Inelastic Electron and Proton Scattering from Nuclei*

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The relation between the inelastic scattering of high-energy electrons and impulse-approximation scattering of protons from light nuclei was investigated for the excitation of collective levels described in a shell-model picture. A search code was developed to find best-fit form factors for the electron scattering to define the nuclear transition density for proton scattering. Calculations were performed for the first 2^+ and 3^- levels of ^{12}C. The data for the two types of probe were found to be consistent as regards enhancement and angular distribution for both transitions. The sensitivity of the comparison was tested by expressing the proton interaction in terms of three different sets of nucleon-nucleon phase shifts. The available data did not allow a choice of a preferred phase-shift set.

I. INTRODUCTION

THE excitation of predominantly collective excited states of nuclei by inelastic electron scattering can be represented in terms of the longitudinal portion of the electron-nucleus interaction.[1] For high-energy electrons and for nuclei whose charge is small compared to the fine-structure constant the scattering is accurately described in the plane-wave limit.[2] The inelastic cross section is then, aside from factors describing the electron-proton interaction, proportional to the square of the Fourier transform of the proton transition density which is characteristic of the nucleus involved. In principle, assuming the restrictions mentioned above, we can find the proton density in configuration space from the electron cross section and if we further restrict our attention to nuclei where the neutron and proton distributions are similar, we can assume that we have obtained the complete nucleon transition density, i.e., the overlap of the final state with the initial (ground) state.

The inelastic scattering of high-energy protons can be described in the distorted-wave impulse approximation (DWIA),[3] wherein the portion of the transition amplitude which depends on the nuclear wave functions involved is just this overlap, provided that one ignores the energy dependence of the nucleon-nucleon interaction. Therefore one can presumably use the inelastic-electron data to predict the inelastic-proton results without assuming a nuclear model as has been done up to now.[4]

We shall be interested in this preliminary investigation in transitions for light, even-even nuclei with $J=0$, $T=0$ ground states. In particular, data for p-p' [5] and e-e' [6] experiments on ^{12}C exist as well as DWIA calculations of the p-p' cross sections where transition densities obtained from structure calculations were used.[7]

The longitudinal portion of the electron-nucleus interaction can only transfer orbital angular momentum to the nucleus: In particular, nuclear spin flip is excluded and therefore we get no information about the spin-flip transition density which can, for proton scattering, contribute to normal parity transitions. However, the spin-flip density is expected to be small for predominantly collective excitations so that we anticipate a reasonable result for p-p' by ignoring it.

II. INELASTIC PROTON SCATTERING

The transition matrix element for inelastic-nucleon scattering can be written as

$$T_{f0}=\sum_{m_a m_b}\int \chi_{m_f m_b}^{(-)*}(\mathbf{r})\langle J_f M_f;\, T_f N_f|\sum_j M_j(\mathbf{r},\mathbf{r}_j) \times|J_0 M_0;\, T_0 N_0\rangle \chi_{m_a m_0}^{(+)}(\mathbf{r})d\mathbf{r}, \quad (1)$$

where $\chi^{(+)}$ and $\chi^{(-)}$ are distorted waves and the interaction produces a nuclear transition $J_0M_0;\ T_0N_0\rightarrow J_fM_f;\ T_fN_f$ and a projectile transition $m_0n_0\rightarrow m_fn_f$. (Capital N's stand for nuclear isospin projections; small n represents the same quantity for the projectile. Since the distorted waves have spin-flip components we use m_a and m_b as intermediate spin states before and after the inelastic scattering.) The interaction, in the distorted-wave impulse approximation, can be written

$$\sum_j M_j(\bar{r},\mathbf{r}_j)=\sum_j \delta(\bar{r}-\bar{r}_j) \times[\sum_{tu}(-1)^u M_t(j)\tau_t{}^u\tau_t{}^{-u}(j)], \quad (2)$$

where $\tau_t{}^u$ is a spherical tensor operator of rank t ($t=0, 1$)

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[1] L. I. Schiff, Phys. Rev. **96**, 765 (1954); J. D. Walecka, *ibid.* **126**, 653 (1962).
[2] See, e.g., R. S. Willey, Nucl. Phys. **40**, 529 (1963).
[3] A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N. Y.) **8**, 551 (1959).
[4] This notion was used by Y. Nishida [Nucl. Phys. **43**, 598 (1963)], in an attempt to test the validity of the impulse approximation.
[5] References for these data are given in Ref. 7.
[6] J. D. Walecka, Phys. Rev. **126**, 653 (1962); J. H. Fregeau, *ibid.* **104**, 225 (1956); H. F. Ehrenberg, R. Hofstadter, U. Meyer-Berkhout, D. G. Ravenhall, and S. E. Sobottka, *ibid.* **113**, 666 (1959).
[7] R. M. Haybron and H. McManus, Phys. Rev. **140**, B638 (1965).

representing projectile isospin. The coefficients $M_t(j)$ are functions of projectile spin σ, target spin $\sigma(j)$, bombarding energy E_0, and momentum transfer $\mathbf{q}$. Their explicit form can be found in Ref. 3.

We wish to evaluate the matrix element in (1) (which we shall label I_{f0}). Substituting (2) into (1) we can get

$$I_{f0}=\sum_u(-1)^u\langle n_f|\tau_t{}^u|n_0\rangle\langle J_fM_f;T_fN_f;m_b|\sum_j\delta(\bar{r}-\bar{r}_j)\times M_t(j)\tau_t{}^{-u}(j)|J_0M_0;T_0N_0;m_a\rangle. \quad (3)$$

We shall restrict our attention here to p-p' scattering for which $n_0=n_f=\frac{1}{2}$. In that case (3) becomes

$$I_{f0}=\langle J_fM_f;T_fN_f;m_b|\sum_j\delta(\bar{r}-\bar{r}_j)M_t(j)\tau_t{}^0(j)\times|J_0M_0;T_0N_0;m_a\rangle\delta_{N_fN_0}. \quad (4)$$

Evaluation of Eq. (4) depends on the ground-state isospin. The important cases are ($N_f=N_0$ for all):

$$\begin{aligned}
&T_0=0: \quad T_f=0, \quad t=0\\
&\qquad\quad T_f=1, \quad t=1;\\
&T_0=\tfrac{1}{2}: \quad T_f=\tfrac{1}{2}, \quad t=0,1\\
&\qquad\quad T_f=\tfrac{3}{2}, \quad t=1;\\
&T_0=1: \quad T_f=0, \quad t=1\\
&\qquad\quad T_f=1, \quad t=0,1.
\end{aligned} \quad (5)$$

The $M_t(j)$'s can be expressed in terms of the target and projectile spin operators. A convenient way to do this is the form[8]

$$M_t(j)=\sum_{ss'\lambda\lambda'}d_{\lambda\lambda'}{}^{ss'}(t)\sigma_\lambda{}^s\sigma_{\lambda'}{}^{s'}(j). \quad (6)$$

The d coefficients depend on E_0 and q and also on the choice of quantization axes.

