Furthermore, it seems to be necessary to consider corrections to these approximations if quantitative agreement is to be obtained at 142 MeV.

We believe that the present calculation somewhat favors the Yale phase-shift set at all energies. This conclusion must remain tentative until the corrections mentioned are considered in detail. It is, of course, possible that all of our fits to the data are fortuitous and that careful examination of the necessary corrections will show them to be major. This seems unlikely, however, in view of the fact that we obtain qualitative description of the data over large angular ranges as well as quantitative fits at small angles. Given the reasonableness of their approximations, as demonstrated in this paper, Saperstein and Feldman have shown that such qualitative fits to the data are *not* characteristic of all N-N phase-shift sets. The use of N-$\mathfrak{N}$ scattering to differentiate between N-N phase-shift sets, while not firmly proven, seems highly plausible at the present time.

PHYSICAL REVIEW VOLUME 156, NUMBER 4 20 APRIL 1967

Optical Potential Correlation Correction from Deuteron-Nucleus Scattering*

J. F. READING†

Laboratory for Nuclear Science and the Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 21 June 1966; revised manuscript received 7 November 1966)

To describe deuteron-nucleus scattering accurately at high energies, one has to correct the approximation that the potential which acts on the deuteron is the sum of the neutron and proton optical potentials. This correction is largely due to nuclear correlations and presents a method of determining the correction to the nucleon optical potential due to correlations. The method is applied to 650-MeV deuteron-carbon scattering. Good agreement is found between theory and experiment.

INTRODUCTION

ONE hopes that from nucleon-nucleus scattering at high energies one can get information about the two-body force and nuclear correlations. The nucleon is scattered in intermediate two-particle states "off its energy shell" and, with a wavelength of a fraction of a fermi, the strongly interacting nucleon multiply scattered from many target nucleons provides an excellent probe for nuclear correlations.[1,2] The off-energy-shell scattering leads to a nonlocality in the Watson potential. This nonlocality, which has been shown by Mulligan[3] and Reading[4] to contribute an important part to the potential, can be shown to be directly related to the derivative of the two-body T matrix for "going off the energy shell."[3,4] If this extremely important information can be extracted from the experimental data, we have the possibility of nucleon-nucleus scattering becoming an extremely important tool in the study of the two-body interaction.[4] Unfortunately, the situation is somewhat complicated by the nuclear correlations, which are expected[5] to give corrections of the order of l/R to the optical potential, where l is the correlation length and R is the nuclear radius. In this note is presented an experimental method for determining the correction to the optical model due to correlations. While some information is necessarily obtained about the correlation function, we should perhaps emphasize that this is not a method for obtaining that function, such as, for example, the methods discussed by Srivastava or Reiner.[6] The pair correlation function enters the optical potential as part of an integrand which is integrated over all the two-body space. There are contributions to this function both from short-range correlations due to the repulsive core, and from long-range correlations due to the exclusion principle and attractive forces. The correlation function for a repulsive core oscillates as the nucleons try to form a crystalline structure, but on the whole it tends to work with the exclusion principle to keep the particles apart whilst the attractive forces

* This work is supported in part through funds provided by the U. S. Atomic Energy Commission under contract AT(30-1)-2098.

† Present address: Department of Physics, University of Washington, Seattle, Washington.

[1] J. S. Levinger, Phys. Rev. **84**, 43 (1951); J. Heidmann, Phys. Rev. **80**, 171 (1951); J. Dabrowski and J. Sawicki, Nucl. Phys. **13**, 621 (1959); J. Dabrowski, Proc. Phys. Soc. (London) **71**, 658 (1958); G. M. Shklyarevsky, Zh. Eksperim. i Teor. Fiz. **41**, 234 (1961); **41**, 451 (1961) [English transl.: Soviet Phys.—JETP **14**, 170 (1961); **14**, 324 (1961)]; Tokuo Terasawa, Nucl. Phys. **39**, 563 (1962); T. I. Kopakishrili and R. I. Jubuti, *ibid.* **44**, 34 (1963); Hidetsugu Ikegami and Takeshi Udagawa, Phys. Rev. **133**, B1388 (1964).

