Surface Delta Interaction and Collective $E2$ and $E3$ Transitions in Rare-Earth Nuclei*

AMAND FAESSLER† AND A. PLASTINO‡
University of California, Los Angeles, California
(Received 21 October 1966)

γ-ray transition probabilities from the ground state into the γ-vibrational band and the $K=0$ octupole vibrational band in even-mass rare-earth nuclei are calculated. The surface delta interaction (SDI) and the pairing-plus-quadrupole (PQF) or -octupole force (POF) are utilized as residual interactions. The quasiparticle random-phase approximation (QRPA) is employed in the calculation. Both residual interactions (SDI and PQF) give good agreement with experiment for electric quadrupole transitions, using an effective charge equal to Z/A. This value is required in a self-consistency argument given by Mottelson. For octupole transitions, good results are also obtained using the same effective charge. The Tamm-Dancoff approximation is compared with the RPA. It is shown that it does not yield reasonable results when used in this kind of calculation.

1. INTRODUCTION

TRANSITION probabilities from the ground state into the vibrational states in heavy deformed nuclei have been calculated by several authors[1] utilizing the pairing-plus-quadrupole- (PQF) or pairing-plus-octupole-force (POF) model.

Recently, the surface delta interaction[2] (SDI) has been successfully applied[3,4] in a calculation of the vibrational energies in heavy deformed nuclei. Here we want to utilize the SDI wave functions[4] to calculate electric quadrupole transition probabilities from the ground state into the γ-vibrational band head and electric octupole transitions into the 3^- state of the $K=0$ octupole vibrational band.

The SDI has, when compared to the PQF or POF model, the advantage of possessing only one free parameter, whose value can be obtained by fitting the odd-even mass differences. One is left, therefore, with no free parameter to fit nuclear spectra. The conventional PQF-POF model has essentially three different parameters: the pairing constant and the quadrupole and octupole coupling parameters.

In this paper, transition probabilities into the γ-band heads and into the $K=0$ octupole vibrational band are calculated within the framework of both the quasiparticle random-phase approximation (QRPA) and the quasiparticle Tamm-Dancoff approximation (QTDA). A comparison of the experimental values for those transitions with the theoretical results shows good agreement for the QRPA. The QTDA underestimates the correlations into the ground state and yields, therefore, too small values.[5]

One has to be very careful in comparing these values with the results for the PQF or POF model given in the literature.[1] The pioneering paper of Marschalek and Rasmussen utilizes the adiabatic model, but it is well known that the results of this approximation are not very accurate. The calculation of Bès, Federman, Maqueda, and Zucker[1] neglects, in the case of the γ vibrations, the two quasiparticle states for which one has to add up the projection quantum numbers $|\Omega_1+\Omega_2|=2$. They consider only those states with $|\Omega_1-\Omega_2|=2$. But they take into account the rotation-vibration interaction due to the Coriolis force and find the same results already given by the phenomenological model.[6] The transition probability is enhanced by about 30%. This enhancement lies normally within the experimental accuracy limits. We have neglected this effect here.

It can be shown, by considering the different summation convention, normalization condition, and modified definitions,[4] that the transition formula for the PQF or POF model given here agrees with the expression which Soloviev and Vogel[1] use. But in their computer program a factor $(2I+1)/2$ was missing.[7] This led to the determination of a too large effective charge. But because the effective charge does not enter linearly into the transition amplitude, one is not able to obtain corrected values from the data given in their publication. We have therefore calculated the transition probabilities also in the PQF and POF model, and the corresponding results are as good as those obtained using the SDI.

* This work was supported in part by the National Science Foundation.

† Present address: University of Freiburg, Freiburg, Germany.

‡ Permanent address: Universidad de la Plata, Argentina. Fellow of the Consejo Nacional de Investigationes Cientificas y Tecnicas of Argentina.

[1] E. R. Marschalek and J. O. Rasmussen, Nucl. Phys. **43**, 438 (1963); D. R. Bès, *ibid.* **49**, 544 (1963); P. Vogel, Yadernaya Fiz. **1**, 752 (1965) [English transl.: Soviet J. Nucl. Phys. **1**, 538 (1965)]; V. G. Soloviev, Nucl. Phys. **69**, 1 (1965); V. G. Soloviev, At. Energy Rev. **3**, 117 (1965); D. R. Bès, P. Federman, M. Maqueda, and A. Zuker, Nucl. Phys. **65**, 1 (1965).