If we put (6) into (4) we get

$$I_{f0}=\sum_{\substack{tss'\\ \lambda\lambda'}}d_{\lambda\lambda'}{}^{ss'}(t)\langle m_b|\sigma_\lambda{}^s|m_a\rangle\langle J_fM_f;T_fN_f|\times\sum_j\delta(\bar{r}-\bar{r}_j)\sigma_{\lambda'}{}^{s'}(j)\tau_t{}^0(j)|J_0M_0;T_0N_0\rangle. \quad (7)$$

The delta function can be expanded in terms of spherical harmonics and if we define a spherical tensor of rank J' by

$$Y_l{}^m(\hat{r}_j)\sigma_{\lambda'}{}^{s'}(j)=\sum_{J'M'}C(ls'J':m\lambda'M')\mathcal{Y}_{J'M'}(l,s'), \quad (8)$$

Eq. (7) becomes

$$I_{f0}=\sum_{\substack{J'M'lm\\ tss'\lambda\lambda'}}d_{\lambda\lambda'}{}^{ss'}(t)C(ls'J':m\lambda'M')\langle m_b|\sigma_\lambda{}^s|m_a\rangle\times Y_l{}^{m*}(\hat{r})\langle J_fM_f;T_fN_0|\sum_j\frac{\delta(r-r_j)}{r_j{}^2}\mathcal{Y}_{J'M'}(l,s')\tau_t{}^0(j)\times|J_0M_0;T_0N_0\rangle. \quad (9)$$

[8] R. M. Haybron and H. McManus, Phys. Rev. 136, B1730 (1964).

The Wigner-Ekhart[9] theorem can be applied to (9) so that the expression becomes finally

$$I_{f0}=\sum_{\substack{J'M'lm\\ tss'\lambda\lambda'}}d_{\lambda\lambda'}{}^{ss'}(t)Y_l{}^{m*}(\hat{r})C(ls'J':m\lambda'M')\times C(J_0J'J_f:M_0M'M_f)C(T_0tT_f:N_0 0N_0)C(\tfrac{1}{2}s\tfrac{1}{2}:m_a\lambda m_b)\times\langle\tfrac{1}{2}\|\sigma^s\|\tfrac{1}{2}\rangle\langle J_f;T_f\|\sum_j\frac{\delta(r-r_t)}{r_j{}^2}\mathcal{Y}_{J'}(l,s')\tau_t(j)\|J_0;T_0\rangle. \quad (10)$$

All the nuclear structure information available to p-p' scattering is contained in the reduced matrix element in (10). Once this entity is determined by one means or another, the transition amplitude in (1) and thus the pertinent observables in the reaction can be computed using existing codes (Ref. 7 and Appendix A). We shall call this quantity the transition density, designated by $F_{s'}{}_t{}^{J'l}(r)$.

It can be seen that for J' and l given, we have at most four densities to determine. For the cases of interest we do not need all these and frequently one density will suffice. For example, let us consider the excitation of ^{12}C($J_0=0^+$, $T_0=0$) to its excited state at 4.43 MeV ($J_f=2^+$, $t_f=0$). Here $l=2$, $t=0$ and $s'=0$ and 1, a mixture of nonspin flip and spin flip. We need in this case at most two densities: In fact, the $s'=1$ density turns out to be small and the excitation is well described by $F_{00}{}^{22}(r)$ only.

Another case is the $0^+\rightarrow 3^-$ transition in ^{40}Ca. Here we must consider admixtures of isospin in the levels involved so that $F_{00}{}^{33}$, $F_{10}{}^{33}$, $F_{01}{}^{33}$, and $F_{11}{}^{33}$ all contribute. (We have written the formulas in our development as though T_0 and T_f were good quantum numbers. They can be generalized for heavier nuclei by summation.) However the isospins are only weakly mixed so that $T_0\simeq T_f\simeq 0$. Also the spin-flip contribution is small once again so that the transition can be described with reasonable accuracy by $F_{00}{}^{33}(r)$.

The low-lying normal parity excitations of light nuclei are usually enhanced by a factor of from 2 to 10 and the spin-flip contributions to these excitations will invariably be small. This is just saying that the transitions are predominantly collective with no change in intrinsic state. Also, contributions from the $t=1$ terms will, in general, be small even when allowed. This is due to the spin-isospin correlation produced by the two nucleon interaction.[10] Transitions are strong for $s'=0$, $t=0$ or $s'=1$, $t=1$. The "mixed" transitions are weak.

The description of enhanced, normal parity transitions to low-lying states in terms of one transition

[9] We use the definitions of D. M. Brink and G. R. Satchler [*Angular Momentum* (Oxford University Press, New York, 1962)], wherein

$$\langle j'm'|\omega_k{}^q|jm\rangle=(-1)^{2k}C(jkj':mqm')\langle j'\|\omega_k\|j\rangle.$$

[10] E. A. Sanderson, Nucl. Phys. 35, 557 (1962).

density which we have just described is very close to the collective theory of excitations[11] in form. On the other hand, we have managed to separate the interaction producing the excitation from the transition density which is not possible in the collective theory. This latter quantity is the most we can learn about nuclear structure from the scattering and its determination is the ultimate purpose of this experiment.

If we were sure that the description of high-energy proton scattering which we have is correct, we could use the available data to determine transition densities for the various excitations. However, a number of uncertainties exist of sufficient importance that such a procedure is perhaps not yet unwarranted. Perhaps the most important of these are: (1) uncertainties in the distorted waves, (2) choice of the two-nucleon interaction, and (3) corrections to the way in which the impulse approximation is applied due to refraction effects, exchange, etc.

We shall not comment further on this list for the moment, but only acknowledge that it exists. The point is that we should like to find some method to test Eq. (10) for some simple collective transitions without using calculated transition densities as has been done previously. The way we shall attempt to do this is generated by the similarity between proton and electron scattering for normal parity transitions. The results of e-e' measurements will be used to determine the required transition density. This will then be used in (10) to compare with available p-p' data. At the least we can look for agreement between the two types of experiments. If data are good enough, we may be in a position to comment on some of the uncertainties previously mentioned.

