(a)=\begin{bmatrix} -X_i\sum_b 5\theta(ab)W(2a2a,b2)F_i(ab)[v_b{}^i(Ia)-v_{ab}{}^i(I)]\Lambda_I{}^i \\ X_i\sum_b 5\theta(ab)W(2a2a,b2)F_i(ab)[u_b{}^i(Ia)-u_{ab}{}^i(I)]\Lambda_I{}^i \end{bmatrix}, \quad (5.26)$$

$$K_2{}^i(a)=X_iF_i(aa)\tau_3\psi_{ai}{}^{(0)}. \quad (5.27)$$

Noting that (5.21) can be written in the form

$$\begin{aligned} r_a{}^i(Ib)&\equiv R_{1a}{}^i(Ib)+R_{2a}{}^i(Ib)\langle I\|\Lambda^i\|I\rangle, \\ s_a{}^i(Ib)&\equiv S_{1a}{}^i(Ib)+S_{2a}{}^i(Ib)\langle I\|\Lambda^i\|I\rangle, \end{aligned} \quad (5.28)$$

Eq. (5.22) takes the final form to be recorded

$$\langle I\|Q^i\|I\rangle+\langle I\|\Lambda^i\|I\rangle\{\tfrac{1}{2}\sum_a F_i(aa)\tilde{\psi}_{ai}{}^{(0)}\tau_1\psi_{ai}{}^{(0)}E_{ai}{}^{-1}\psi_{ai}{}^{(0)}i\tau_2K_2{}^i(a)+\sum_{a\neq b}F_i(ab)[v_{bi}R_{2a}{}^i(Ib)-u_{bi}S_{2a}{}^i(Ib)]\}$$
$$=-\tfrac{1}{2}\sum_{abc}5\theta(ac)F_i(ab)W(2a2b,c2)[\tilde{\psi}_{ca}{}^i(I)\tau_3\psi_{cb}{}^i(I)+\tilde{\psi}_a{}^i(Ic)\tau_3\psi_b{}^i(Ic)]$$
$$+\tfrac{1}{2}\sum_{ab}F_i(aa)5\theta(ab)W(2a2a,b2)\tilde{\psi}_{ai}{}^{(0)}\tau_3\psi_{ai}{}^{(0)}[\tilde{\psi}_{ba}{}^i(I)\psi_{ba}{}^i(I)+\tilde{\psi}_a{}^i(Ib)\psi_a{}^i(Ib)]$$
$$-\sum_{a\neq b}F_i(ab)[v_{bi}R_{1a}{}^i(Ib)-u_{bi}S_{1a}{}^i(Ib)]-\tfrac{1}{2}\sum_a F_i(aa)\tilde{\psi}_{ai}{}^{(0)}\tau_1\psi_{ai}{}^{(0)}E_{ai}{}^{-1}\tilde{\psi}_{ai}{}^{(0)}[i\tau_2K_1{}^i(a)]. \quad (5.29)$$

With (5.23), (5.29) constitutes a pair of linear inhomogeneous equations for the $\langle I\|Q^i\|I\rangle$ which can be solved explicitly, since all coefficients and the inhomogeneous term are known either from the BCS theory of Sec. III or from the RPA of Sec. IV. The results of Eq. (5.29) will be compared with what is normally taken as the RPA value,[4] which in our notation is

$$\langle I\|Q^i\|I\rangle=-\tfrac{1}{2}\sum 5\theta(ac)F_i(ab)W(2a2b,c2)$$
$$\times[\tilde{\psi}_{ca}{}^i(I)\tau_3\psi_{cb}{}^i(I)+\tilde{\psi}_a{}^i(Ic)\tau_3\psi_b{}^i(Ic)]. \quad (5.30)$$

In terms of these quantities, the quadrupole moment in the state I, the observable of interest, is given by the formula

$$Q_I=\frac{1}{e}\left(\frac{16\pi}{5}\right)^{1/2}\sum_{i=n,p}e_i\langle II|[r^2Y_{20}]_{\mathrm{op}}{}^i|II\rangle$$
$$=\frac{2}{e}\sum_i e_i\nu_i\langle I\|Q^i\|I\rangle(2222|20), \quad (5.31)$$

where in the first version the subscript "op" refers to the second-quantized form of the operator for the type of particle in question.

VI. RESULTS AND DISCUSSION

The main new result of this paper is the expression (5.29) which determines the quadrupole moment of the first excited 2^+ state of "spherical" nuclei according to an apparently well-defined approximation method. In (5.29) we have a pair of linear inhomogeneous equations in which the driving terms are indeed of second order and the determinant of the coefficients of zero order.

We must first ask whether the solution of (5.20) is guaranteed by the nonvanishing of the determinant of the coefficients. In this connection the following interesting result can be proven: The determinant in question has the value $\mathfrak{F}(0)$, where the condition $\mathfrak{F}(\omega^2)=0$ [Eq. (4.11)] is the RPA equation for determining the excitation energies. Since the condition $\mathfrak{F}(0)=0$ must be interpreted here as yielding those values of the parameters for which the 2^+ state can experience a self-sustaining deformation, it is an interesting addendum to Thouless's theorem[11] that in the approximation of this paper, this does not occur until the 2^+ excitation energy vanishes.

