Double-Scattering Corrections to Medium- and High-Energy Inelastic Nucleon Scattering

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We calculate double-scattering corrections to the differential cross section for the inelastic scattering of nucleons from complex nuclei. The Watson formalism of multiple scattering is employed. Numerical computations are performed for the reaction $^{12}C(p,p')^{12}C^*$ (2^+ state, 4.43 MeV) for the incident proton energies $E_0=156$, 100, and 90 MeV. Our 2^+-state eigenvector is taken from a microscopic random-phase-approximation model calculation. The computed double-scattering corrections to $\sigma(\theta)$ are appreciable and negative at small θ (at 156 MeV of the order of $-10\sim-20\%$). They are rather sensitive to the details of the final-state nuclear wave function.

1. INTRODUCTION

MOST calculations of elastic and inelastic scattering of nuclear systems at medium and high energies as in the up-to-date literature are based on the usual or distortion-modified impulse approximation. This approximation includes only the leading term of the Watson series expansion of the actual many-body scattering matrix called the multiple-scattering series. The leading term is a simple sum of the t matrices describing the pair interactions of the projectile with the individual nucleons of the target (free two-body scatterings or, if a part of the effect of the medium of all the remaining target nucleons is included, we have two-body scatterings with a distortion effect). A rapid convergence of the Watson series is guaranteed only at quite high energies. In the case of elastic scattering, the double-scattering, target-exchange, and binding corrections are known to contribute importantly to the optical-model potential $\mathcal{V}_{\rm opt}$ at energies of the incoming nucleon $E_0\lesssim 50$ MeV. Some corrections to $\mathcal{V}_{\rm opt}$ in this case, corresponding to the first (double) cluster terms in the Watson series, have been only very roughly estimated.[1,2] (For the case of $\mathcal{V}_{\rm opt}$ for complex targets see also general discussions by Fowler and Watson,[3] by Johnston and Watson,[4] by Johnston,[5] and, most recently, by McDonald and Hull.[6]) Only very few rough calculations of such scattering corrections have been published on the elastic and quasielastic scattering from deuterons.[7,8]

From the experimental side there have been several examples of a situation where the impulse approximation is insufficient independently of the details of the nuclear form factor and for any "reasonable" nucleon-nucleon free scattering t matrix. One such example seems to be the polarization of 155-MeV protons inelastically scattered from C^{12} leaving the residual nucleus in its first excited state (2^+, $T=0$, 4.43 MeV), especially in the region of small scattering angles θ.[9]

In the following we shall present a calculation of the double-scattering correction in the sense of the Watson multiple-scattering series for a complex spherical nucleus. In spite of several drastic approximations employed, the final expressions are rather complex and cumbersome for numerical computations.

2. CALCULATION

Our first and basic approximation is that of neglecting the initial, intermediate, and final momenta of the target nucleons involved. This is valid certainly only for sufficiently high energies ($\gtrsim 150$ MeV). Our correction scattering amplitude ΔT corresponds to exciting the target nucleus from its ground state 0 to the final state f; the incident nucleon momentum is $\mathbf{k}_0$, and the final one of the same is $\mathbf{k}_0'$; at the beginning we suppress in our notation the spins involved, for the sake of simplicity. With the above we can write:

$$\langle \mathbf{k}_0'|\Delta T|\mathbf{k}_0\rangle \cong \frac{2m}{\hbar^2}A(A-1)\sum_{n\neq 0}\int d\mathbf{k}\langle\tfrac{1}{2}\mathbf{k}_0'|\mathbf{t}_{01}|\tfrac{1}{2}\mathbf{k}\rangle \times\left(k_0^2-k^2-\frac{2m}{\hbar^2}\Delta\epsilon_n+i\delta\right)^{-1}\langle\tfrac{1}{2}\mathbf{k}|\mathbf{t}_{02}|\tfrac{1}{2}\mathbf{k}_0\rangle \times\int d\mathbf{r}_1 e^{i\mathbf{q}\cdot\mathbf{r}_1}\rho_{nf}(\mathbf{r}_1)\int d\mathbf{r}_2 e^{i\mathbf{q}\cdot\mathbf{r}_2}\rho_{0n}(\mathbf{r}_2)\,. \quad (1)$$

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[1] N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953).
[2] J. Dabrowski and J. Sawicki, Nucl. Phys. **13**, 621 (1959).
[3] T. K. Fowler and K. M. Watson, Nucl. Phys. **13**, 549 (1959).
[4] R. R. Johnston and K. M. Watson, Nucl. Phys. **28**, 583 (1961).
[5] R. R. Johnston, Nucl. Phys. **36**, 368 (1962).
[6] F. A. McDonald and H. H. Hull, Jr., Phys. Rev. **143**, 838 (1966).
[7] A. Everett, Phys. Rev. **126**, 831 (1962).
[8] N. M. Queen, Nucl. Phys. **55**, 177 (1964); **66**, 673 (1965).
[9] B. Tatischeff *et al.*, Phys. Letters **16**, 282 (1965); M. Perrin and N. Vinh Mau, *ibid.* **14**, 236 (1965).

Here $\mathbf{t}_{0i}$ is the antisymmetrized t-matrix operator for the nucleon pair $(0,i)$; $\Delta\epsilon_n=\epsilon_n-\epsilon_0$ is the excitation energy of the target nucleus in the intermediate state n; $\mathbf{q}'=\mathbf{k}-\mathbf{k}_0'$, $\mathbf{q}=\mathbf{k}_0-\mathbf{k}$; and the one-nucleon density matrices are defined as

$$\rho_{0n}(\mathbf{r}_2)\equiv\int d(1)d(3)\cdots d(A)\times\psi_n{}^*(1,\mathbf{r}_2,3,\cdots,A)\psi_0(1,\mathbf{r}_2,3,\cdots,A)\,, \quad (2)$$

$$\rho_{nf}(\mathbf{r}_1)\equiv\int d(2)\cdots d(A)\times\psi_f{}^*(\mathbf{r}_1,2,\cdots,A)\psi_n(\mathbf{r}_1,2,\cdots,A)\,. \quad (3)$$

It is only thanks to the neglect of the momenta of the target nucleons 1 and 2 relative to $\mathbf{k}_0$, $\mathbf{k}$, and $\mathbf{k}_0'$ of the scattering nucleon 0, that we obtain nuclear form factors as Fourier transforms of $\rho_{ab}(\mathbf{r}_i,\mathbf{r}_i)\equiv\rho_{ab}(\mathbf{r}_i)$.[10] Otherwise, the form factors are transforms of the single-nucleon density matrices "mixed" in the coordinates, $\rho_{ab}(\mathbf{r}_i,\mathbf{r}_j)$, as in the exact original expressions, which are in our approximation reduced to their diagonal components only.

This equation corresponds to Eq. (4.5) of Johnston and Watson[4] except that their final ground state $\langle 0|$ is replaced by an excited state f (inelastic scattering). It corresponds also to Eq. (3) of Ref. 6 in the case of the optical model. This is a variant of the Watson multiple-scattering series which probably converges rapidly. The exclusion of the ground state 0 from the intermediate states n is related to a "coherent scattering" (optical-model) distortion operator (it has only diagonal matrix elements) in the propagators for the states n. Such propagators are called e by Watson (compare, e.g., Ref. 1). We also consider a multiple-scattering series based on Watson's propagators a (without the distortion potential) and without the projection operator excluding the ground state from $\{|n\rangle\}$. In the former variant, the distortion operator is treated as if it were a small perturbation, and as if it could be neglected (one retains the zero-order terms of an expansion). It means that in Eq. (1) we use the Watson series expansion with the propagators unperturbed for the particle 0; in other words, the basic states for the particle 0 are plane waves rather than any distorted waves.

Our second approximation is closure. For a light nucleus it may be that only one level or a group of levels n are important for our reaction channel so that one can replace $\Delta\epsilon_n$ in Eq. (1) by a corresponding fixed $\Delta\epsilon_{n0}$ appropriate to a given incident energy E_0; for a heavy nucleus with a high level density, one can replace $\Delta\epsilon_{n0}$ by an "average" $\langle\Delta\epsilon\rangle_{\text{av}}$; if E_0 is high enough we can even neglect the nuclear excitation energy altogether $(\langle\Delta\epsilon\rangle_{\text{av}}\ll E_0,\hbar^2k^2/2m)$. The summation over n yields then:

$$\langle\mathbf{k}_0'|\Delta T|\mathbf{k}_0\rangle\cong\frac{2m}{\hbar^2}A(A-1)\times\int d\mathbf{k}\left(k_0{}^2-k^2-\frac{2m}{\hbar^2}\langle\Delta\epsilon\rangle_{\text{av}}+i\delta\right)^{-1}\langle\tfrac{1}{2}\mathbf{k}_0'|\mathbf{t}_{01}|\tfrac{1}{2}\mathbf{k}\rangle\times\langle\tfrac{1}{2}\mathbf{k}|\mathbf{t}_{02}|\tfrac{1}{2}\mathbf{k}_0\rangle F(\mathbf{q},\mathbf{q}')\,, \quad (4)$$

where

$$F(\mathbf{q},\mathbf{q}')\equiv\int d\mathbf{r}_1d\mathbf{r}_2e^{i(\mathbf{q}'\cdot\mathbf{r}_1+\mathbf{q}\cdot\mathbf{r}_2)}[\rho_{0f}{}^{(2)}(\mathbf{r}_1,\mathbf{r}_2)-\rho_{0f}(\mathbf{r}_1)\rho_{00}(\mathbf{r}_2)]\,, \quad (5)$$

$$\rho_{0f}{}^{(2)}(\mathbf{r}_1,\mathbf{r}_2)\equiv\int d(3)d(4)\cdots d(A)\times\psi_f{}^*(\mathbf{r}_1,\mathbf{r}_2,3,\cdots A)\psi_0(\mathbf{r}_1,\mathbf{r}_2,3,\cdots A)\,. \quad (5')$$

The quantity $\rho_{0f}{}^{(2)}$ is a $(0\rightarrow f)$ two-nucleon density matrix.