III. INELASTIC ELECTRON SCATTERING

The cross section for the inelastic scattering of a high-energy electron from a nucleus due to the Coulomb portion of the interaction is given in the Born approximation, for a transition of multipolarity λ, by (Ref. 2)

$$d\sigma|_{c\lambda}=\left(\frac{1}{Ze}\right)^2 d\sigma|_M\left[\frac{4\pi}{[(2\lambda+1)!!]^2}q^{2\lambda}\right]B(c\lambda,q). \quad (11)$$

The quantity $d\sigma|_M$ is the Mott scattering from a point nucleus with charge Z, given by

$$d\sigma|_M=\left(\frac{Ze^2}{\hbar c}\right)^2\frac{\cos^2(\frac{1}{2}\theta)}{4k^2\sin^4(\frac{1}{2}\theta)}.$$

The rest mass of the electron has been neglected in these expressions: k is the incident wave number of the electron and q is the momentum transfer. We shall ignore the energy loss of the electron so that

$$q^2=4k^2\sin^2(\tfrac{1}{2}\theta),$$

where θ is the scattering angle. The reduced transition amplitude $B(c\lambda,q)$ is given by

$$B(c\lambda,q)=\left(\frac{2J_f+1}{2J_0+1}\right)|\langle f\|M(c\lambda,q)\|0\rangle|^2, \quad (12)$$

where

$$\langle f|M(c\lambda,u,q)|0\rangle = \frac{(2\lambda+1)!!}{q^\lambda}\int j_\lambda(qr)Y_\lambda{}^u(\hat{r})\langle f|\rho_N|0\rangle d\mathbf{r}, \quad (13)$$

and

$$\langle f|\rho_N|0\rangle=|e|\langle f|\sum_{j=1}^{A}\epsilon_j\delta(\mathbf{r}-\mathbf{r}_j)|0\rangle. \quad (14)$$

These expressions describe the excitation of a nucleus from its ground state $|0\rangle$ with spin J_0 to an excited state $|f\rangle$ with spin J_f. The quantity ϵ_j in (14) normally equals 1 for a proton and 0 for a neutron. [The expression (12) appears to be $(2J_f+1)$ times the corresponding quantity in Ref. 2, because of our choice of definition for the reduced matrix element.]

We want to find the nuclear transition density in terms of the inelastic electron-scattering data. Using (12) and (13) in (11), one can obtain the relation

$$\left|\int j_\lambda(qr)Y_\lambda{}^u(\hat{r})\langle f|\rho_N|0\rangle d\mathbf{r}\right| = (-1)^{2\lambda}C(J_0\lambda J_f:M_0uM_f) \times\left(\frac{2J_0+1}{2J_f+1}\right)^{1/2}\left(\frac{Ze}{\sqrt{4\pi}}\right)\left[\frac{d\sigma|_{c\lambda}}{d\sigma|_M}\right]^{1/2}, \quad (15)$$

where we have used the expression for a reduced matrix element given in Ref. 9. Taking the expression for the charge-transition density in (14) and expanding the delta function contained there, we obtain an expression which can be transformed to

$$\langle J_fM_f;T_fN_f|\sum_{j=1}^{A}\epsilon_jY_\lambda{}^u(\hat{r}_j)\frac{\delta(r-r_j)}{r_j^2}|J_0M_0;T_0N_0\rangle = (-1)^{2\lambda}C(J_0\lambda J_f:M_0uM_f)\left(\frac{2J_0+1}{2J_f+1}\right)^{1/2}\left(\frac{Z}{\sqrt{4\pi}}\right)\left(\frac{2}{\pi}\right) \times\int_0^\infty j_\lambda(qr)\left[\frac{d\sigma|_{c\lambda}}{d\sigma|_M}\right]^{1/2}q^2dq. \quad (16)$$

We have now specified $|0\rangle$ and $|f\rangle$ with their quantum numbers for what follows.

The operator ϵ_j can be written in the form

$$\epsilon_j=\tfrac{1}{2}(\epsilon_p+\epsilon_n)+\tfrac{1}{2}(\epsilon_p-\epsilon_n)\tau_1{}^0(j), \quad (17)$$

where ϵ_p and ϵ_n are the fractional charges on the proton and neutron. We should expect to set $\epsilon_p=1$, $\epsilon_n=0$, but in some cases, as briefly discussed in Ref. 2, another

[11] R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).

choice is indicated. Equation (17) can be rewritten for convenience as

$$\epsilon_j=\sum_t \alpha_t{}^0\tau_t{}^0(j), \tag{18}$$

where

$$\alpha_0{}^0=\tfrac{1}{2}(\epsilon_p+\epsilon_n)$$

and

$$\alpha_1{}^0=\tfrac{1}{2}(\epsilon_p-\epsilon_n).$$

Equation (16) becomes, using (18) and applying the Wigner-Eckart theorem,

$$\sum_t C(T_0tT_f;N_00N_f)$$
$$\times\langle J_f;T_f\|\sum_{j=1}^{A}\frac{\delta(r-r_j)}{r_j{}^2}\mathcal{Y}_\lambda(\lambda,0)\tau_t(j)\|J_0;T_0\rangle\alpha_t{}^0$$
$$=\left(\frac{2J_0+1}{2J_f+1}\right)^{1/2}\left(\frac{Z}{\sqrt{4\pi}}\right)\left(\frac{2}{\pi}\right)\int_0^\infty j_\lambda(qr)\left[\frac{d\sigma|_{c\lambda}}{d\sigma|_M}\right]^{1/2}q^2dq, \tag{19}$$

where we have used notation chosen to correspond closely to Eq. (10) and some factors have been cancelled.

Several remarks are in order about the form of Eq. (19). First of all we note that in terms of the definition introduced after Eq. (10), the left-hand side can be written as

$$\sum_t C(T_0tT_f:N_00N_f)\alpha_t{}^0F_{0t}{}^{\lambda\lambda}(r),$$

in other words in terms of the non-spin-flip transition densities, so that we are assured that we can compute at least some of the transition densities for inelastic proton scattering from the electron data. Now if the nuclear states have mixed isospins or if the ground-state isospin is nonzero, then in general (19) gives us a linear combination for the $t=0$ and $t=1$ densities which is not separable except in terms of a model. We shall therefore be most interested in $N=Z$ nuclei which do not have appreciable isospin mixing. (This latter restriction more or less covers the nuclei for which the Born approximation is good.)

We have just remarked that Eq. (19) applies only to nuclear isospin changes of 0 or 1. This is a direct result of the neglect of multiple scattering of the electron: Since it collides with only one nucleon the isospin change is thus limited.