[2] B. W. Reisenfeld and K. N. Watson, Phys. Rev. **102**, 1151 (1956); H. A. Bethe, Ann. Phys. (N. Y.) **3**, 190 (1958); A. K. Kerman, H. McManus, and R. M. Thaler, *ibid.* **8**, 551 (1959).

[3] B. Mulligan, Ann. Phys. (N. Y.) **26**, 159 (1964).

[4] J. F. Reading, Phys. Letters **20**, 518 (1966); J. F. Reading, following paper, Phys. Rev. **156**, 1116 (1967).

[5] R. Glauber, *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1958), Vol. 1.

[6] Y. N. Srivastava, Bull. Am. Phys. Soc. **9**, 15 (1964); A. Reiner (private communication).

keep them together. Thus there is a great deal of cancellation involved, which makes it desirable to determine the correlation correction experimentally rather than calculate it from some more basic premise[7]—though of course once the answer is found one can compare it with such calculations.

The trouble with determining the nuclear correlation correction comes when comparing a theoretical calculation with experiment. A calculation of the elastic differential cross-section using the zero-range approximation for the nucleon-nucleon force, while reproducing the general details of the experiment, typically differs from it by about 10% at small angles, and this disagreement tends to get worse at large-angle scattering. The difficulty is to determine what causes this. One has the choice of nonlocality, finite range, correlations, or the failure of the impulse approximation at the two-body level.

We propose to compare deuteron-nucleus scattering with nucleon-nucleus scattering. Given the nucleon amplitude for elastic scattering, Glauber[5] has shown how to derive the deuteron differential cross section. The derivation depends only on the validity of the high-energy approximation applied to the optical potential. As the potential is well behaved with no singularities, has a range of a few fermis, and can be considered local,[4] there is no difficulty with the derivation. If deuteron scattering can be described by scattering from the sum of the neutron and proton potentials V_n and V_p, this method should give extremely good answers. It has been applied by Franco and Glauber[8] to deuteron-proton scattering and found to work well. It has been applied to deuteron-nucleus scattering[9–11] and found to be bad. It is suggested that the reason for this is that deuteron scattering cannot be described by the sum of the proton and neutron potentials—a well-known fact that has been pointed out by Zamick[9] and Mcauley,[12] and also by Campbell and Kerman.[11] It will always be true whenever the deuteron interacts with a system which has internal degrees of freedom, and the deuteron can excite the system to an intermediate state without being broken up.[13] For an independent-particle model of the nucleus, the correction is of order $\sigma_T/2\pi r_d^2$, where σ_T is the nucleon-nucleon total cross section and r_d is the radius of the deuteron. This is usually a negligibly small correction at high energies, when $\sigma_T \sim 40$ mB—which is just a statement of the fact that the deuteron is so loosely bound that it is usually broken up when it excites a nucleus. However, nuclear correlations increase the correction by a factor of 5, and we use this fact to study them. As the correlations represent such a large proportion of this correction term, it is hoped that this will provide a useful method for sorting out the various corrections to the nucleon-nucleus case.

The method has been applied to deuteron-carbon scattering at 650 MeV. Assuming that the only other correction term for nucleon-nucleus scattering is due to the nonlocality, and that this term is well reproduced by a hard-core potential, we can calculate a correlation correction. This is in agreement with the correlation correction that is needed to fit the deuteron data.[10]

DEUTERON-NUCLEUS SCATTERING

The derivation given here follows closely that for proton-nucleus scattering given in Glauber's paper,[5] and also the derivation given by Zamick.[9] Further details, together with a complete discussion of the high-energy approximation and its application to scattering from compound systems, may be found in Ref. 5.

In the following, it is assumed that the high-energy approximation can be applied to nucleon-nucleon scattering. This is not true because of the hard core. Most of what we derive here can be obtained in ways which do not depend on the high-energy approximation. However, this is undoubtedly a problem which should be looked into before the method is applied to studying the nucleon-nucleon interaction. At the moment we have a theory which is only strictly valid for soft-core potentials and sufficiently high energies.