The numerical consequences of Eq. (5.29) turn out to be rather striking. In Table I, we exhibit some results obtained for Cd^{114} as follows: for a given X_{np}, $X_p=X_n$ was determined from Eq. (4.11) to give the correct 2^+ excitation energy. The effective charge was then computed using Eq. (4.21) for the $E2$ transition probability. Finally, Q_{2^+} was computed from Eqs. (5.29) and (5.30), respectively, for the present theory and for the RPA approximation to it. We remark that the values of Q_2

[10] Considerable simplification results here from the relation $\sum_b\theta(ab)(2b+1)W(2a2a,b2)=0$, see Ref. 8.

[11] D. J. Thouless, Nucl. Phys. 22, 78 (1961).

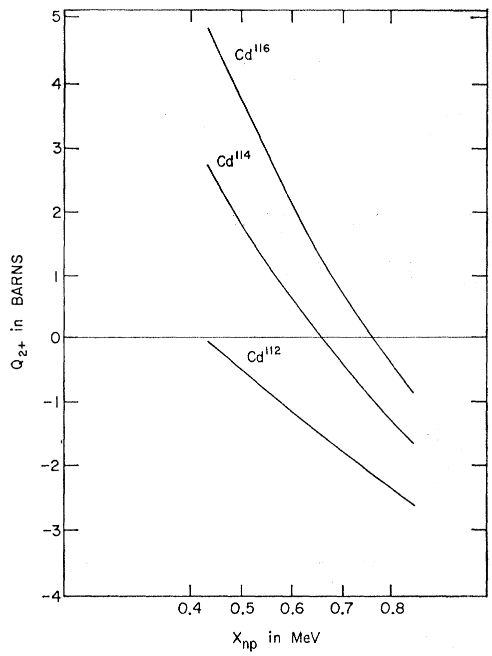

FIG. 1. Quadrupole moment (Q_{2^+}) of the first excited, 2^+ state for three isotopes of Cd in units of 10^{-24} cm² plotted versus X_{np}, the strength of the neutron-proton quadrupole force. Each point of any curve is associated with values of the remaining parameters chosen to fit the excitation energy of the state and its $E2$ transition probability to the ground state.

found by us are extremely sensitive functions of the quadrupole parameter with quite large values (including the experimental one) easily obtainable. For the usual choice $X_{np}=X_n=X_p$, we would be well beyond the experimental value

$$Q_2=(-0.50\pm0.25)\times10^{-24}\ \mathrm{cm}^2.$$

The most important conclusion to be drawn from these results, however, is the inadequacy of the present theory since for a quantity which was supposed nominally to be of second order, we obtained numerical values comparable with first-order quantities. This demonstrates the inadequacy of the usual linearized theory and shows that the diagonal elements of the quadrupole operators must be treated on an equal footing[12] with the nondiagonal elements. Put otherwise, it appears that the theory has within it at least one mechanism for achieving qualitative accord with experiment, but any serious comparison must await a consistent nonlinear intermediate-coupling calculation.

TABLE I. Quadrupole force parameters (X_{np}, $X_n=X_p$) obtained by fitting first 2^+ energy, effective charge (e_{eff}) needed to describe $E2$ transition to the ground state, and quadrupole moments (Q_{2^+}) determined by the RPA on the one hand and by a self-consistent calculation on the other. The additional parameters such as single-particle energies and pairing forces are taken from K.S.

			Q_{2^+} in barns	
X_{np}	$X_p=X_n$	e_{eff}	RPA	Present theory
0.50	1.21	0.57	−0.046	0.98
0.60	1.16	0.59	−0.066	−0.092
0.64	1.12	0.60	−0.072	−0.50
0.70	1.01	0.61	−0.082	−1.01

We have also done calculations for the neighboring isotopes of Cd. That these give qualitatively similar behavior is shown in Fig. 1, where Q_2 is plotted for several isotopes as a function of X_{np}. We emphasize again that *no effort should be made to compare these results with experiment.*

We may finally remark that the nature of the results is easily explained from the properties of the expression (5.29). The fluctuation in sign is a consequence of the sensitivity of the driving terms to the quadrupole parameter. These terms never become large in absolute value, however. The enhancement arises from the fact that $\mathfrak{F}(0)$ is sensibly smaller than unity hinting the approach to a self-sustaining deformation.

ACKNOWLEDGMENTS

One of the authors (A. K.) is delighted to acknowledge the hospitality of the Faculty of Sciences of Orsay (Physique Théorique et Hautes Energies), and particularly of Professor B. Jancovici, during the course of some of this work.

[12] Such a point of view is to be found in the version of the phenomenological intermediate coupling theory of V. K. Thankappan and W. W. True [Phys. Rev. **137**, B793 (1965)]. These authors found that in order to fit the one-phonon, one-particle states of Cu^{63} they needed to invoke comparable diagonal and off-diagonal quadrupole matrix elements for the first 2^+ state of Ni^{62}.