It has been previously noted that Eq. (19) only gives the non-spin-flip transition densities. We could, if we wished, work out expressions for the spin-flip transitions as well, which would be expressed in terms of the cross section for magnetic transitions. These are however of much less importance, since the number of spin-flip transitions which can be observed with the high-energy particles we are considering is quite small and we leave this for another time.

IV. CALCULATION OF TRANSITION DENSITIES

The quantity which must be computed in order to use (19) which we will call $I_\lambda(r)$ is the integral

$$I_\lambda(r)=\int_0^\infty j_\lambda(qr)\left[\frac{d\sigma|_{c\lambda}}{d\sigma|_M}\right]^{1/2}q^2dq, \tag{20}$$

which must be computed from the experimental determination of the electron cross section.

It is apparent that experiment will not provide the cross section at enough points to allow an accurate evaluation of (20), so that to perform the integral, we must fit the electron data with a continuous function of q. We have chosen the expansion

$$\left[\frac{d\sigma|_{c\lambda}}{d\sigma|_M}\right]^{1/2}=\left(\sum_{i=1}^{N}c_iq^i\right)e^{-(\beta/2)q^2}\equiv f_\lambda(q), \tag{21}$$

which is essentially the form one obtains if the nuclear wave functions are represented by oscillator functions in Eq. (11). The coefficients in the expansion (21) are then determined by finding a minimum χ^2 fit to the electron data, that is, the c_i's are determined by minimizing the quantity

$$\chi^2=\sum_{j=1}^{p}\left[\frac{f_{\exp}{}^2(q_j)-f_\lambda{}^2(q_j)}{\Delta(f_{\exp}{}^2(q_j))}\right]^2. \tag{22}$$

In Eq. (22),

$$f_{\exp}{}^2(q_j)=\left[\frac{d\sigma|_{c\lambda}}{d\sigma|_M}\right]_{q=q_j},$$

as measured by experiment at $q=q_j$, and

$$\Delta(f_{\exp}{}^2(q_j))$$

is the experimental error attached to that point.

The parameter β in (21) is simply related to the range parameter for the underlying oscillator functions. If the nuclear wave functions are of the asymptotic form

$$\varphi(r)\rightarrow e^{-\alpha r^2}$$

then a simple computation shows that

$$\beta=1/4\alpha. \tag{23}$$

In fact, the value of α used must be corrected for the finite size of the proton and center-of-mass effects. If one determines a range parameter α' which yields a fit to the elastic electron data, the value of α which should be used in (23) is related to α' via

$$\alpha=\alpha'\left(\frac{1}{1+2a_p{}^2\alpha'-1/A}\right),$$

where

$$a_p{}^2=0.43.$$

A computer code has been written which performs the calculations just described. This code allows a

TABLE I. The expansion coefficients for the electron form factors for the 4.43-MeV level (2^+) of ^{13}C are shown, along with the corresponding χ^2 and β_2's. Analysis of proton-scattering data yields β_2 in the range 0.60–0.67.

Search No.	C_2	C_4	C_6	C_8	β	χ^2	β_2
1	0.2197	0.0	0.0	0.0	1.434	130.0	0.439
2	0.2589	−0.01693	0.0	0.0	1.434	47.9	0.517
3	0.2300	0.01043	−0.005224	0.0	1.434	23.1	0.461
4	0.2215	0.02597	−0.01179	0.000701	1.434	22.5	0.443
5	0.2211	0.03276	−0.01194	0.000615	1.485	22.4	0.442

simultaneous search on up to fifteen powers of q and β in (21). When an acceptable fit to the electron data has been obtained, $I_\lambda(r)$ in (20) is output. This brief description will be augmented by an example in the next section.

V. TRANSITION DENSITY FOR THE FIRST LEVEL OF ^{12}C

We shall discuss in this section the calculation of the transition density for the 2^+, $T=0$, 4.43-MeV level of ^{12}C. This has been the most popular transition for high-energy studies with both protons and electrons so that a reasonable amount of data is available for both types of probe.

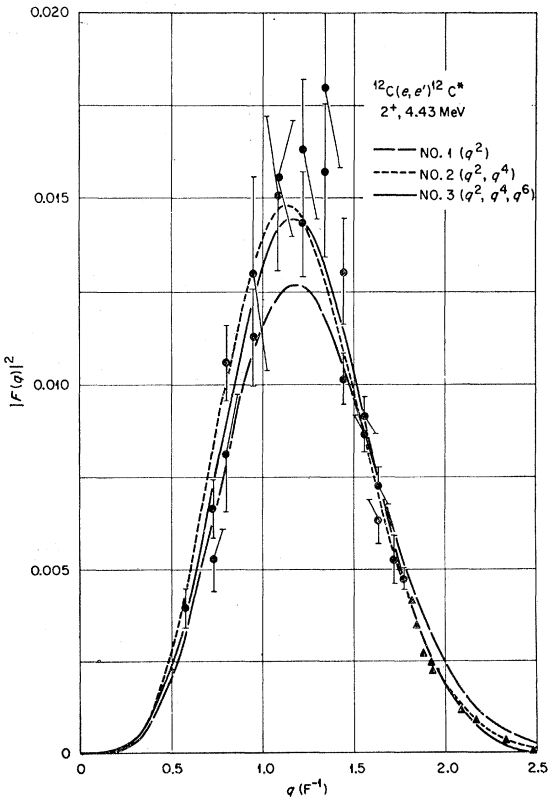

FIG. 1. Some of the fits to the e-e' data are shown for the first excited state of ^{12}C. The curve labeled No. 3 was used in subsequent calculations. The triangular points were assigned errors of 100% in the fitting.

The electron data we shall attempt to fit were taken at energies from 80 to 600 MeV (Ref. 6). Points for $q>2.5$ F^{-1} were omitted. The points for $0.5\leq q\leq 1.8$ were assigned the experimental errors. The large q points were assigned errors of 100%. The fitting was thus biased to smaller q values.

With the shell-model picture and oscillator functions, the lowest power of q allowed in Eq. (21) for a normal parity transition of multipolarity L is q^L. Therefore, the first nonzero coefficient for the transition we are considering is c_2. Further, only coefficients c_n are allowed with indices $n=L, L+2, L+4, \cdots$.

The general procedure we have adopted is to start with the lowest allowed power of q and obtain a best fit, introduce the next allowed power of q and obtain a new fit (searching on both coefficients) and so on until one or all of the following criteria are met: (1) The computed curve "looks" like the data, (2) the percent reduction in χ^2 is small when an additional power of q is introduced, and/or (3) too many powers of q are required. Statement (3) is essentially a shell-model requirement, that is, by inspecting the single-particle levels which may participate in the transition, one may limit the largest power of q allowed. However, higher powers of q than expected on a shell-model argument can appear due to the neglect of distortions in the electron scattering and also due to the inadequacy of the form of the expansion in Eq. (21).