If we exclude in our $\langle\mathbf{k}_0'|\Delta T|\mathbf{k}_0\rangle$ only the initial target (ground) state in the summation over intermediate states, this corresponds to what one obtains from the multiple-scattering expansion as given, e.g., by Watson[11] and Takeda and Watson[12] or to the expansion given in pp. 790–791 of Ref. 13 and in Refs. 4–6. If no such exclusion is made, we omit the term $\rho_{0f}\rho_{00}$ in Eq. (5). In the following we concentrate on the case of a spherical even-even nucleus. Our final state f is supposed to be made of a superposition of particle-hole pairs coupled to a definite spin J_f (projection M_f) and isospin T_f (projection T_{3f}); i.e., our excitations are described by spherical shell-model configuration mixing as given by the random phase approximation (RPA) or the Tamm-Dancoff (TD) methods. Using second quantization, we can write in this case

$$\rho_{0f}{}^{(2)}(1,2)=\frac{1}{A(A-1)}\sum_{\alpha\alpha'}\sum_{\beta\beta',\gamma\gamma'}(-)^{j_\alpha-m_\alpha}(j_{\alpha'}j_\alpha;m_{\alpha'}-m_\alpha|J_fM_f)\times(-)^{\frac{1}{2}-t_\alpha}(\tfrac{1}{2}\,\tfrac{1}{2};t_{\alpha'}-t_\alpha|T_fT_{3f})\chi^{(f)*}(\alpha\alpha')\langle\tilde{O}|c_\alpha{}^\dagger c_{\alpha'}c_{\beta'}{}^\dagger c_{\gamma'}{}^\dagger c_\gamma c_\beta|\tilde{O}\rangle\varphi_{\beta'}{}^*(1)\varphi_{\gamma'}{}^*(2)\varphi_\beta(1)\varphi_\gamma(2)\,, \quad (6)$$

where $\{\varphi_\alpha(1)\}$ is a complete set of single-particle wave functions of the shell model; the $\chi^{(f)}(\alpha\alpha')$ are the hole-particle components of the given eigenvector f; $|\tilde{O}\rangle$ is the "vacuum" (ground state) correlated or uncorrelated for

[10] Actually, the "density matrix" $\rho_{ab}(\mathbf{r})$ is usually defined with the extra normalization factor A in front of the integral $\int d(2)\cdots d(A)$ of Eqs. (2)–(3).
[11] K. M. Watson, Phys. Rev. **89**, 575 (1953).
[12] G. Takeda and K. M. Watson, Phys. Rev. **97**, 1336 (1955).
[13] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964).

the RPA and TD cases, respectively. The $\langle\tilde{O}|\cdots|\tilde{O}\rangle$ element is evaluated by contractions in the sense of the Hartree-Fock ground state $|\tilde{O}\rangle$ as is usual in the RPA method. This "recipe" constitutes our "first variant."

For a general spin and isospin component, we can write in this case our nuclear form factor of Eq. (5) in the form:

$$\langle s_1't_1',s_2't_2'|F(\mathbf{q},\mathbf{q}')|s_1t_1,s_2t_2\rangle=\frac{1}{A(A-1)}\sum_{\nu\nu'}\sum_{\beta(\mathrm{occ})}\chi^{(f)*}(\nu\nu')(-)^{j_\nu-m_\nu+\frac{1}{2}-t_\nu}(j_{\nu'}j_\nu;m_{\nu'}-m_\nu|J_fM_f)$$
$$\times(\tfrac{1}{2}\tfrac{1}{2};t_{\nu'}-t_\nu|T_fT_{3f})[-E_{\nu'\beta,\beta\nu}+E_{\beta\nu',\beta\nu}-E_{\beta\nu',\nu\beta}]\equiv\sum_{\nu\nu'}\sum_{\beta(\mathrm{occ})}[-F_{\nu'\beta,\beta\nu}+F_{\beta\nu',\beta\nu}-F_{\beta\nu',\nu\beta}],\quad(7)$$

where

$$E_{\alpha\beta,\gamma\delta}\equiv\langle\alpha;s_1't_1'|e^{i\mathbf{q}'\cdot\mathbf{r}_1}|\gamma;s_1t_1\rangle\times\langle\beta;s_2't_2'|e^{i\mathbf{q}\cdot\mathbf{r}_2}|\delta;s_2t_2\rangle.\quad(8)$$

If the term $\rho_{0f}\rho_{00}$ is left out in Eq. (5), we have to add on a term $F_{\nu'\beta,\nu\beta}$ in Eq. (7). The first scattering is related to $\mathbf{k}_0$ and to the target nucleon 2; in this sense the second and the third E in Eq. (7) correspond to two very simple diagrams perfectly reasonable physically; the first term $-E_{\nu'\beta,\beta\nu}$ can be obtained from the third one by interchanging the target particles 1 and 2 and is a consequence of the mathematical symmetries of our Eq. (4); unfortunately, physically it means that the projectile of momentum $\mathbf{k}_0$ brings the target nucleon 2 from a single nucleon state ν to an occupied state β; this would involve a violation of the Pauli exclusion principle unless our ground state is correlated as in the RPA. However, the contractions in Eq. (6) should in this case be better performed allowing for the true partial occupations of the hole and particle states; consequently, neither β nor ν will be exactly hole states as in the TD case. This situation in Eq. (7) is a consequence of the closure approximation and of the contractions in Eq. (6). Suppose now that we go back to our Eq. (1) and introduce for the complete set of intermediate states n the complete set of the RPA.

We perform contractions in the corresponding formula as we did them in Eq. (6) and then we apply the closure approximation to the RPA set. We obtain a result of the form of our Eq. (7) with the only difference that the term $-E_{\nu'\beta,\beta\nu}$ appears with the summation over β unoccupied (lying above the Fermi level); in this "second variant" of our theory we no longer have any apparent violation of the exclusion principle. The two variants mentioned correspond to two different approximations. They are discussed below in connection with our numerical results.

One very serious difficulty with a numerical calculation is the $\mathbf{k}$ integration. In particular, it is generally very difficult to determine the off-energy-shell propagation of the nucleon-nucleon t matrix. In the following, we replace our t-matrix elements by their on-energy-shell values (the intermediate-state kinetic energy of the nucleon 0 is set equal to that of the initial state of the same $=E_0$). This fixing of k as equal to k_0[14] constitutes our third major approximation analogous to that of Refs. 3 and 4–6 for the elastic nucleon-nucleus scattering. The actual off-energy-shell propagation of t would be generally expected to reduce somewhat the magnitude of our correction, although the most important contribution should be expected naturally from k close to k_0.

In the case of the scattering of 145-MeV neutrons from deuterons, Everett[7] has found that the off-energy-shell propagation of t matrices involved in the double-scattering terms could be limited to an interval of (−40, 20 MeV) of our intermediate state energy $\hbar^2k^2/2m$ about the incident laboratory energy $\hbar^2k_0^2/2m$. The nature of our nuclear form factor in the case of C^{12} is still more restrictive, and reduces that interval considerably.

Recently, Nishida[15] has investigated the off-energy-shell propagation of the t matrix for a simple but "reasonable" nucleon-nucleon potential in a wide energy range. He finds that up to the scattering angle of about 90° the initial energy on-shell amplitudes almost coincide with the corresponding off-shell amplitudes in a wide energy range between 30 and 150 MeV. Obviously, the approximation is the better the higher the incident energy. In order to estimate an upper limit to the error of our approximation we have chosen $E=90$ MeV. Actually the situation is better at 150 MeV as for our results of Table II. We have exploited the results of Fig. 6 of Ref. 15 which compares the off-shell amplitudes of $E_0=90$ MeV to $E_0=70$ MeV and of $E_0=90$ MeV to $E_0=110$ MeV with the corresponding on-shell amplitude of $E_0=90$ MeV.

We have estimated the upper limit of our error by suppressing the effect of our nuclear form factor (this overestimates the effect considerably). We numerically calculate the integral $\int d\mathbf{k}$ over the interval D (the solid angle 4π and 70 MeV $\leqslant\hbar^2k^2/2m\leqslant$ 110 MeV) as

$$\int_D d\mathbf{k}\frac{\left\langle\frac{\mathbf{k}_0'}{2}\middle|t_{01}\middle|\frac{\mathbf{k}}{2}\right\rangle\left\langle\frac{\mathbf{k}}{2}\middle|t_{02}\middle|\frac{\mathbf{k}_0}{2}\right\rangle}{k_0^2-k^2+i\delta}$$

and compare it with the corresponding one with $\mathbf{k}=\mathbf{k}_0$ in the t elements. For $\mathbf{k}_0\simeq\mathbf{k}_0'$ where the effect is the most important we find that the absolute value of the difference between the two integrals is of only 2%.