We use the following notation. In cylindrical coordinates the position vectors of the proton, neutron, and the jth nuclear particle in the nucleus of A particles are

$$\mathbf{P}=(\mathbf{p},z_p)\,,\quad \mathbf{N}=(\mathbf{n},z_n)\,,\quad \boldsymbol{\xi}_j=(\mathbf{s}_j,z_j)\,,$$

respectively, with center-of-mass and relative coordinates of the deuteron satisfying the usual identities,

$$\mathbf{R}=(\mathbf{B},Z)=\tfrac{1}{2}(\mathbf{P}+\mathbf{N})\,,\quad \mathbf{r}=(\mathbf{b},z)=\mathbf{P}-\mathbf{N}.$$

Elastic scattering amplitudes will be denoted by $F_p(Q)$ or $f_p(q)$, the subscript p, n, d denoting neutron, proton, and deuteron amplitudes, respectively; F for interactions with a nucleus, and f for interactions with a nucleon. The only difference we shall consider between f_n and f_p is that due to the Coulomb force, and we shall neglect this when convenient. (We shall use nucleon and neutron interchangeably.) Deuteron momentum transfers are denoted by Q,

$$Q=2K\sin\tfrac{1}{2}\theta_d\,,$$

where θ_d is the deuteron scattering angle and K is the wave number. We shall always be concerned with nucleon scattering at wave numbers of $K/2$ and

[7] R. T. Eden and V. T. Emery, Proc. Roy. Soc. (London) **242**, 266 (1958), **253**, 177, 186 (1959); K. Gottfried, Nucl. Phys. **5**, 557 (1948).
[8] V. Franco and R. Glauber, Phys. Rev. **142**, 1195 (1966).
[9] L. Zamick, Ann. Phys. (N. Y.) **21**, 551 (1963).
[10] L. M. C. Dutton, J. D. Jafar, H. B. Van der Raay, D. G. Ryan, J. A. Stiegelmair, R. K. Tandon, and J. F. Reading, Phys. Letters **16**, 331 (1965).
[11] A. K. Kerman and L. Campbell (unpublished).
[12] G. Mcauley (private communication).
[13] E. S. Abers, H. Burkhardt, V. L. Teplitz, and C. Wilkin, Nuovo Cimento **42**, 365 (1966).

momentum transfers q, where

$$q=2k\sin\tfrac{1}{2}\theta_n=K\sin\tfrac{1}{2}\theta_n\,,$$
$$2k=K\,.$$

In the high-energy approximation we have the quantities Γ_n, x_n which are related to f_n and each other by the following equations:

$$f_n(q)=\frac{ki}{2\pi}\int e^{-i\mathbf{q}\cdot\mathbf{b}}\Gamma_n(b)d^2b$$

$$\Gamma_n(b)=1-e^{ix_n(b)}=\frac{1}{2\pi ki}\int e^{i\mathbf{q}\cdot\mathbf{b}}f_n(q)d^2q\,.$$

The complete set of nuclear wave functions ψ_m, with $m=0$ denoting the ground state, are written as

$$\psi_m=|m\rangle=\psi_m(\xi_1,\cdots,\xi_j,\cdots,\xi_A)\,.$$

The nuclear density functions are defined in the usual way as

$$\rho^{(A)}(\xi_1,\xi_2,\cdots,\xi_A)=|\psi_0(\xi_1,\xi_2,\cdots,\xi_A)|^2$$

$$\rho^{(j-1)}(\xi_1,\xi_2,\cdots,\xi_{j-1})=\int\rho^j(\xi_1,\xi_2,\cdots,\xi_j)d^3\xi_j\,.$$

For convenience we introduce the quantities n, $n^{(2)}$ such that

$$n(\xi)=A\rho^{(1)}(\xi)\equiv A\rho(\xi)$$
$$n^{(2)}(\xi_1,\xi_2)=A(A-1)\rho^{(2)}(\xi_1,\xi_2)\,.$$

As usual for the limit of large A, the correlation function g is defined as

$$n^{(2)}(\xi_1,\xi_2)\approx n(\xi_1)n(\xi_2)g(\xi_1-\xi_2)\,.$$

Finally, the deuteron wave function is $\phi(r)$.