Distortion effects are certainly present in the electron wave functions although they are ignored here. Inclusion of distortion effects in general shifts the peak cross section to smaller q without essentially changing the magnitude or shape of the angular distribution,[12] and most of this effect seems to be described if one picks a slightly larger effective radius for the transition (smaller β). For the light nuclei with which we are concerned such corrections should be minimal and we are assuming the plane-wave limit is adequate.

The inelastic form factors for the level under consideration which were obtained at various levels of the search procedure are shown with the data in Fig. 1. The curves are labeled according to the key in Table I where the parameter values are displayed along with the value of χ^2 obtained in each case. Not all the form factors

[12] D. S. Onley, J. T. Reynolds, and L. E. Wright, Phys. Rev. 134, B945 (1964). We should note that this remark is only true at q values below the first diffraction minimum.

TABLE II. The expansion coefficients for the electron form factors for the 9.6-MeV level (3^-) of ^{12}C are shown. Analysis of proton-scattering data yields β_3 in the range 0.44–0.57.

Search No.	C_3	C_5	C_7	C_9	β	χ^2	β_3
1	0.1010	0.0	0.0	0.0	1.434	31.9	0.475
2	0.1275	−0.01616	0.0	0.0	1.434	16.4	0.599
3	0.1390	−0.03232	0.00481		1.434	15.99	0.653
4	0.1491	−0.05373	0.01845	−0.00265	1.434	15.91	0.700
5	0.1509	−0.04299	0.01836	−0.00279	1.650	15.85	0.710

have been shown in Fig. 1, since No. 3, No. 4, and No. 5 are essentially identical.

These data are very creditably fit by No. 3, with three powers of q and β as inferred from the elastic-electron data. Higher powers of q, or the variation of β does not improve the fit appreciably. If we now included the very high q points which have been ignored ($q>2.5$ F^{-1}) or increase the weighting of the points for $1.8\leq q\leq 2.5$ F^{-1} the story might be different. However, large momentum transfers for either electron or proton scattering can reveal complexities for which our present picture must fail, so that we leave such an investigation for a time when the quantity and quality of the data require it.

The procedure just described was repeated for the 3^-, $T=0$, 9.6-MeV level of ^{12}C, yielding the results shown in Fig. 2 and Table II. It is obvious that these data do not define the form factor for this level (at least in terms of our parametrization) as accurately as was the case for the 2^+ level.

Data also are available for the 0^+, 7.6-MeV level of ^{12}C. Unfortunately, they are not consistent enough to define the form factor to an extent which would justify attempting to find a fit.

VI. REDUCED TRANSITION PROBABILITY

The reduced transition probability $B(\lambda)$ for radiative decay is given by

$$B(\lambda)=\lim_{q\to 0}B(c\lambda,q), \tag{24}$$

where $B(c\lambda,q)$ is expressed in terms of the reduced nuclear matrix element in Eq. (12). The lifetime τ_λ for radiative decay is then given by

$$\tau_\lambda^{-1}=\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}\left(\frac{E^*}{\hbar c}\right)^{2\lambda+1}B(\lambda), \tag{25}$$

where E^* is the deexcitation energy. Using (11) and (21) the reduced transition probability may be expressed as

$$B(\lambda)=\lim_{q\to 0}\left\{(Ze)^2\left[\frac{[(2\lambda+1)!!]^2}{4\pi}\right]\frac{[f_\lambda(q)]^2}{q^{2\lambda}}\right.$$
$$\left.=(Ze)^2\frac{[(2\lambda+1)!!]^2}{4\pi}|c_\lambda|^2. \right. \tag{26}$$

Thus, one may compute the reduced transition probability from the value of the first coefficient obtained in the search on the electron data. Alternately, one could use values of $B(\lambda)$ to determine this coefficient and perhaps reduce the numbers of independent parameters needed in the expansion of the form factor.

The predominantly collective transitions we are interested in can be characterized by a deformation β_λ. The reduced transition probability can be expressed in terms of this quantity as

$$B(\lambda)=(3/4\pi)^2(Ze)^2R_c^{2\lambda}\beta_\lambda^2, \tag{27}$$

where R_c is the nuclear radius. The deformation in (27) is associated with axially symmetric deformations of the nuclear charge distribution. In fact, the deformation parameters measured with nuclear probes (which presumably "see" the entire nucleus) indicate that the charge deformation follows the nuclear deformation[13] so that β_λ can be determined either by electron scattering or, say, proton scattering.

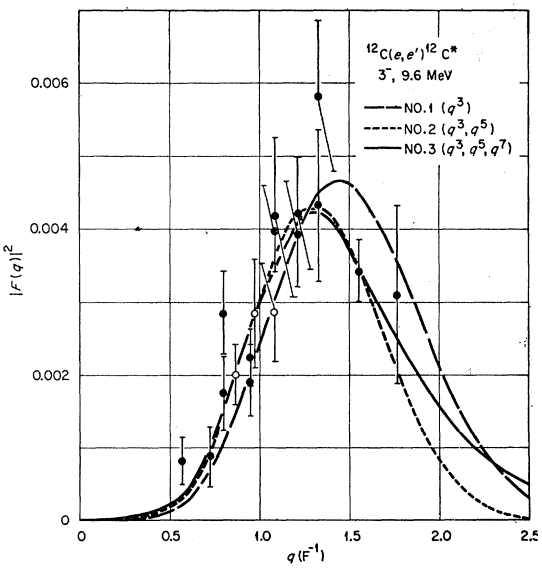

FIG. 2. The fits to the e-e' data for the 9.6 MeV, octupole level of ^{12}C are shown. The solid points were taken at 187 MeV; the points shown with circles were taken at 150 MeV.

[13] For a review of work in this field, see G. R. Satchler, in *Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962*, edited by E. Clementel and C. Villi (Gordon and Breach Science Publishers, Inc., New York, 1963), p. 80.