[14] $k\simeq k_0$ in t is somewhat arbitrary; no essential difference would result, however, if we took $k\simeq k_0'$ or even $k_0'\simeq k\simeq k_0$ in the t elements involved at our high-energy E_0.

[15] Y. Nishida, Nucl. Phys. **82**, 385 (1966).

Consequently, we feel that the on-shell approximation is very probably not important in our energy range.

In order to perform the remaining **k** integration it is practical to work with harmonic-oscillator wave functions $\varphi_\alpha(i)$ for which the Talmi transformation can be performed. In this way, by introducing the center-of-mass system, and the relative coordinates, $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/\sqrt{2}$ and $\mathbf{r}=(\mathbf{r}_1-\mathbf{r}_2)/\sqrt{2}$, respectively, we can put all the **k** dependence of F into the matrix elements of the relative coordinates only. Using the Brody-Moshinsky transformation brackets,[16] we can write

$$\langle n_al_a\mu_a|e^{i\mathbf{q}'\cdot\mathbf{r}_1}|n_bl_b\mu_b\rangle\langle n_cl_c\mu_c|e^{i\mathbf{q}\cdot\mathbf{r}_2}|n_dl_d\mu_d\rangle$$
$$=\sum(l_al_c;\mu_a\mu_c|\lambda'\mu')(l'L';m'M'|\lambda'\mu')(l_bl_d;\mu_b\mu_d|\lambda\mu)$$
$$\times(lL;mM|\lambda\mu)\langle'|ac;\lambda'\rangle\langle\,|bd;\lambda\rangle$$
$$\times\langle N'L'M'|e^{i\mathbf{K}\cdot\mathbf{R}}|NLM\rangle\langle n'l'm'|e^{i\mathbf{Q}'\cdot\mathbf{r}}|nlm\rangle,\quad(9)$$

where

$$\langle'|ac;\lambda'\rangle\equiv\langle n'l'N'L';\lambda'|n_al_an_cl_c;\lambda'\rangle,$$
$$\langle\,|bd;\lambda\rangle\equiv\langle nlNL;\lambda|n_bl_bn_dl_d;\lambda\rangle,$$

and

$$\mathbf{K}=(\mathbf{k}_0-\mathbf{k}_0')/\sqrt{2},\quad \mathbf{Q}'=\mathbf{Q}-\mathbf{k}\sqrt{2},\quad \mathbf{Q}=(\mathbf{k}_0+\mathbf{k}_0')/\sqrt{2}.$$

Our next approximation is to suppress the **k**-angular dependence of our products of t-matrix elements. Following, e.g., Ref. 6, we replace each such element by its value at forward angles. This approximation tends to overestimate our second-order correction. The relative "flatness" of the angular dependence of the nucleon-nucleon t-matrix elements makes this Ansatz not unreasonable. Another procedure would be to perform the **k** angular averaging of our products of t elements.

In our numerical work, we confine ourselves to small $(\mathbf{k}_0,\mathbf{k}_0')$ scattering angles θ. In the case of deuterium, Everett[7] finds that it is enough to consider for the angle $\measuredangle(\mathbf{k},\mathbf{k}_0)$ the interval $\theta-20°\leqslant\measuredangle(\mathbf{k},\mathbf{k}_0)\leqslant\theta+20°$. Because of the nature of our nuclear form factor of C^{12}, this interval is even narrower. We feel that our angular approximation could not, in general, introduce errors higher than a few percent. Consequently, the only **k** integration remaining can be performed analytically in a closed form for each partial configuration:

$$\int\frac{d\mathbf{k}}{k_0^2-k^2-(2m/\hbar^2)\langle\Delta\epsilon\rangle_{\rm av}+i\delta}\langle n'l'm'|e^{i\mathbf{Q}'\cdot\mathbf{r}}|nlm\rangle$$
$$=-\frac{2\pi^2}{\sqrt{2}}\delta_{mm'}\hat{l}\hat{l}'(-)^m\sum_{l_1}i^{l_1}(ll';-mm|l_10)$$
$$\times(ll';00|l_10)g_{l_1}{}^{nln'l'}(Q),\quad(10)$$

where

$$g_{l_1}{}^{nln'l'}(Q)\equiv\int_0^\infty R_{n'l'}(r)R_{nl}(r)j_{l_1}(Qr)e^{i\zeta r}rdr,\quad(11)$$

and

$$\zeta\equiv\sqrt{2}\left(k_0^2-\frac{2m}{\hbar^2}\langle\Delta\epsilon\rangle_{\rm av}\right)^{1/2};\quad \hat{l}=(2l+1)^{1/2}.$$

Until now we have worked with $\mathbf{t}_{0i}$ and F elements of definite s and t substates with the corresponding summations. It is actually more practical to work with the $\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_i$ operators and to perform the spin and isospin integrations after all the spatial integrations. Some of the corresponding final expressions for the j-j coupling shell model are presented in Appendix I. They involve quite an amount of Racah algebra and rather complex geometrical factors. In order to simplify the following discussion, we prefer to consider two extreme simple coupling models.

In the first of these models we suppose our J_f, M_f to be given by the vector addition of the orbital angular momenta of the particle-hole pairs; T_f and T_{3f} are projected out as before; the spins of the particle-hole pairs are coupled to $S=0$. It is then a restricted model in which we exclude the $S=1$ particle-hole pairs, and we have $J_f=L_f$. This oversimplification of the physical reality should not alter the general characteristics of our correction effects, except for any possible polarization effects. In the place of our F's of Eq. (7), we can work now with the corresponding **k** integrals as in Eq. (11):

$$\phi_{\gamma\gamma',\delta\delta'}{}^{f(S=0,T)}(k_0,k_0',\cos\vartheta)$$
$$\equiv\int\frac{d\mathbf{k}}{k_0^2-k^2-(2m/\hbar^2)\langle\Delta\epsilon\rangle_{\rm av}+i\delta}\times F_{\gamma\gamma',\delta\delta'}{}^{f(S=0,T)}(\mathbf{q},\mathbf{q}'),\quad(12)$$

where $\vartheta=\measuredangle(\mathbf{k}_0,\mathbf{k}_0')$. In particular,

$$\phi_{\nu'\beta,\beta\nu}{}^{f(0,0)}=-8\pi\left(\frac{\pi}{2}\right)^{3/2}\hat{J}_f(-)^{l_\nu+l_\beta}\chi_{\nu'\nu}{}^{(f)*}\sum(-)^{l_1+L}\langle\,|\nu'\nu;\lambda=J_f\rangle\langle'|\beta\beta;\lambda'=0\rangle$$
$$\times i^{L_1}\hat{L}\hat{L}'\hat{L}_1\begin{pmatrix}L&L_1&L'\\0&0&0\end{pmatrix}h_{L_1}{}^{NLN'L'}(K)i^{l_1}\hat{l}_1{}^2\hat{l}\hat{l}'\begin{pmatrix}l&l_1&l'\\0&0&0\end{pmatrix}g_{l_1}{}^{nln'l'}(Q)$$
$$\times\begin{Bmatrix}L_1&l_1&J_f\\l&L&l'\end{Bmatrix}\delta_{L'l'}\begin{pmatrix}l_1&L_1&J_f\\0&M_f&-M_f\end{pmatrix}(-)^{M_f}Y_{L_1M_f}{}^*(\omega_{\mathbf{k}}),\quad(13)$$

[16] T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets* (Monografias del Instituto de Fisica, Mexico, 1960).

$$\phi_{\beta\nu',\beta\nu}{}^{f(0,0)}=-8\pi\left(\frac{\pi}{2}\right)^{3/2}\hat{J}_f(-)^{l_\nu+l_\beta}\chi_{\nu'\nu}{}^{(f)*}\sum(-)^{L+l+\lambda}\hat{\lambda}^2\hat{\lambda}'^2\langle\,|\beta\nu;\lambda\rangle\langle'|\beta\nu';\lambda'\rangle$$

$$\times i^{L_1}\hat{L}_1\hat{L}\hat{L}'\begin{pmatrix}L & L_1 & L'\\ 0 & 0 & 0\end{pmatrix}h_{L_1}{}^{NLN'L'}(K)i^{l_1}\hat{l}_1{}^2\hat{l}\hat{l}'\begin{pmatrix}l & l_1 & l'\\ 0 & 0 & 0\end{pmatrix}g_{l_1}{}^{nln'l'}(Q)$$

$$\times\begin{Bmatrix}\lambda & \lambda' & J_f\\ l_{\nu'} & l_\nu & l_\beta\end{Bmatrix}\begin{Bmatrix}l & l' & l_1\\ L & L' & L_1\\ \lambda & \lambda' & J_f\end{Bmatrix}\begin{pmatrix}l_1 & L_1 & J_f\\ 0 & M_f & -M_f\end{pmatrix}(-)^{M_f}Y_{L_1M_f}{}^*(\omega_k),\quad(14)$$

and

$$\phi_{\beta\nu',\nu\beta}{}^{f(0,0)}=\sum_{l_1}\phi_{\nu'\beta,\beta\nu}{}^{f(0,0)}(l_1)(-)^{l_1},\quad(15)$$

where

$$\phi_{\alpha\alpha',\beta\beta'}{}^f=\sum_{l_1}\phi_{\alpha\alpha',\beta\beta'}{}^f(l_1),$$

$$h_{L_1}{}^{NLN'L'}(K)\equiv\int_0^\infty R_{N'L'}(R)R_{NL}(R)j_{L_1}(KR)R^2dR,\quad(16)$$

and g_{l_1} is defined by Eq. (11). $\phi_{\nu'\beta,\nu\beta}$ is related to $\phi_{\beta\nu',\beta\nu}$ by a relation similar to Eq. (15).