The deuteron scattering amplitude can be written in the high-energy approximation as

$$F_d(Q)=\frac{K}{2\pi i}\int e^{-i\mathbf{Q}\cdot\mathbf{B}}|\phi(r)|^2\times\left[|\psi_0|^2\left\{\exp\left[i\sum_{j=1}^{A}x_p(\mathbf{p}-\mathbf{s}_j)+i\sum_{j=1}^{A}x_n(\mathbf{n}-\mathbf{s}_j)\right]-1\right\}\prod_{j=1}^{A}d^3\xi_j\right]d^3r d^2B\,. \quad (1)$$

The first term in square brackets in the integrand of Eq. (1) can be written as

$$\left\langle 0\left|\exp\left(i\sum_{j=1}^{A}x_p(\mathbf{p}-\mathbf{s}_j)\right)\exp\left(i\sum_{j=1}^{A}x_n(\mathbf{n}-\mathbf{s}_j)\right)\right|0\right\rangle. \quad (2)$$

A similar equation to Eq. (1) for the neutron-nucleus amplitude $F_n(q)$ is

$$F_n(q)=\frac{k}{2\pi i}\int e^{-i\mathbf{q}\cdot\mathbf{n}}d^2n\times\left[|\psi_0|^2\{\exp[i\sum_j x_n(\mathbf{n}-\mathbf{s}_j)]-1\}\prod_{j=1}^{A}d^3\xi_j\right]$$
$$=\frac{k}{2\pi i}\int e^{-i\mathbf{q}\cdot\mathbf{n}}\{\exp[ix_n^{\text{op}}(\mathbf{n})]-1\}d^2n\,, \quad (3)$$

where we have written

$$\exp[ix_n^{\text{op}}(\mathbf{n})]=\langle 0|\exp[i\textstyle\sum_j x_n(\mathbf{n}-\mathbf{s}_j)]|0\rangle$$

and $x_n^{\text{op}}(\mathbf{n})$ is the phase given by the optical potential $V_n(\mathbf{N})$.

$$x_n^{\text{op}}(\mathbf{n})=\frac{-1}{hv}\int_{-\infty}^{+\infty}V_n(\mathbf{n},z)dz\,, \quad (4)$$

where v is the velocity of the neutron.

The usual approximation is to write

$$\begin{aligned}\exp(ix_d)&\equiv\langle 0|\exp[i\textstyle\sum_j x_n(\mathbf{n}-\mathbf{s}_j)]\\&\quad\times\exp[i\textstyle\sum_j x_p(\mathbf{p}-\mathbf{s}_j)]|0\rangle\\&\equiv\textstyle\sum_m\langle 0|\exp[i\sum_j x_n(\mathbf{n}-\mathbf{s}_j)]|m\rangle\\&\quad\times\langle m|\exp[i\textstyle\sum_j x_p(\mathbf{p}-\mathbf{s}_j)]|0\rangle\\&\approx\langle 0|\exp[i\textstyle\sum_j x_n(\mathbf{n}-\mathbf{s}_j)]|0\rangle\\&\quad\times\langle 0|\exp[i\textstyle\sum_j x_p(\mathbf{p}-\mathbf{s}_j)]|0\rangle\\&=\exp[ix_n^{\text{op}}(\mathbf{n})]\exp[ix_p^{\text{op}}(\mathbf{p})]\,,\end{aligned}$$

or

$$x_d\approx x_p^{\text{op}}(\mathbf{p})+x_n^{\text{op}}(\mathbf{n})\,. \quad (5)$$

This is, as can be seen from Eq. (4), equivalent to writing the potential acting on the deuteron as the sum of the neutron and proton potentials.

The approximation explicitly neglects all intermediate excited states. The physical reason for hoping this is a good approximation is that the deuteron cannot excite the nucleus without a high probability of being broken up because it is so loosely bound.

We have

$$\langle 0|\exp[i\sum_j x_n(\mathbf{n}-\mathbf{s}_j)]\exp[i\sum_j x_p(\mathbf{p}-\mathbf{s}_j)]|0\rangle$$
$$=\langle 0|\prod_j[1-\Gamma_n(\mathbf{n}-\mathbf{s}_j)]\prod_{j'}[1-\Gamma_p(\mathbf{p}-\mathbf{s}_{j'})]|0\rangle.$$

Thus x_d is given by a sum of terms $t_{uu'}$, where

$$t_{uu'}=\langle 0|\Gamma_n(\mathbf{n}-\mathbf{s}_j)\cdots\Gamma_n(\mathbf{n}-\mathbf{s}_u)\Gamma_p(\mathbf{p}-\mathbf{s}_{j'})\cdots\Gamma_p(\mathbf{p}-\mathbf{s}_{u'})|0\rangle.$$