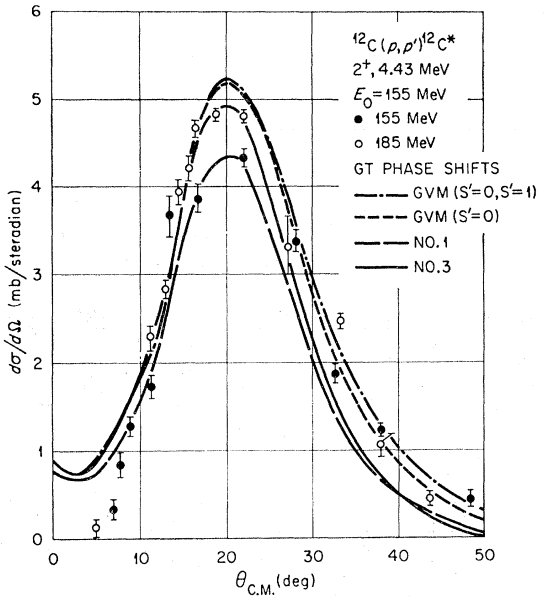

FIG. 3. The p-p' cross section for the 4.43-MeV level of ^{12}C as predicted from the electron data is compared to the proton data and also to the predictions obtained from the GVM transition densities, both with and without spin flip.

The deformations obtained for the various form factors are tabulated in Tables I and II.

VII. INELASTIC PROTON SCATTERING

The inelastic transition densities obtained in Sec. V were used to evaluate Eq. (10) and compute the cross section for inelastic proton scattering at 156 MeV for these two levels.

The first feature of interest is the comparison of the results obtained here to those of previous computations

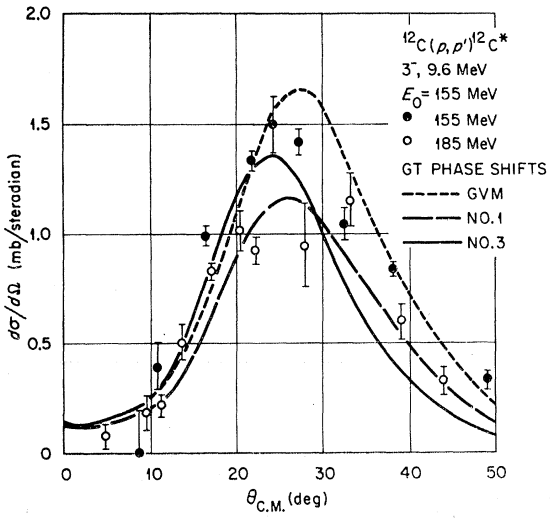

FIG. 4. The p-p' cross section obtained from density No. 1 does not have the same angular dependence as does that obtained from the GVM density due to the contributions of spin flip in the latter density which are not entirely negligible for this level.

where the Gillet–Vinh-Mau[14] (GVM) hole-particle transition densities were used. This comparison is shown in Figs. 3 and 4. The Gammel-Thaler phase shifts[15] were used to obtain these curves. The experimental data are described in Ref. 7. To get an idea of the importance of the spin-flip contributions, which we have neglected, calculations were performed for just the non-spin-flip GVM density and for the mixture for the quadrupole level. The results of both computations are shown. It can be seen that the spin-flip contributions are small as we anticipated.

We have shown the results for the No. 1 and No. 3 form factors described in Tables I and II to get an idea of the sensitivity of the p-p' cross section to how well the electron data are fit. For the 2^+ level density No. 1 is just that which would be obtained for a single-particle

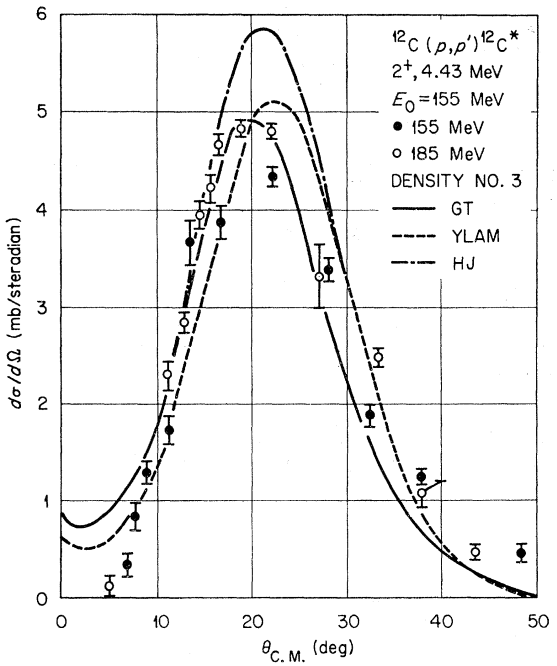

FIG. 5. The p-p' cross section for the 4.43-MeV level obtained using the "best-fit" electron form factor with the three choices of phase shifts considered.

representation of the transition ($1p_{3/2} \rightarrow 1p_{1/2}$ for j-j coupling, or a recoupling in the p shell for L-S coupling) with an adjustable normalization. The results obtained here indicate that such a picture is inadequate. If one tries to obtain the proper enhancement for the electron data, the shape of the form factor will not resemble the data. Therefore, additional configurations are clearly required.

Similar remarks may be made for the 3^- level. The simplest description involves a $1p$-$1d$ transition which produces a q^3 dependence for the electron form factor. Inspection of Figs. 2 and 4 indicate an insufficiency for

[14] V. Gillet and N. Vinh-Mau, Nucl. Phys. **54**, 321 (1964).
[15] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957).

such a description. Here, however, the best fit to the electron data still does not agree very well with the proton data. In particular, the peak cross section is 15%–20% too small. Inspection of Fig. 2 suggests that such an increase in normalization could be supported by the electron data with some adjustment in the relative weighting of the data points, since none of the fits reproduce the maximum value of the data. Such an adjustment, however, is outside the spirit of this work, and we only note that it could be in order.

It was previously mentioned that one reason for using electron data to compute transition densities for protons is to try and improve the proton calculations. Various phase-shift sets for two-nuclear scattering yield different results when used to compute p-p' scattering in the impulse approximation. In order to determine whether the transition densities obtained from the electron data can serve to select a "best" set of two-nuclear phase shifts, the proton calculations were done using the No. 3 densities for three sets of phase shifts and the results are shown in Figs. 5, 6, and 7. The sets used were those of Gammel and Thaler (GT), Yale (YLAM and YLAM3),[16] and Hamada and Johnson (HJ).[17]

The agreement between calculation and experiment is unfortunately mixed for the two levels we are discussing. The cross sections obtained for the HJ set seem too large at the peak and we (tentatively) exclude this set. For the 2^+ level both GT and Yale give the right order of magnitude for the peak cross section but the Yale set seems to move the peak to too large an angle. The cross section for the 3^- level is best for the Breit set as far as normalization is concerned. Once again, however, the peak occurs at an angle somewhat larger than that required by experiment.