In our numerical work we shall use an approximate form of the t_{0i} operators which is sufficient for our computation of the parameters α, β, etc., as defined below. The meaning of this form of t_{0i} is explained more in detail in Sec. 3. We put t_{0i} in the form:

$$t_{0i}\equiv t_{0i}{}^{(0)}+t_{0i}{}^{(n)}(\sigma_{0n}+\sigma_{in})+t_{0i}{}^{(\tau)}\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_i+t_{0i}{}^{(n\tau)}(\sigma_{0n}+\sigma_{in})\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_i+t_{0i}{}^{(\sigma)}\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_i+t_{0i}{}^{(\sigma\tau)}(\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_i)(\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_i),\quad(17)$$

where $\sigma_{jn}=\boldsymbol{\sigma}_j\cdot\mathbf{n}$, $\mathbf{n}=\mathbf{k}_{0i}\times\mathbf{k}_{0i}'/|\mathbf{k}_{0i}\times\mathbf{k}_{0i}'|$. The vector $\mathbf{n}$ is the vector perpendicular to the $(0,i)$ scattering plane. Actually, in practical applications the contributions of $t^{(n)}$ and $t^{(n\tau)}$ are quite small compared with any other.

After the summations over the spins and isospins of the particles 1 and 2, the entire double-scattering correction amplitude becomes, in the case of our model ($S=0$) and for $T=0$,

$$\langle\mathbf{k}_0'|\Delta T|\mathbf{k}_0\rangle_{if}=-2\frac{2m}{\hbar^2}\sum_{\nu\nu'}\sum_{\beta(\mathrm{occ})}[(\alpha+\alpha_\sigma\sigma_{0n})(\phi_{\nu'\beta,\beta\nu}{}^{f(0,0)}+\phi_{\beta\nu',\nu\beta}{}^{f(0,0)})+(\beta+\beta_\sigma\sigma_{0n})\phi_{\beta\nu',\beta\nu}{}^{f(0,0)}]$$

$$\equiv\sum_{l_1L_1}[\psi_{l_1L_1}{}^{(0)}(k_0,k_0',\cos\vartheta)+\psi_{l_1L_1}{}^{(\sigma)}(k_0,k_0',\cos\vartheta)\sigma_{0n}]\begin{pmatrix}l_1 & L_1 & J_f\\ 0 & M_f & -M_f\end{pmatrix}(-)^{M_f}Y_{L_1M_f}{}^*(\omega_k),\quad(18)$$

where

$$\alpha\equiv\langle t_{01}{}^{(0)}t_{02}{}^{(0)}+3t_{01}{}^{(\sigma)}t_{02}{}^{(\sigma)}+3t_{01}{}^{(\tau)}t_{02}{}^{(\tau)}+9t_{01}{}^{(\sigma\tau)}t_{02}{}^{(\sigma\tau)}+2t_{01}{}^{(n)}t_{02}{}^{(n)}+6t_{01}{}^{(n\tau)}t_{02}{}^{(n\tau)}\rangle_0,\quad(19a)$$

$$\beta\equiv-4\langle t_{01}{}^{(0)}t_{02}{}^{(0)}+t_{01}{}^{(n)}t_{02}{}^{(n)}\rangle_0,\quad(19b)$$

$$\alpha_\sigma\equiv\langle t_{01}{}^{(0)}t_{02}{}^{(n)}+t_{01}{}^{(n)}t_{02}{}^{(0)}+t_{01}{}^{(n)}t_{02}{}^{(\sigma)}+t_{01}{}^{(\sigma)}t_{02}{}^{(n)}+3t_{01}{}^{(\tau)}t_{02}{}^{(n\tau)}+3t_{01}{}^{(n\tau)}t_{02}{}^{(\tau)}+3t_{01}{}^{(n\tau)}t_{02}{}^{(\sigma\tau)}+3t_{01}{}^{(\sigma\tau)}t_{02}{}^{(n\tau)}\rangle_0,\quad(19c)$$

$$\beta_\sigma\equiv-4\langle t_{01}{}^{(0)}t_{02}{}^{(n)}+t_{01}{}^{(n)}t_{02}{}^{(0)}\rangle_0.\quad(19d)$$

If we drop the term $\rho_{0f}\rho_{00}$ in Eq. (5), $\phi_{\beta\nu',\beta\nu}{}^{f(0,0)}$ has to be replaced by $\phi_{\beta\nu',\beta\nu}{}^{f(0,0)}+\phi_{\nu'\beta,\nu\beta}{}^{f(0,0)}$. The products of two t's are taken, as explained above, with the ansatz $\mathbf{k}\approx\mathbf{k}_0(\approx\mathbf{k}_0')$ which corresponds to the notation $\langle\ \rangle_0$. To be consistent, we now have to write the (main) term, which is first order in t matrix (impulse approximation) in the same model approximation:

$$\langle\mathbf{k}_0'|T|\mathbf{k}_0\rangle_{if}=2\langle\tfrac{1}{2}\mathbf{k}_0'|t_{01}|\tfrac{1}{2}\mathbf{k}_0\rangle(\boldsymbol{\sigma}_0)(4\pi)^{1/2}\sum_{\mu\mu'}\chi_{\mu'\mu}{}^{(f)*}(-)^{l_{\mu'}}\hat{l}_\mu\hat{l}_{\mu'}\begin{pmatrix}l_\mu & l_{\mu'} & J_f\\ 0 & 0 & 0\end{pmatrix}$$

$$\times h_{J_f}{}^{n_{\mu'}l_{\mu'}n_\mu l_\mu}(\sqrt{2}K)i^{J_f}Y_{J_fM_f}{}^*(\omega_k)\equiv[A^{(0)}(k_0,k_0',\cos\vartheta)+A^{(\sigma)}(k_0,k_0',\cos\vartheta)\sigma_{0n}]i^{J_f}Y_{J_fM_f}{}^*(\omega_k),\quad(20)$$

where $\langle\frac{1}{2}\mathbf{k}_0'|t_{01}|\frac{1}{2}\mathbf{k}_0\rangle(\boldsymbol{\sigma}_0)$ is the $\boldsymbol{\sigma}_1$-averaged t_{01} element explicitly defined in Sec. 3.

In the following, we are not interested in the absolute values of the cross sections but rather in the relative contribution to T coming from ΔT. In particular, we study the angular distributions and the order of magnitude of the ΔT correction terms. The angular distribution of the outgoing nucleon 0′ is, with an arbitrary normalization, given by

$$\frac{4\pi}{2J_f+1}\mathrm{Tr}_{(\sigma_0)}\sum_{M_f}\langle T+\Delta T\rangle^\dagger\langle T+\Delta T\rangle=|A^{(0)}|^2+|A^{(\sigma)}|^2+\hat{J}_f^{-1}2\,\mathrm{Re}\left\{\sum_{l_1L_1}\hat{L}_1\begin{pmatrix}L_1 & J_f & l_1\\ 0 & 0 & 0\end{pmatrix}i^{-J_f}\right.$$
$$\left.\times(A^{(0)}\psi_{l_1L_1}{}^{(0)*}+A^{(\sigma)}\psi^{(\sigma)*})P_{l_1}(\cos\vartheta_{\mathbf{k}})\right\}+\sum_{l_1L_1l_1'L_1'}(\psi_{l_1L_1}{}^{(0)}\psi_{l_1'L_1'}{}^{(0)*}+\psi_{l_1L_1}{}^{(\sigma)}\psi_{l_1'L_1'}{}^{(\sigma)*})$$
$$\times\sum_{\mathcal{L}}(-)^{J_f+\mathcal{L}}\hat{J}_f^{-2}(2\mathcal{L}+1)\hat{L}_1\hat{L}_1'\begin{pmatrix}L_1' & L_1 & \mathcal{L}\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}l_1 & l_1' & \mathcal{L}\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}l_1 & l_1' & \mathcal{L}\\ L_1' & L_1 & J_f\end{Bmatrix}P_{\mathcal{L}}(\cos\vartheta_{\mathbf{k}}).\quad(21)$$

The actual physical situation as outlined in the Appendix is probably intermediate between our first model with the restriction $S=0$ and the second model in which we assume the complete degeneracy in spin (for an even-even nucleus). In this model, any spin orientation is equivalent. As for the components $\chi_{(\nu'\nu)}{}^{(f)}$, they are essentially the same as before; even in the case of an RPA diagonalization for spinless nucleons, the $\chi_{(\nu'\nu)}{}^{(f)}$ remain unchanged provided one confines oneself to a simple Wigner force, and the coupling constant (well depth) is an adjustable parameter to fit the experimental energy of the level in question.