As Γ is nothing more than the two-body T matrix in configuration space, we see that the above term represents a process in which the neutron interacts with u, and the proton with u', target nucleons. Assuming an *independent*-particle model for the nucleus, we see that only terms for which the proton and neutron scatter from the same target nucleon are approximated by neglecting the intermediate excited states. This statement follows from merely noting that for an independent model

$$t_{uu'}\equiv\langle 0|\Gamma_n(\mathbf{n}-\mathbf{s}_j)\cdots\Gamma_n(\mathbf{n}-\mathbf{s}_u)|0\rangle \times\langle 0|\Gamma_p(\mathbf{p}-\mathbf{s}_{j'})\cdots\Gamma_p(\mathbf{p}-\mathbf{s}_{u'})|0\rangle$$

if none of the labels $j, k, \cdots, u$ are equal to any of the labels $j', k', \cdots, u'$. It is a peculiarity, and one that is often missed, of the high-energy approximation that an incident particle never interacts with the same nucleon twice. To suppose that it does so, when one assumes forward scattering together with the assumption that the wavelength of the incident particle is much less than the average spacing between the target nucleons, would be a contradiction in terms. (This is the reason we limit ourselves to double scattering events in discussing deuteron-potential scattering.) The incident particle is moving so fast that the target nucleons are frozen in position. After interacting with one of them, the incident particle travels on in the forward direction, as it does in all subsequent interactions, and it is not possible for it to interact with the same nucleon twice. We have called this effect a kinematic correlation and have discussed it in detail elsewhere.[14] However, one consequence of the effect is that those terms of the type we have been considering which have any of the labels $j, k, \cdots, u, j', k', \cdots, u'$ repeated are smaller by a numerical factor of uu'/A than those for which the label is not repeated if we assume that $u, u'\ll A$.

As the latter condition is very well satisfied in practice because of the essential *weakness* of the two-body T matrix together with its short range, we are making an approximation by neglecting the intermediate excited states in terms whose numerical weight is small. However, for a correlated nuclear model this condition is no longer satisfied, and we can expect nonnegligible corrections in this case, which therefore come largely from correlations. Thus in $(x_d-x_p^{\text{op}}-x_n^{\text{op}})$ we have a quantity which is easily measurable and is directly related to the correlation function.

Returning to Eq. (1), which can be written as

$$F_d(Q)=\frac{K}{2\pi i}\int e^{-i\mathbf{Q}\cdot\mathbf{B}}|\phi(r)|^2(e^{ix_d}-1)d^2Bd^3r, \qquad (6)$$

we have for x_d

$$x_d=-i\ln\int\rho^{(A)}(\xi_1,\cdots\xi_A) \times\sum_{j=1}^{A}\left[1-\Gamma_p(\mathbf{p}-\mathbf{s}_j)-\Gamma_n(\mathbf{n}-\mathbf{s}_j) +\Gamma_n(\mathbf{n}-\mathbf{s}_j)\Gamma_p(\mathbf{p}-\mathbf{s}_j)\right]\prod_j d^3\xi_j.$$

[14] J. F. Reading, Ph.D. thesis, Birmingham, United Kingdom (unpublished).

Expanding the logarithm, keeping just two-body correlations, we obtain

$$\begin{aligned}x_d=i\int n(\xi)&\times[\Gamma_p(\mathbf{p}-\mathbf{s})+\Gamma_n(\mathbf{n}-\mathbf{s})-\Gamma_p(\mathbf{p}-\mathbf{s})\Gamma_n(\mathbf{n}-\mathbf{s})]d^3\xi\\ &-\tfrac{1}{2}\int n(\xi_1)n(\xi_2)[g(\xi_1-\xi_2)-1]\\ &\times[\Gamma_p(\mathbf{p}-\mathbf{s}_1)+\Gamma_n(\mathbf{n}-\mathbf{s}_1)-\Gamma_p(\mathbf{p}-\mathbf{s}_1)\Gamma_n(\mathbf{n}-\mathbf{s}_1)]\\ &\times[\Gamma_p(\mathbf{p}-\mathbf{s}_2)+\Gamma_n(\mathbf{n}-\mathbf{s}_2)-\Gamma_n(\mathbf{n}-\mathbf{s}_2)\Gamma_p(\mathbf{p}-\mathbf{s}_2)]\\ &\times d^3\xi_1d^3\xi_2.\end{aligned}$$