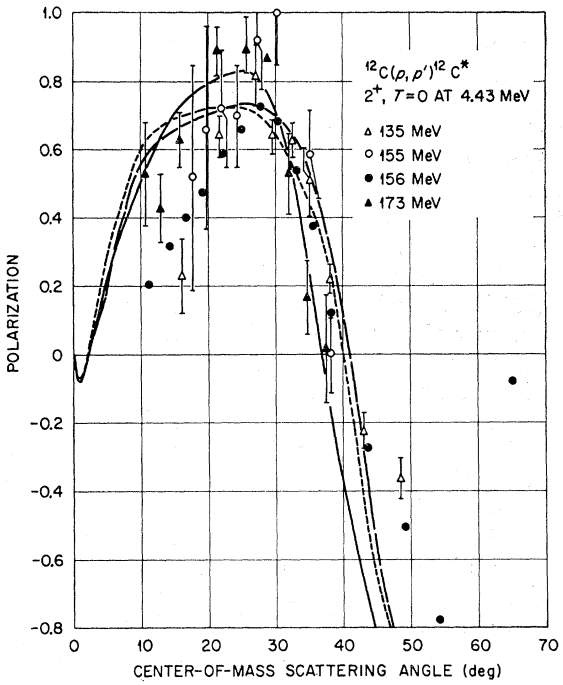

FIG. 6. The polarization results shown here correspond to the calculations described in the caption on Fig. 5. The curve drawn with long dashes should have been drawn dash-dot-dash, i.e., it was obtained with the HJ phase shifts.

VIII. DISCUSSION OF REDUCED TRANSITION PROBABILITIES

The inelastic form factor $f_\lambda(q)^2$ used to define the reduced transition probability in (26) is related to the square of the Fourier transform of the radial proton density. If a proton is initially described by $u_0(r)$ and finally by $u_f(r)$, we may write

$$f_\lambda(q)\sim\int_0^\infty j_\lambda(qr)u_f^*(r)u_0(r)r^2dr. \tag{28}$$

In the limit of $q=0$,

$$\frac{|f_\lambda(q)|^2}{q^{2\lambda}}\bigg|_{q=0}\sim\left|\int_0^\infty r^{\lambda+2}u_f^*(r)u_0(r)dr\right|^2. \tag{29}$$

Thus, the value of the reduced transition probability is related to a moment of the radial density and will obviously be affected by the behavior of the tail of the wave functions used. We have found (Table II) that the β_2 for the quadrupole level is too small for any of our form factors. (These comments are not applied to the octupole level since we apparently do not have enough data for a "stable" fit.) This says that the value obtained for the second moment of the transition den-

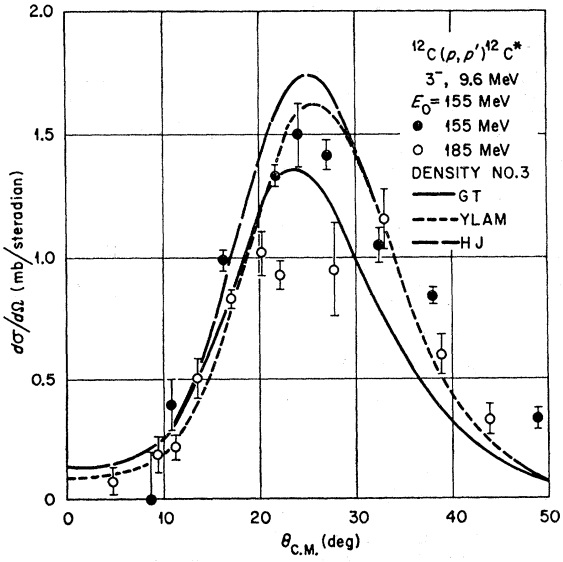

FIG. 7. The p-p' cross section for the 9.6-MeV level obtained using the "best-fit" electron form factor with the three phase-shift sets.

[16] G. Breit, M. H. Hull, K. E. Lassila, K. D. Pyatt, and H. M. Ruppel, Phys. Rev. **128**, 826 (1962); M. H. Hull, K. E. Lassila, H. M. Ruppel, F. A. MacDonald, and G. Breit, *ibid.* **128**, 830 (1962).

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sity in the fitting program is too small. This may be a reflection of the fact that the parametrization in Eq. (21) results from a Gaussian dependence on radius in the nuclear wave functions which is known to fall off too fast outside the nucleus. In order to extract accurate $B(\lambda)$ values from the electron data by our procedure [or conversely to use measured $B(\lambda)$'s reduce the number of parameters in the expansion], it may be necessary to choose a different expansion in (21). On the other hand, for large q the oscillations of the Bessel function in (28) for r, also large, probably tend to cancel out the importance of the wave-function tails, insofar as fitting the electron data is concerned.

Of course, one can fit a set of data to an arbitrary degree of accuracy using the expansion in (21) by using enough powers of q. The point is, the first coefficient in the expansion cannot assume its "correct" value unless the expansion is of a form appropriate to the underlying physical situation. The form used also forces the transition densities obtained in (20) to have a Gaussian behavior for large r. Therefore, no matter how many powers of q are determined in the search, there will be a value of radius beyond which the $I_\lambda(r)$ obtained from the search will fall off too rapidly with radius.

It is not expected that the difficulties mentioned will have too much effect on the proton calculations at these energies. However, the calculations should be redone with Saxon-Woods functions, or more generally with basis functions falling off exponentially for large r. The GT prediction is too small. However, we have previously noted that an increase in normalization might be in order for the density we have used which would, if applied, render the GT fit acceptable and the Breit result too large.

The comparison of calculation and experiment for the polarization is essentially indeterminate as regards a choice between phase-shift sets. In the angular region 30°–40° where spin-orbit distortion effects are small, the data may favor Yale or HJ somewhat. Effects of distortion are not small, and are probably only qualitatively correct in these calculations, since the parameters of the spin-orbit part of the optical potential are not well determined.

Spin-flip contributions to the results should, as we have said, be small. The transition densities of Gillet and Vinh-Mau (Ref. 14) yield very small spin-flip densities for both these levels. In addition, all three two-nucleon amplitudes we have considered have small $\Delta S=1$ parts, so that inclusion of the spin-flip part of these transitions should produce small corrections.