In this model one has to add to the coefficients α, β, α_σ, and β_σ of Eq. (19) the respective correction terms:

$$\Delta\alpha=\langle t_{01}{}^{(0)}t_{02}{}^{(n)}+t_{01}{}^{(n)}t_{02}{}^{(0)}+t_{01}{}^{(\sigma)}t_{02}{}^{(n)}+t_{01}{}^{(n)}t_{02}{}^{(\sigma)}+3t_{01}{}^{(\tau)}t_{02}{}^{(n\tau)}+3t_{01}{}^{(n\tau)}t_{02}{}^{(\tau)}+3t_{01}{}^{(\sigma\tau)}t_{02}{}^{(n\tau)}+3t_{01}{}^{(n\tau)}t_{02}{}^{(\sigma\tau)}\rangle_0,\quad(22\text{a})$$

$$\Delta\beta=-4\langle t_{01}{}^{(0)}t_{02}{}^{(n)}+t_{01}{}^{(n)}t_{02}{}^{(\sigma)}\rangle_0,\quad(22\text{b})$$

$$\Delta\alpha_\sigma=\langle 2t_{01}{}^{(n)}t_{02}{}^{(n)}+6t_{01}{}^{(n\tau)}t_{02}{}^{(n\tau)}+t_{01}{}^{(0)}t_{02}{}^{(\sigma)}+t_{01}{}^{(\sigma)}t_{02}{}^{(0)}+3t_{01}{}^{(\tau)}t_{02}{}^{(\sigma\tau)}+3t_{01}{}^{(\sigma\tau)}t_{02}{}^{(\tau)}\rangle_0,\quad(22\text{c})$$

$$\Delta\beta_\sigma=-4\langle t_{01}{}^{(n)}t_{02}{}^{(n)}+t_{01}{}^{(0)}t_{02}{}^{(\sigma)}\rangle_0.\quad(22\text{d})$$

Actually, $\langle\Delta T\rangle$ of our second model, if calculated with the same over-all normalization as in Eq. (18), would correspond to the right-hand sides of Eqs. (18) and (20) multiplied by $(-\sqrt{2})$; we drop this factor everywhere as we work with an arbitrary normalization.

3. NUMERICAL CALCULATIONS AND DISCUSSION

We perform our numerical computations on the example of the 4.43-MeV state of C^{12} with $J_f{}^\pi=2^+$, $T=0$.

TABLE I. The RPA eigenvector of the 2^+, $T=0$ state of C^{12} for the asymptotic model of no spin-orbit coupling; the coupling constant of a zero-range Wigner force is adjusted so as to fit the 4.43-MeV energy.

$(\nu\nu')$	$1p^{-1}1p$	$1p^{-1}1f$	$1p^{-1}2p$	$1s^{-1}1d$
$\chi_{\nu'\nu}{}^{(f)}$	−1.125	0.305	−0.199	0.235
$\chi_{\nu\nu'}{}^{(f)}$	−0.577	0.225	−0.146	0.183

The RPA eigenvectors of this state for both our models ($S=0$ and "random spins") are obtained directly by diagonalizing the corresponding secular matrices for a simple zero-range Wigner force. The coupling-constant parameter is taken as adjustable so as to fit the energy 4.43 MeV. The two model solutions are identical in this case. The eigenvector obtained is given in Table I. It is quite similar to what one obtains if one averages arbitrarily over and renormalizes the corresponding components $\chi_{(\nu'\nu)}{}^{(f)}$ of the RPA eigenvector obtained by Goswami and Pal[17] for the complete j-j coupling. Our corresponding Wigner force coupling constant V_0 for the "random spins" model is reasonable ($|V_0|/4\pi b_0{}^3=1.944\times3\sqrt{\pi}\cong10.3$ MeV, in agreement with the value 10.2 MeV as given by Vinh Mau[18]), and it is three times smaller for the $S=0$ model. As in the quoted j-j coupling RPA calculations we find an important fraction of the "backward-going graphs," or the component $\chi_{(\nu\nu')}{}^{(f)}$.

In order to compute the coefficients α, β, α_σ, and β_σ, and $\langle\frac{1}{2}\mathbf{k}_0'|t_{01}|\frac{1}{2}\mathbf{k}_0\rangle(\boldsymbol{\sigma}_0)$ we have chosen the on-energy-shell elements of the free nucleon-nucleon scattering[19] matrix $M(\theta)$ corresponding to a realistic potential containing a hard-core repulsive part. In particular, Kerman *et al.*[20] tabulate all the components of $M(\theta)$ for the Gammel-Thaler potential at the nucleon laboratory energies of 156 and 90 MeV just appropriate to our application. As one can see from Table III of Ref. 20 the coefficient B_T $(T=0,1)$ of the operator $\sigma_{0n}\sigma_{1n}$ is at small scattering angles θ numerically rather close to the coefficients F_T and E_T of the operator $\sigma_{0p}\sigma_{1p}$ and $\sigma_{0q}\sigma_{1q}$, respectively, where $\mathbf{p}$ and $\mathbf{q}$ are vectors which form with $\mathbf{n}$ an orthogonal frame. To a reasonable approximation, one can work then with $B_T(\theta)\approx F_T(\theta)\approx E_T(\theta)$ which simplifies greatly our formulas and permits us to apply directly Eqs. (18), (23), and (24). In terms of the quantities A_T, B_T, F_T, and E_T of Ref. 20,

[17] A. Goswami and M. Pal, Nucl. Phys. 44, 294 (1963).
[18] N. Vinh Mau, thesis, University of Paris, 1963 (unpublished).
[19] We neglect the difference between k_0 and k_0', which is rather small at 156 MeV.
[20] A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N. Y.) **8**, 551 (1959).

TABLE II. The differential cross section $\sigma(\theta)$, in the center-of-mass system, for the $^{12}C(p,p')^{12}C^*$ (2^+ state, 4.43-MeV) reaction (in mb/sr) for the incident proton energy=156 MeV. The impulse-approximation results and the double-scattering corrections are given for our two models: the model with $S=0$ and that of "random spins." The nucleon-nucleon potential is that of Gammel and Thaler. Results for the variant of our theory where the term $\rho_{0f}\rho_{00}$ is omitted in Eq. (5) are given in the last two lines.

$\theta_{(c.m.)}$		10.50°	15.68°	20.76°	30.54°
$\sigma(\theta)$, impulse approximation	Model $S=0$	2.923	7.772	10.346	6.625
	Model of random spins	2.118	6.785	10.957	6.943
Double-scattering corrections $\Delta\sigma(\theta)$	Model $S=0$	−0.827	−1.419	+1.205	−1.312
	Model of random spins	−0.339	−0.139	+2.858	−1.314
Double-scattering corrections(%)	Model $S=0$	−28.3	−18.3	+11.6	−19.8
	Model of random spins	−16.0	−2.0	+26.1	−18.9
$\Delta\sigma(\theta)$ when $\rho_{0f}\rho_{00}$ is omitted in Eq. (5) (model $S=0$)		−0.569	+0.146	+2.001	−0.959
Double-scattering corrections (%) when $\rho_{0f}\rho_{00}$ is omitted in Eq. (5) (model $S=0$)		−19.5	+1.9	+19.3	−14.5

we have with $E_T \approx F_T \approx B_T$:

$$M^{(0)}=\tfrac{1}{4}(A_0+3A_1)\,,\quad M^{(\sigma)}=\tfrac{1}{4}(B_0+3B_1)\,,\quad M^{(\tau)}=\tfrac{1}{4}(A_1-A_0)\,,$$

$$M^{(\sigma\tau)}=\tfrac{1}{4}(B_1-B_0)\,,\quad M^{(n)}=\tfrac{1}{4}(C_0+3C_1)\,,\quad M^{(n\tau)}=\tfrac{1}{4}(C_1-C_0)\,.$$

The t-matrix elements are obtained from the corresponding M elements by multiplying them by the normalization factor $-2\hbar^2/(2\pi)^2m$. With the help of Eqs. (19) and (22), we finally calculate the coefficients α, β, α_σ, and β_σ.

Another calculation was performed with $M(\theta)$ for the Yale YLAM+YLAN 3M potentials for $E_0=100$ MeV as tabulated by De Bouard *et al.*[21] Using the same notations of Ref. 20, we find for t elements of the first-order term the expressions:

(1) In the case of our model with $S=0$ and for $T=0$:

$$\langle\tfrac{1}{2}\mathbf{k}_0'|t_{01}|\tfrac{1}{2}\mathbf{k}_0\rangle(\boldsymbol{\sigma}_0)=-\frac{2\hbar^2}{(2\pi)^2m}[M^{(0)}+M^{(n)}\sigma_{0n}],\quad(23)$$

and (2) for the model of "random spins" ($T=0$):

$$\langle\tfrac{1}{2}\mathbf{k}_0'|t_{01}|\tfrac{1}{2}\mathbf{k}_0\rangle(\boldsymbol{\sigma}_0)=-\frac{2\hbar^2}{(2\pi)^2m}[(M^{(0)}+M^{(n)})+(M^{(\sigma)}+M^{(n)})\sigma_{0n}].\quad(24)$$

The neglect of the differences between the components B, E, and F corresponds to the neglect of the splitting effects of the two-nucleon tensor force (as if we did not have any tensor forces). In our numerical results as given below we have arbitrarily taken $B(\approx E\approx F)$ $\simeq\frac{1}{3}(B+E+F)$. Had we chosen only the B of Ref. 20 instead of the arithmetic mean of the three, our final net double-scattering corrections would be slightly increased. All the quantities involving the elements t_{0i} are computed numerically from the tables of Ref. 20.

The components of our vector $|2^+\rangle$ are taken from Table I; the usual harmonic-oscillator constant is $\sqrt{\nu}=b_0^{-1}=0.62\ \mathrm{F}^{-1}$.

All our numerical computations have been performed on the Univac 1107 computer of the Faculte des Sciences d'Orsay.

In Table II we give our final results for the two models. The contributions to the differential cross section $\sigma(\theta)$ from the impulse term and the double-scattering corrections are tabulated in mb/sr.