This expresses x_d in terms of the two-body amplitude functions Γ. Thus, in principle, we can calculate x_d directly from two-body data. We break x_d up into four parts.

$$x_d=x_n^{\text{op}}(n)+x_p^{\text{op}}(p)+S+C. \qquad (7)$$

We have, by comparison with Eq. (3),

$$x_n^{\text{op}}(\mathbf{n})=i\int n(\xi)\Gamma_n(\mathbf{n}-\mathbf{s})d^3\xi-\tfrac{1}{2}i\int n(\xi_1)n(\xi_2) \times[g(\xi_1-\xi_2)-1]\Gamma_n(\mathbf{n}-\mathbf{s}_1)\Gamma_n(\mathbf{n}-\mathbf{s}_2)d^3\xi_1d^3\xi_2, \qquad (8)$$

and a similar expression for $x_p^{\text{op}}(\mathbf{p})$. The two correction terms to Eq. (5) are S and C, where as a first approximation

$$S=-i\int n(\xi)\Gamma_p(\mathbf{p}-\mathbf{s})\Gamma_n(\mathbf{n}-\mathbf{s})d^3\xi$$

and

$$C=-i\int n(\xi_1)n(\xi_2)[g(\xi_1-\xi_2)-1] \times\Gamma_n(\mathbf{n}-\mathbf{s}_1)\Gamma_p(\mathbf{p}-\mathbf{s}_2)d^3\xi_1d^3\xi_2.$$

Let us first study S, which is present whether or not there are any correlations in the nucleus. The term represents scattering processes in which both the neutron and proton interact with the same target nucleon. As such, it may be interpreted as an internal shadow effect. It will contribute a correction which is roughly the same size as the shadow effect in deuteron-nucleon scattering. Ignoring the correlation term in $x_n^{\text{op}}(\mathbf{n})$ and assuming zero range for the nucleon-nucleon interaction, we obtain

$$x_n^{\text{op}}(\mathbf{n})=\frac{2\pi f_n(0)A}{k}\int_{-\infty}^{+\infty}\rho(\mathbf{n},z)dz.$$

In the same approximation

$$S=\frac{iA}{k^2}\int_{-\infty}^{+\infty}\rho(\mathbf{B},z)dz\int e^{i\mathbf{q}\cdot\mathbf{b}}f_p(\mathbf{q})f_n(\mathbf{q})d^2q.$$

To get some idea of the magnitude and sign of S, we assume f_p is mostly imaginary and $f_p(q)$ is given by

$$f_p(\mathbf{q})=f_n(\mathbf{q})=f(0)\exp(-q^2a^2/4).$$

Then,

$$S=\frac{A}{k^2}[\operatorname{Im}f(0)]^2\frac{2\pi}{a^2}\left[\exp\left(-\frac{b^2}{2a^2}\right)\right]\int_{-\infty}^{+\infty}\rho(\mathbf{B},z)dz.$$

We see that $x_n^{\mathrm{op}}(\mathbf{n})$, or equivalently $x_n^{\mathrm{op}}(\mathbf{B}-\mathbf{b}/2)$, has the same shape as S, as a function of B, when $b=0$. The ratio of the strength of these two terms when the deuteron radius is much smaller than the nuclear size is given by

$$\frac{S}{x_n^{\mathrm{op}}}=\frac{\sigma_T}{4\pi a^2}\exp\left(-\frac{b^2}{2a^2}\right). \qquad (9)$$

Because in Eq. (6), where this term enters, we still have an integral over B to do, S contributes a term of order $\sigma_T/2\pi r_d^2$ to the integral. This is small at high energies, but may not be completely negligible at low energies, if one attempts to apply the impulse approximation there. As we have written it, the ratio of S to x_n^{op} has been determined in terms of σ_T, the two-body cross section. We can equally well find it in terms of the optical potential V_n. Assuming the nucleus has a square-well shape—which is not a bad approximation for most nuclei—and that the optical potential has a strength V_0 with a radius $R(=R_0A^{1/3})$—we obtain

$$\frac{S}{x_n^{\mathrm{op}}}=\frac{\sigma_T}{4\pi a^2}\exp\left(-\frac{b^2}{2a^2}\right)=\frac{8V_0\hbar^2R_0^3}{3kma^2}\exp\left(-\frac{b^2}{2a^2}\right).$$

This puts the correction in terms of the optical potential which is directly obtainable from the scattering amplitude F_n. At 300 MeV, we have $\sigma_T/4\pi a^2\approx 0.1$.