IX. CONCLUSIONS

The inelastic electron and proton data for the two levels considered here seem to be consistent with one another, assuring us that the essential assumptions connecting the two processes are correct and that at least the main features of these two types of process are correctly treated in their separate descriptions. However, only qualitative statements regarding this consistency are supported by these results. Investigation of several levels in different nuclei would have been desirable since one is interested in the systematics of the comparison of the electron and proton data. It is likely that only such a survey could reveal, for instance, a definite preference for one set of nucleon-nucleon phase shifts or demonstrate the deficiency of all the sets available. Of course, other data exist. We have looked at some of the data on levels of ^{40}Ca, but the quality of the data and the relatively small number of points taken did not allow a meaningful search. One needs a sizable number of data points to make a search procedure such as we have adopted have much significance. The use of 17 points to determine three coefficients for the 3^- level of carbon is a debatable procedure statistically, and here we are on surer ground than for a heavier nucleus like calcium where the form-factor expansion may have to account for deviations from the underlying harmonic-oscillator representation.

The use of the electron-scattering form factors to define the proton interaction has been emphasized in the foregoing. Of course, the form factors determined from the search procedure are of value in their own right, especially if one is ultimately concerned with the shell-model behavior of these transitions which underlies the collective features most often emphasized in scattering analyses to date.

The utility and significance of the procedure outlined here are apparent. Active exploitation of this method must however await more experimental information, both for electron and proton scattering. It is hoped that in this way the inelastic-proton interaction can be reliably determined, both at the energies considered here and at lower energies where the impulse approximation is liable to fail. It is also hoped that the comparison of results for the two types of nuclear probe can finally yield a quantitative picture of the nuclear transition density with all that implies regarding the structure of the target nucleus.

ACKNOWLEDGMENTS

The authors are grateful for the comments of G. R. Satchler and H. McManus. We should also like to thank D. Slanina (Michigan State University) who provided us with a computer code to construct the two-nucleon t matrix in an appropriate form from the two-nucleon phase-shift sets.

APPENDIX A

The distorted waves $\chi_{mm'}{}^{(\pm)}(\mathbf{r})$ in Eq. (1) satisfy the Schrödinger equation

$$\left[\nabla^2+k^2-\frac{2\mu}{\hbar^2}(U+U_s\mathbf{L}\cdot\boldsymbol{\sigma})-V_c\right]\chi_{mm'}(r)=0, \quad \text{(A1)}$$

where the choice of + or − represents either outgoing or incoming boundary conditions on the distorted waves. k is the wave number of the relative motion of the target and projectile, μ is the reduced mass, and V_c is the Coulomb potential produced by a uniform spherical charge distribution of radius

$$R_c=r_cA^{1/3}, \tag{A2}$$

where A is the atomic weight of the target.

The optical potentials in (A1) are defined by

$$U=-V/(1+e^x)-iW/(1+e^{x'}), \tag{A3}$$

where

$$x=(r-r_0A^{1/3})/a \tag{A4}$$

and

$$x'=(r-r_0'A^{1/3})/a'. \tag{A5}$$

U_s is given by

$$U_s=-2(V_s+iW_s)\frac{1}{r}\frac{d}{dr}\left(\frac{1}{1+e^x}\right). \tag{A6}$$

The parameters used in the calculation presented here were $V=22.1$ MeV, $W=15.9$ MeV, $W_s=0$ MeV, $V_s=4.31$ MeV, $W_s=-0.11$ MeV, $r_0=0.902$ F, $a=0.452$ F, $r_0'=1.19$ F, $a'=0.556$ F, and $r_c=1.33$ F. For the present calculations Eq. (A1) was solved in a manner appropriate to the evaluation of Eq. (1) using an adaptation of the Oak Ridge code JULIE due to R. M. Drisko.

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Levels of F^{18} from the $O^{16}(He^3,p\gamma)F^{18}$ Reaction*

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Angular correlations in the $O^{16}(He^3,p\gamma)F^{18}$ reaction have been studied through two-parameter analyses of proton–gamma-ray coincidences at He^3 bombarding energies of 4.65, 5.40, and 6.40 MeV. Protons were detected by an annular solid-state detector centered at $\theta_p=180°$; gamma rays were detected with a NaI(Tl) spectrometer for five angles in the range $0°\leq\theta_\gamma\leq90°$. Branching ratios were determined for nine of the ten F^{18} levels of $3.4<E_{ex}<4.9$ MeV; the exception is the 4.74-MeV level, which was not observed. From analysis of angular-correlation data the following spin-parity limitations are obtained for the triplet of levels at $E_{ex}\sim3.8$ MeV: 3.72-MeV level ($J=1$); 3.79-MeV level ($J=1$, 2, or 3); and 3.84-MeV level ($J^\pi=2^{(+)}$). The preference for an even-parity assignment for the 3.84-MeV level is quite strong. The decay modes of the 3.79-MeV level are consistent with the assumption that this is the 3^- member of the group of odd-parity levels which includes those at 1.08 MeV (0^-), 2.10 MeV (2^-), and 3.13 MeV (expected 1^-). For those levels with $E_{ex}>4$ MeV, the correlation analysis provides restrictions on possible level spins and on multipole-mixing amplitudes for some of the principal deexcitation transitions. These results are consistent with other available information. The 4.65-MeV level of F^{18} is found to have $J\geq3$, and decays by transitions to the 0.94-MeV level ($J^\pi=3^+$) and to the 1.13-MeV level [$J^\pi=(5)^+$] with branching ratios of 15 and 85%, respectively. This appears to be the most likely candidate for the $J^\pi=4^+$, $T=1$ analog of the 3.55-MeV 4^+ state of O^{18}. The F^{18} 4.40-MeV level is also observed to decay to the 1.13-MeV level, and is a possible but less likely candidate for the analog of this O^{18} level.

I. INTRODUCTION

In a recent publication[1] we reported an investigation of the first 10 excited states of F^{18} ($E_{ex}<3.4$ MeV) from the $O^{16}(He^3,p\gamma)F^{18}$ reaction ($Q=2.021$ MeV). In the present paper we report an extension of these measurements into the range of excitation $3.4<E_{ex}<4.9$ MeV. For convenience, previously available[1–9] information on the F^{18} level structure is summarized in Fig. 1 for levels of $E_{ex}<4.9$ MeV. Excitation energies and branching ratios for those levels of $E_{ex}<4$ MeV are those summarized in a recent report[9] of high-resolution Ge(Li) studies of F^{18} gamma rays, and incorporate the earlier results of Refs. 1–8. The position of those levels of $E_{ex}>4$ MeV are taken primarily from the compilation of Ajzenberg-Selove and Lauritsen,[2] but include some results from the present work. Indicated spin assignments are from the information presented, or reviewed, in Refs. 1–9. We note in particular that the $J=0$ assignment for the 1.08-MeV level is from the results of

* Work performed under the auspices of the U. S. Atomic Energy Commission.

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