The contributions quadratic in the double-scattering corrections are generally of the same order of magnitude as the crossed (interference) terms, and of the opposite sign; consequently, they cancel most of the (negative) contribution of the latter to our final $\Delta\sigma(\theta)$. Quite similar results are obtained (not presented here) for the variant of our theory discussed in Sec. 2, where we do not include any Pauli principle violating component.

From Table II we see that the double-scattering corrections are generally considerable. In particular, they reduce the absolute values of $\sigma(\theta)$ at very small and at large ($\gtrsim 30°$) angles. The detailed numerical results for our two extreme model approximations ($S=0$ and random spins) differ considerably; a more realistic calculation based on the j-j scheme as indicated in Appendix I could be interesting. We conclude that the double-scattering corrections to inelastic nucleon scattering are rather sensitive to the details of the nuclear wave functions involved.

If we omit the term $\rho_{0f}\rho_{00}$ in Eq. (5), the absolute values of $\Delta\sigma(\theta)$ are generally smaller. The corrections $\Delta\sigma(\theta)$ at $E_0=100$ and 90 MeV are of the same order of magnitude as those at 156 MeV. We find no appreciable difference between the $\Delta\sigma(\theta)$ computed with $M(\theta)$ of the Gammel-Thaler potential and the Yale phase parameters.

[21] X. deBouard, B. Tatischeff, A. Willis, N. Marty, C. Rolland, and B. Geoffrion (private communication).

Actually, the ^{12}C nucleus most probably has a stable strong (negative) deformation,[22] and any detailed RPA (TD) calculation of the wave functions should be based on a deformed (Nilsson-type) scheme.[23] Unfortunately, no RPA eigenvectors of this nature are as yet available in the literature for the 2^+ states.

Our calculated absolute values of $\sigma(\theta)$ are much larger (by about a factor of 2) than the measured values.[24] It is known from the results of Haybron and McManus[25] that the effect of an elastic distortion on the impulse approximation term is quite important for the absolute values of the cross sections even at incident proton energies as high as 150 MeV.

The distortion is relatively unimportant for the angular distributions. Our calculations could be redone, e.g., with a simple JWKB approximation distortion. On the other hand, it is just our case with the term $\rho_{0f}\rho_{00}$ omitted in Eq. (5) which seems to include the first correction of the distortion in the sense of the multiple-scattering series with undistorted (plane) waves. However, the main effect of the distortion appears to be a certain renormalization of the "effective" plane waves of the scattering nucleon; this reduces considerably the absolute values of $\sigma(\theta)$ as given by the first-order term, and may reduce even more the double-scattering term, so that the application of distorted waves in the place of plane waves could possibly tend to reduce the ratio $\Delta\sigma(\theta)/\sigma(\theta)$. The convergence of the multiple-scattering series would tend to be better in the former case.

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APPENDIX: ELEMENTS $\langle k_0'|\Delta T|k_0\rangle$ IN THE CASE WHEN THE TARGET STATES ARE DESCRIBED BY *j-j* COUPLING

In this case we limit ourselves to final states with $T=0$; thus the isospin summations are identical with those of Sec. 2. On using the representation of the operator t_{0i} of Eq. (19), we can write the correction to our transition amplitude in the form:

$$\langle \mathbf{k}_0'|\Delta T|\mathbf{k}_0\rangle = -\frac{2m}{\hbar^2}\left(\Delta^{(0)}+\sum_i(-)^i\sigma_0{}^i\Delta_{-i}{}^{(\sigma)}\right), \quad \text{(A1)}$$

where the $\sigma_0{}^i$ $(i=-1,0,1)$ are the usual tensor components of the spin operator of the projectile. The terms $\Delta^{(0)}$ and $\Delta_i{}^{(\sigma)}$ are obtained by regrouping the various terms of the expression

$$\{-\langle\nu'|t_{01}|\beta\rangle\langle\beta|t_{02}|\nu\rangle + \langle\beta|t_{01}|\beta\rangle\langle\nu'|t_{02}|\nu\rangle - \langle\beta|t_{01}|\nu\rangle\langle\nu'|t_{02}|\beta\rangle\}.$$

If the term $\rho_{0f}\rho_{00}$ is omitted in Eq. (5) the term

$$\langle\nu'|t_{01}|\nu\rangle\langle\beta|t_{02}|\beta\rangle$$

must be included. In the following we do not give explicit formulas for this case. The corresponding extra terms $\psi_{\nu'\beta,\nu\beta}$ have a form similar to that of Eq. (A7).

Let us define the following quantities (averaged over $\omega_{\mathbf{k}}$):

$$\begin{aligned}\alpha_1{}^{(0)} &\equiv -\sqrt{2}\langle t_{01}{}^{(0)}t_{02}{}^{(0)}+3t_{01}{}^{(\tau)}t_{02}{}^{(\tau)}\rangle_{\text{av}},\\ \alpha_1{}^{(\sigma1)} &\equiv -\sqrt{2}\langle t_{01}{}^{(\sigma)}t_{02}{}^{(0)}+3t_{01}{}^{(\sigma\tau)}t_{02}{}^{(\tau)}\rangle_{\text{av}},\\ \alpha_1{}^{(\sigma2)} &\equiv -\sqrt{2}\langle t_{01}{}^{(0)}t_{02}{}^{(\sigma)}+3t_{01}{}^{(\tau)}t_{02}{}^{(\sigma\tau)}\rangle_{\text{av}},\\ \alpha_1{}^{(\sigma1\sigma2)} &\equiv -\sqrt{2}\langle t_{01}{}^{(\sigma)}t_{02}{}^{(\sigma)}+3t_{01}{}^{(\sigma\tau)}t_{02}{}^{(\sigma\tau)}\rangle_{\text{av}},\end{aligned} \quad \text{(A2)}$$

and

$$\begin{aligned}\alpha_2{}^{(0)} &= -2\sqrt{2}\langle t_{01}{}^{(0)}t_{02}{}^{(0)}\rangle_{\text{av}},\\ \alpha_2{}^{(\sigma1)} &\equiv -2\sqrt{2}\langle t_{01}{}^{(\sigma)}t_{02}{}^{(0)}\rangle_{\text{av}},\\ \alpha_2{}^{(\sigma2)} &\equiv -2\sqrt{2}\langle t_{01}{}^{(0)}t_{02}{}^{(\sigma)}\rangle_{\text{av}},\\ \alpha_2{}^{(\sigma1\sigma2)} &\equiv -2\sqrt{2}\langle t_{01}{}^{(\sigma)}t_{02}{}^{(\sigma)}\rangle_{\text{av}}.\end{aligned} \quad \text{(A3)}$$

One then finds

$$\Delta^{(0)}=\sum_{\nu\nu'}\sum_{\beta_{\text{occ}}}(\phi_{\nu'\beta\beta\nu}{}^{(0)}+\psi_{\beta\nu'\beta\nu}{}^{(0)}), \quad \text{(A4)}$$

$$\phi_{\nu'\beta\beta\nu}{}^{(0)}=\sum_{\lambda,L,L_1}A_{\nu'\beta\beta\nu}(\lambda,L,L_1)\left\{\alpha_1{}^{(0)}[(-)^{M_f}Y_{L_1}{}^{M_f*}(\omega_{\mathbf{k}})+(-)^{\lambda+L}Y_{L_1}{}^{-M_f*}(\omega_{\mathbf{k}})]\right.$$
$$\left.+6\alpha_1{}^{(\sigma1\sigma2)}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & \lambda\\ \frac{1}{2} & \frac{1}{2} & 1\end{Bmatrix}[(-)^{M_f+\lambda}Y_{L_1}{}^{M_f*}(\omega_{\mathbf{k}})+(-)^{l_\nu+L}Y_{L_1}{}^{-M_f*}(\omega_{\mathbf{k}})]\right\}, \quad \text{(A5)}$$

[22] A. B. Volkov, Nucl. Phys. **74**, 33 (1965); H. Morinaga, Phys. Letters **21**, 78 (1966); W. A. Lochstet and W. E. Stephens, Phys. Rev. **141**, 1002 (1966); B. C. Cook *et al.*, *ibid.* **143**, 724 (1966); S. C. Fultz *et al.*, *ibid.* **143**, 790 (1966).

[23] S. G. Nilsson, J. Sawicki, and N. K. Glendenning, Nucl. Phys. **33**, 239 (1962).

[24] J. C. Jacmart, J. P. Garron, M. Riou, and C. Ruhla, in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach Science Publishers, Inc., New York, 1963).