We now turn out attention to C, the remaining term in Eq. (6). This term is only present if there are nuclear correlations.

$$C=-i\int n(\xi_1)n(\xi_2)[g(\xi_1-\xi_2)-1]\times\Gamma_n(\mathbf{n}-\mathbf{s}_1)\Gamma_p(\mathbf{p}-\mathbf{s}_2)d^3\xi_1d^3\xi_2. \qquad (10)$$

This should be compared with the correlation term in $x_n^{\mathrm{op}}(\mathbf{n})$, the last term in Eq. (8), which we call C_N.

$$C_N=-\tfrac{1}{2}i\int n(\xi_1)n(\xi_2)[g(\xi_1-\xi_2)-1]\times\Gamma_n(\mathbf{n}-\mathbf{s}_1)\Gamma_n(\mathbf{n}-\mathbf{s}_2)d^3\xi_1d^3\xi_2. \qquad (11)$$

We see that C_N and C are very similar. A good value of either is a good estimate of the other. In describing the correction C_N to the optical potential due to correlations, it is not necessary to find g, however interesting that function may be for other purposes.

In fact, writing C as a function of B and b, we have $2C_N(B)=C(B,0)$.

If we assume the correlation function and the two-body force have a range small compared to the radius of the nucleus, we have

$$C=-i\int_{-\infty}^{+\infty}[n(\mathbf{B},Z)]^2dZ\int[g(\mathbf{s}_1-\mathbf{s}_2,z)-1]\times\Gamma_n(\mathbf{n}-\mathbf{s}_1)\Gamma_p(\mathbf{p}-\mathbf{s}_1)d^2s_1d^2s_2dz.$$

Given a square-well distribution for the nucleus, we have

$$\rho(r)=\rho^2(r)4\pi R^3/3.$$

This gives

$$\frac{C}{x_n^{\mathrm{op}}(B)}=-i\frac{3k}{2\pi f(0)4\pi R_0^3}\int[g(\mathbf{s}_1-\mathbf{s}_2,z)-1]\times\Gamma_n(\mathbf{n}-\mathbf{s}_1)\Gamma_p(\mathbf{p}-\mathbf{s}_2)d^2s_1d^2s_2.$$

The assumption

$$g(r)=1+e^{-r^2/l^2} \qquad (12)$$

gives as an estimate of the ratio C/x_n^{op},

$$\frac{C}{x_n^{\mathrm{op}}(B)}=-\frac{\sigma_T}{4\pi(a^2+\frac{1}{2}l^2)}\exp\left(\frac{-b^2}{2(a^2+\frac{1}{2}l^2)}\right)\frac{3\pi l^3}{4R_0^3}. \qquad (13)$$

With such a crude expression for $g(r)$, Eq. (12), we should not take Eq. (13) seriously or use it to calculate l and expect to get a reasonable answer for a correlation length. Attractive forces give positive values for $g-1$, while repulsive forces and the exclusion principle tend to give negative values, though the correlation function for repulsive forces tends to oscillate as the nucleons try to build up a crystalline type of structure. Thus in the integral of Eq. (10) there will be a great deal of cancellation. With our limited theoretical knowledge of what the correlation function is, it is essential, therefore, to have an experimental measurement of $C(B,0)$ rather than to rely on calculating it from some more fundamental approach.[7] Once we have $C(B,0)$, we can calculate the correction to the optical potential due to correlations. With l presumably of the order of R_0, we see from Eq. (13) that such a correction may by no means be negligible.

APPLICATION TO DEUTERON-CARBON SCATTERING

We have applied the method to analyze the experimental data of Dutton *et al.*[10] on deuteron-carbon scattering and have compared the results with the data of Ashmore *et al.*[15] for neutron scattering.