[25] R. M. Haybron and H. McManus, Phys. Rev. **136**, B1730 (1965); **140**, B638 (1965); R. M. Haybron *et al.*, Phys. Rev. Letters **12**, 249 (1964).

$$A_{\nu'\beta\beta\nu}(\lambda,L,L_1)\equiv(8\pi^5)^{1/2}\sum_{\substack{NN'L'nl\\ n'l'l_1\lambda}}(-)^{\frac{3}{2}+j_\beta+l_\nu+\lambda'+L+l+J_f}\chi_{\nu'\nu}{}^{(f)*}i^{L_1+l_1}$$
$$\times\langle'|\nu'\nu\lambda'\rangle\langle\ |\beta\beta\lambda\rangle\hat{L}\hat{L}'\hat{L}_1\hat{l}_1{}^2\hat{l}\hat{l}'\hat{J}_f\hat{j}_\beta{}^2\hat{j}_\nu\hat{j}_{\nu'}\hat{\lambda}^2\hat{\lambda}'^2\begin{pmatrix}L' & L_1 & L\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}l' & l_1 & l\\ 0 & 0 & 0\end{pmatrix}$$
$$\times\begin{pmatrix}l_1 & L_1 & J_f\\ 0 & M_f & -M_f\end{pmatrix}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & \lambda\\ l_\beta & l_\beta & j_\beta\end{Bmatrix}\begin{Bmatrix}l_1 & L_1 & J_f\\ l & L & \lambda\\ l' & L' & \lambda'\end{Bmatrix}\begin{Bmatrix}J_f & \lambda' & \lambda\\ j_{\nu'} & l_{\nu'} & \frac{1}{2}\\ j_\nu & l_\nu & \frac{1}{2}\end{Bmatrix}h_{L_1}{}^{NLN'L'}(K)g_{l_1}{}^{nln'l'}(Q)\,,\qquad\text{(A6)}$$

where $g(Q)$ and $h(K)$ are defined by Eqs. (11) and (16a) of the main text, and

$$\psi_{\beta\nu'\beta\nu}{}^{(0)}=\sum_{\lambda\lambda'}B_{\beta\nu'\beta\nu}(\lambda,\lambda')\Bigg[-\alpha_2{}^{(0)}\begin{Bmatrix}l_\nu & l_{\nu'} & J_f\\ j_{\nu'} & j_\nu & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix}\lambda & J_f & \lambda'\\ l_{\nu'} & l_\beta & l_\nu\end{Bmatrix}+(-)^{j_{\nu'}+j_\beta+\lambda'}$$
$$\times6\alpha_2{}^{(\sigma_1\sigma_2)}\begin{Bmatrix}l_\beta & 1 & l_\beta\\ \frac{1}{2} & j_\beta & \frac{1}{2}\end{Bmatrix}\sum_{\mathcal{K}}\hat{\mathcal{K}}^2\begin{Bmatrix}J_f & 1 & \mathcal{K}\\ j_{\nu'} & \frac{1}{2} & l_{\nu'}\\ j_\nu & \frac{1}{2} & l_\nu\end{Bmatrix}\begin{Bmatrix}1 & \mathcal{K} & J_f\\ l_\beta & l_{\nu'} & \lambda'\\ l_\beta & l_\nu & \lambda\end{Bmatrix}\Bigg]\,,\qquad\text{(A7)}$$

where

$$B_{\beta\nu'\beta\nu}(\lambda,\lambda')\equiv(8\pi^5)^{1/2}\sum_{\substack{NLN'L'L_1\\ nln'l'l_1}}\chi_{\nu'\nu}{}^{(f)*}(-)^{\frac{1}{2}+j_\nu+J_f+M_f+L+l'+l_\nu+l_{\nu'}+l_\beta+\lambda}$$
$$\times\langle'|\beta\nu'\lambda'\rangle\langle\ |\beta\nu\lambda\rangle i^{l_1+L_1}\hat{L}\hat{L}'\hat{L}_1\hat{l}_1{}^2\hat{l}\hat{l}'\hat{J}_f\hat{j}_\beta{}^2j_\nu\hat{j}_{\nu'}\hat{\lambda}\hat{\lambda}'\begin{pmatrix}L' & L_1 & L\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}l' & l_1 & l\\ 0 & 0 & 0\end{pmatrix}$$
$$\times\begin{pmatrix}l_1 & L_1 & J_f\\ 0 & M_f & -M_f\end{pmatrix}\begin{Bmatrix}l_1 & L_1 & J_f\\ l & L & \lambda\\ l' & L' & \lambda'\end{Bmatrix}Y_{L_1}{}^{M_f*}(\omega_{\mathrm{k}})h_{L_1}{}^{NLN'L'}(K)g_{l_1}{}^{nln'l'}(Q)\,.\qquad\text{(A8)}$$

For $\Delta_i{}^{(\sigma)}$ one obtains the following expressions:

$$\Delta_i{}^{(\sigma)}\equiv D_i+E_i\,,\qquad\text{(A9)}$$

$$D_i=\sum_{\nu\nu'}\sum_{\beta_{\mathrm{occ}}}\Big[\alpha_1{}^{(\sigma_1)}\sum_{l_1}M_{\nu'\beta\beta\nu}{}^i(l_1)+\alpha_1{}^{(\sigma_2)}\sum_{l_1}(-)^{l_1}M_{\nu'\beta\beta\nu}{}^i(l_1)+\alpha_1{}^{(\sigma_1\sigma_2)}N_{\nu'\beta\beta\nu}{}^i\Big]\,,\qquad\text{(A10)}$$

$$E_i=\sum_{\nu\nu'}\sum_{\beta_{\mathrm{occ}}}\Big[\alpha_2{}^{(\sigma_1)}P_{\beta\nu'\beta\nu}{}^i+\alpha_2{}^{(\sigma_2)}Q_{\beta\nu'\beta\nu}{}^i+\alpha_2{}^{(\sigma_1\sigma_2)}R_{\beta\nu'\beta\nu}{}^i\Big]\,,\qquad\text{(A11)}$$

where

$$M_{\nu'\beta\beta\nu}{}^i(l_1)=\sum_{\lambda\lambda'L_1l_2}C_{\nu'\beta\beta\nu}{}^i(\lambda\lambda'L_1l_2;l_1)\Bigg[(-)^{M_f+j_\nu+j_{\nu'}+i}Y_{L_1}{}^{(-M_f+i)*}(\omega_{\mathrm{k}})$$
$$\times\begin{Bmatrix}j_\nu & l_\nu & \frac{1}{2} & \frac{1}{2}\\ j_{\nu'} & l_{\nu'} & \frac{1}{2} & l_2\\ J_f & \lambda' & 1 & \lambda\end{Bmatrix}-(-)^{J_f+\lambda'+L'+l'}Y_{L_1}{}^{(M_f-i)*}(\omega_{\mathrm{k}})\begin{Bmatrix}j_{\nu'} & l_{\nu'} & \frac{1}{2} & \frac{1}{2}\\ j_\nu & l_\nu & \frac{1}{2} & l_2\\ J_f & \lambda' & 1 & \lambda\end{Bmatrix}\Bigg]\,.\qquad\text{(A12)}$$

Equation (A12) involves 12-j symbols as defined, e.g., by Jahn and Hope.[26] The coefficients C are defined as

$$C_{\nu'\beta\beta\nu}{}^{i}(\lambda\lambda' L_1 l_2; l_1) = -2(6\pi^5)^{1/2}\chi_{\nu'\nu}{}^{(f)*}(-)^{\frac{1}{2}-j_\beta+l_\nu} \sum_{\substack{NLN'L'\\ nln'l'}} i^{l_1+L_1}\langle' |\nu'\nu\lambda'\rangle$$

$$\times\langle\ |\beta\beta\lambda\rangle \hat{L}\hat{L}'\hat{L}_1\hat{l}_1{}^2\hat{l}\hat{l}'\hat{J}_f\hat{j}_\beta{}^2\hat{j}_\nu\hat{j}_{\nu'}\hat{\lambda}^2\hat{\lambda}'^2\hat{l}_2{}^2 \begin{pmatrix} L' & L_1 & L \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l' & l_1 & l \\ 0 & 0 & 0\end{pmatrix}$$

$$\times\begin{pmatrix} l_1 & L_1 & l_2 \\ 0 & -M_f+i & M_f-i\end{pmatrix}\begin{pmatrix} 1 & l_2 & J_f \\ i & M_f-i & -M_f\end{pmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & \lambda \\ l_\beta & l_\beta & j_\beta\end{Bmatrix}\begin{Bmatrix} l_1 & L_1 & l_2 \\ l & L & \lambda \\ l' & L' & \lambda'\end{Bmatrix} h_{L_1}{}^{N'L'NL}(K) g_{l_1}{}^{n'l'nl}(Q)\,, \quad \text{(A13)}$$

$$N_{\nu'\beta\beta\nu}{}^{i} = \sum F_{\nu'\beta\beta\nu}{}^{i}(Ll l' L_1\lambda l_2 l_3)[(-)^{l_\nu+L+l+\lambda} Y_{L_1}{}^{M_f-(j+k)*}(\omega_k) - (-)^{M_f+L_1+l'+l_2+l_3+(j+k)} Y_{L_1}{}^{(j+k)-M_f*}(\omega_k)]\,, \quad \text{(A14)}$$

where

$$F_{\nu'\beta\beta\nu}{}^{i}(Ll l' L_1\lambda l_2 l_3) = -36(8\pi^5)^{1/2}\chi_{\nu'\nu}{}^{(f)*}(-)^{J_f-j_\beta+1/2}\sum i^{l_1+L_1}\langle' |\nu'\nu\lambda'\rangle$$

$$\times\langle\ |\beta\beta\lambda\rangle \hat{L}\hat{L}'\hat{L}_1\hat{l}_1{}^2\hat{l}\hat{l}'\hat{J}_f\hat{j}_\beta{}^2\hat{j}_\nu\hat{j}_{\nu'}\hat{\lambda}^2\hat{\lambda}'^2\hat{l}_2{}^2\hat{l}_3{}^2 \begin{pmatrix} L' & L_1 & L \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l' & l_1 & l \\ 0 & 0 & 0\end{pmatrix}$$

$$\times\begin{pmatrix} l_1 & L_1 & l_3 \\ 0 & (j+k-M_f) & (M_f-[j+k])\end{pmatrix}\begin{pmatrix} J_f & l_3 & 1 \\ -M_f & M_f-(j+k) & (j+k)\end{pmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & \lambda \\ l_\beta & l_\beta & j_\beta\end{Bmatrix}\begin{Bmatrix} J_f & l_3 & 1 \\ \lambda & l_2 & \lambda'\end{Bmatrix}$$

$$\times\begin{Bmatrix} J_f & \lambda' & l_2 \\ j_{\nu'} & l_{\nu'} & \frac{1}{2} \\ j_\nu & l_\nu & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix} l_2 & \lambda & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1\end{Bmatrix}\begin{Bmatrix} l_1 & L_1 & l_3 \\ l & L & \lambda \\ l' & L' & \lambda'\end{Bmatrix} h_{L_1}{}^{N'L'NL}(K) g_{l_1}{}^{n'l'nl}(Q)\,. \quad \text{(A15)}$$

Here (i,j,k) form an even permutation of $(-1,0,1)$. The quantities P^i, Q^i, and R^i are defined by

$$P_{\beta\nu'\beta\nu}{}^{i} = -4(3\pi^5)^{1/2}\chi_{\nu'\nu}{}^{(f)*}\sum(-)^{j_\nu-j_\beta+l_\beta+l_{\nu'}+J_f+L+\lambda+\lambda'+l'} i^{l_1+L_1}$$

$$\times\langle' |\beta\nu'\lambda'\rangle\langle\ |\beta\nu\lambda\rangle \hat{L}\hat{L}'\hat{L}_1\hat{l}_1{}^2\hat{l}\hat{l}'\hat{J}_f\hat{j}_\beta{}^2\hat{j}_\nu\hat{j}_{\nu'}\hat{\lambda}^2\hat{\lambda}'^2\hat{l}_2{}^2 \begin{pmatrix} L' & L_1 & L \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l' & l_1 & l \\ 0 & 0 & 0\end{pmatrix}$$

$$\times\begin{pmatrix} 1 & J_f & l_2 \\ i & -M_f & M_f-i\end{pmatrix}\begin{pmatrix} l_1 & L_1 & l_2 \\ 0 & M_f-i & i-M_f\end{pmatrix}\begin{Bmatrix} l_\beta & 1 & l_\beta \\ \frac{1}{2} & j_\beta & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix} l_\nu & l_{\nu'} & J_f \\ j_{\nu'} & j_\nu & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix} 1 & J_f & l_2 \\ l_\beta & l_{\nu'} & \lambda' \\ l_\beta & l_\nu & \lambda\end{Bmatrix}$$

$$\times\begin{Bmatrix} l_1 & L_1 & l_2 \\ l & L & \lambda \\ l' & L' & \lambda'\end{Bmatrix} h_{L_1}{}^{N'L'NL}(K) g_{l_1}{}^{n'l'nl}(Q) Y_{L_1}{}^{(M_f-i)*}(\omega_k)\,, \quad \text{(A16)}$$

$$Q_{\beta\nu'\beta\nu}{}^{i} = -4(3\pi^5)^{1/2}\chi_{\nu'\nu}{}^{(f)*}\sum(-)^{j_\beta-j_{\nu'}+J_f+L+l'+l_{\nu'}+l_\beta+\lambda} i^{l_1+L_1}\langle' |\beta\nu'\lambda'\rangle\langle\ |\beta\nu\lambda\rangle \hat{L}\hat{L}'\hat{L}_1\hat{l}_1{}^2\hat{l}\hat{l}'\hat{J}_f\hat{j}_\beta{}^2\hat{j}_\nu\hat{j}_{\nu'}\hat{\lambda}^2\hat{\lambda}'^2\hat{l}_2{}^2$$

$$\times\frac{1}{\hat{l}_\beta{}^2}\begin{pmatrix} L' & L_1 & L \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l' & l_1 & l \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} 1 & J_f & l_2 \\ i & -M_f & M_f-i\end{pmatrix}\begin{pmatrix} l_1 & L_1 & l_2 \\ 0 & M_f-i & i-M_f\end{pmatrix}\begin{Bmatrix} l_2 & \lambda & \lambda' \\ l_\beta & l_{\nu'} & l_\nu\end{Bmatrix}$$

$$\times\begin{Bmatrix} J_f & 1 & l_2 \\ j_{\nu'} & \frac{1}{2} & l_{\nu'} \\ j_\nu & \frac{1}{2} & l_\nu\end{Bmatrix}\begin{Bmatrix} l_1 & L_1 & l_2 \\ l & L & \lambda \\ l' & L' & \lambda'\end{Bmatrix} h_{L_1}{}^{N'L'NL}(K) g_{l_1}{}^{n'l'nl}(Q) Y_{L_1}{}^{(M_f-i)*}(\omega_k)\,, \quad \text{(A17)}$$

[26] H. A. Jahn and J. Hope, Phys. Rev. **93**, 318 (1954); R. J. Ord-Smith, *ibid.* **94**, 1227 (1954).

$$R_{\beta\nu'\beta\nu}{}^{i}=\sum_{l_2l_3} G_{\beta\nu'\beta\nu}{}^{i}(l_2l_3)\Bigg[(-)^{j}\begin{pmatrix} J_f & 1 & l_3 \\ -M_f & k & M_f-k\end{pmatrix}\begin{pmatrix} 1 & l_3 & l_2 \\ j & k-M_f & M_f-(j+k)\end{pmatrix}-(-)^{k} \times\begin{pmatrix} J_f & 1 & l_3 \\ -M_f & j & M_f-j\end{pmatrix}\begin{pmatrix} 1 & l_3 & l_2 \\ k & j-M_f & M_f-(j+k)\end{pmatrix}\Bigg], \quad \text{(A18)}$$

where (i,j,k) is again an even permutation of $(-1,0,1)$, and

$$G_{\beta\nu'\beta\nu}{}^{i}(l_2l_3)=-6(8\pi^5)^{1/2}\chi_{\nu'\nu}{}^{(f)*}\sum(-)^{j_\nu+j_{\nu'}+j_\beta+\frac{1}{2}+l_\nu+l_{\nu'}+l_\beta+M_f+\lambda+\lambda'+L+l'+l_3}\langle'|\beta\nu'\lambda'\rangle\langle|\beta\nu\lambda\rangle$$
$$\times i^{l_1+L_1}\hat{L}\hat{L}'\hat{L}_1\hat{l}_1{}^2\hat{l}\hat{l}'\hat{J}_f\hat{j}_\beta{}^2\hat{j}_\nu\hat{j}_{\nu'}\hat{\lambda}^2\hat{\lambda}'^2\hat{l}_2{}^2\hat{l}_3{}^2\begin{pmatrix} L' & L_1 & L \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l' & l_1 & l \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l_2 & L_1 & l_1 \\ M_f-(j+k) & j+k-M_f & 0\end{pmatrix}\begin{Bmatrix} l_\beta & 1 & l_\beta \\ \frac{1}{2} & j_\beta & \frac{1}{2}\end{Bmatrix}$$
$$\times\begin{Bmatrix} J_f & 1 & l_3 \\ j_{\nu'} & \frac{1}{2} & l_{\nu'} \\ j_\nu & \frac{1}{2} & l_\nu\end{Bmatrix}\begin{Bmatrix} 1 & l_3 & l_2 \\ l_\beta & l_{\nu'} & \lambda' \\ l_\beta & l_\nu & \lambda\end{Bmatrix}\begin{Bmatrix} l_1 & L_1 & l_2 \\ l & L & \lambda \\ l' & L' & \lambda'\end{Bmatrix} h_{L_1}{}^{N'L'NL}(K)g_{l_1}{}^{n'l'nl}(Q)Y_{L_1}{}^{M_f-(j+k)*}(\omega_{\mathbf{k}}). \quad \text{(A19)}$$

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Projected Hartree-Fock Spectra of 2s–1d-Shell Nuclei

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The Hartree-Fock calculations with a phenomenological internucleon residual interaction are carried out for the 2s–1d-shell nuclei up to ^{25}Al. The low-lying excited states of these nuclei are obtained by projecting out good angular-momentum states from the deformed Hartree-Fock states. The energy spectra thus obtained are in good agreement with the experimental results. From the study of the odd-odd nuclei, it is found that the employed residual interaction correctly reproduces the ground-state spins of these nuclei. The binding energies calculated from the projected ground-state energies and a naive model are in very good agreement with the experimental binding energies.

1. INTRODUCTION

THE complexity of shell-model calculations for more than three nucleons prohibits any such calculations for many-nucleon systems. For nuclei in the 2s–1d shell, the shell-model calculations have been carried out in the case of at the most four particles outside the ^{16}O core.[1] Since the exact shell-model calculations for many-nucleon systems are prohibitive, attempts have been made to do the next best thing. Redlich[2] showed for the first time that the results of shell-model calculations can be reproduced by projecting the good angular-momentum states from an intrinsic determinantal state. These observations of Redlich in the 2s–1d shell were confirmed by Kurath and Picman,[3] who showed that for nuclei in the 1p shell as well, the projection method is a good approximation to the configuration-mixing calculations. This success of the projection method in obtaining shell-model wave functions implies that there is an underlying independent-particle behavior in these wave functions. The natural tool to study this independent-particle behavior is the Hartree-Fock (HF) method. The recent calculations of Bassichis, Giraud, and Ripka[4] clearly demonstrate that one can derive the energy spectra of nuclei in the 2s–1d shell by projecting out good angular-momentum states from an intrinsic (HF) state composed of the deformed single-particle orbitals.

With this success of the projection prescription, we

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