In Fig. 1 we plot the deuteron elastic differential cross section. The broken line is calculated under the assumption that

$$x_d=x_p^{\mathrm{op}}+x_n^{\mathrm{op}}.$$

[15] A. Ashmore, D. S. Mather, and S. K. Sen, Proc. Phys. Soc. (London) **71**, 552 (1958).

It was assumed that $F_n(q)=F_p(q)$ and that both were purely imaginary and could be represented by a single Gaussian. The agreement with experiment is fairly good, but the calculated curve is consistently higher than the experimental one.

We next try to fit the experimental elastic differential cross section with one parameter y, where now we use

$$x_d=x_p{}^{\text{op}}(\mathbf{p})+x_n{}^{\text{op}}(\mathbf{n})+\frac{y}{L^2}x_n{}^{\text{op}}(\mathbf{B})\exp\left(-\frac{b^2}{2L^2}\right).$$

The quantity L would be given by $L^2=a^2+\frac{1}{2}l^2$, if we were to make the Gaussian behavior assumption above.

The solid line in Fig. 1 represents a calculation for which y was chosen to be 1.0 F^2. The calculation is insensitive to the choice of L; it was taken to be 1.4 F. Only linear powers of y/L^2 were kept, and all contributions from $[1-\exp ix_n{}^{\text{op}}(n)]$ raised to powers higher than 2 were neglected. These approximations seem fairly reasonable, considering the others. The fit to the data is encouraging, considering that we only have one parameter.

However, as pointed out above, y can be determined from the correction to the optical potential due to correlations. Mulligan has already corrected the optical potential for the nonlocality. The correction to the imaginary part of the potential is small, and he gives the corrected imaginary part as 23 MeV. The experimental value of the optical potential is 15±5 MeV. This gives y as 1.6±1.0. Thus, at the moment, the deuteron and the nucleon data seem to be in reasonable agreement.

CONCLUSION

We have presented a method for determining the correction to the optical potential due to correlations. Once the difficulty of making this correction has been overcome, one can use nucleon-nucleus scattering to study the two-body T matrix. The method applied to deuteron-carbon scattering removes the discrepancy between proton-nucleus scattering and the Watson theory at 300 MeV.

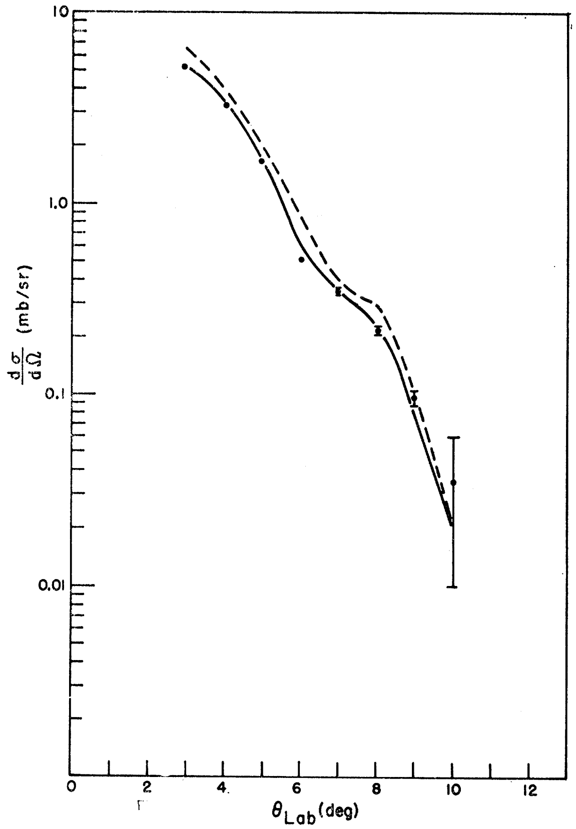

FIG. 1. Comparison of experimental data of Dutton *et al.* on elastic differential cross section for deuteron-carbon scattering with two calculations. Dashed line: $y=0.0$; solid line: $y=1.0$ F^2.

ACKNOWLEDGMENTS

I wish to thank Professor L. Castillejo for a long series of discussions on deuteron-nucleus scattering, and Professor L. Wilets for a useful conversation which prompted this work.