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[6] A. Faessler, W. Greiner, and R. K. Sheline, Nucl. Phys. **70**, 33 (1965).

[7] We thank Dr. Soloviev and Dr. Vogel for communications about this point. Their best fit for the effective charge is after the above correction $e_{\text{eff}}=0.2-0.3$. [See also V. G. Soloviev and P. Vogel, Dubna report No. E-2561, 1966 (unpublished).] This value agrees fairly well with our effective charge $e_{\text{eff}}=Z/A$.

2. FORMALISM AND DETAILS OF THE CALCULATION

The surface delta interaction[2] and the pairing-plus-quadrupole or -octupole force have been utilized as the residual interaction. In the expression for the SDI

$$V(1,2)=-4\pi F(u_0R_0)^{-4}\delta(r_1-R_0)\delta(r_2-R_0)\delta(\theta_{12}),$$

we assume that the amplitude $u_0\equiv u(R_0)$ of the radial function at the nuclear surface R_0 is constant. Numerical calculations utilizing the Saxon-Woods potential confirm this assumption.[8] The radial integrals and the factor $(R_0u_0)^{-4}$ give thus a factor unity.

The energies and eigenfunctions used in calculating transition probabilities in heavy deformed nuclei are taken from a publication of Faessler, Plastino, and Moszkowski.[3,4] The reduced electric multipole transition probability from a state $|I_ig\rangle$ in the ground-state rotational band into a vibrational band is given by the expression

$$B(E\lambda;I_ig\rightarrow I_fK)=C^2(I_i\lambda I_f;0KK)M^2, \quad (1)$$

with the matrix element

$$M=\sum_{\mu\geq\nu>0}(u_\mu v_\nu+u_\nu v_\mu)(e_p+e_{\text{eff}})[Q_{\mu\nu}{}^{\lambda K}(1-\delta_{\mu,\nu})(\xi_{\mu\nu}+\eta_{\mu\nu})$$
$$+Q_{\mu\bar{\nu}}{}^{\lambda K}[(1+\delta_{K,0})/(1+\delta_{\mu,\nu})]^{1/2}(\xi_{\mu\bar{\nu}}+\eta_{\mu\bar{\nu}})], \quad (2)$$

and the charge

$$e_p=1 \quad \text{for protons}$$
$$=0 \quad \text{for neutrons}. \quad (3)$$

The multipole matrix elements

$$Q_{\mu\nu}{}^{\lambda K}=-\langle\mu|r^\lambda Y_{\lambda K}|\bar{\nu}\rangle, \quad Q_{\mu\bar{\nu}}{}^{\lambda K}=\langle\mu|r^\lambda Y_{\lambda K}|\nu\rangle, \quad (4)$$

and the mixing coefficients $\xi_{\mu\nu}$, $\eta_{\mu\nu}$ for the two-quasiparticle states are given in Ref. 4 in formulas (6), (7), and (9). (The Q's are named D's in this reference.) The minus sign in formula (4) has been introduced as in Refs. 3 and 4, to make the expressions more symmetric. The u's and the v's are the coefficients of the Bogoliubov-Valatin transformation. The bar above a state ($\bar{\nu}$) indicates the time-reversed Nilsson state. If not specifically indicated by the notation $\nu>0$, the summation runs over the time-reversed states too, as, for example, in Eq. (6) for the subscript ν. (In the QTDA one has to put all the η's equal to zero.) For the PQF or the POF model, one gets the same formula. But now the mixing coefficients ξ and η are differently defined.[4]

The $E2$ transition probabilities from the ground state to the γ-vibrational band head are shown in Figs. 1 and 2. They are given in single-particle units:

$$B_{\text{SP}}(E\lambda)=\frac{2\lambda+1}{4\pi}\left[\frac{3}{\lambda+3}R_0{}^\lambda\right]^2 e^2\ \text{cm}^{2\lambda}. \quad (5)$$

In obtaining the values shown in Fig. 1, the SDI has been utilized, while in Fig. 2 the PQF has been employed. The method (a) utilizes the parameters of Table I. These parameters are fitted independently by the odd-even mass differences for protons and neutrons, by the γ-vibrational energies and by the $K=0$ octupole vibrational band heads. For the pairing-plus-quadrupole force (PQF) and the pairing-plus-octupole force (POF) model one has essentially three different parameters. The important point to be noted is that for the SDI one gets, from totally independent fits, the same parameter $A\times F$ (for the RPA) within $\pm 20\%$.

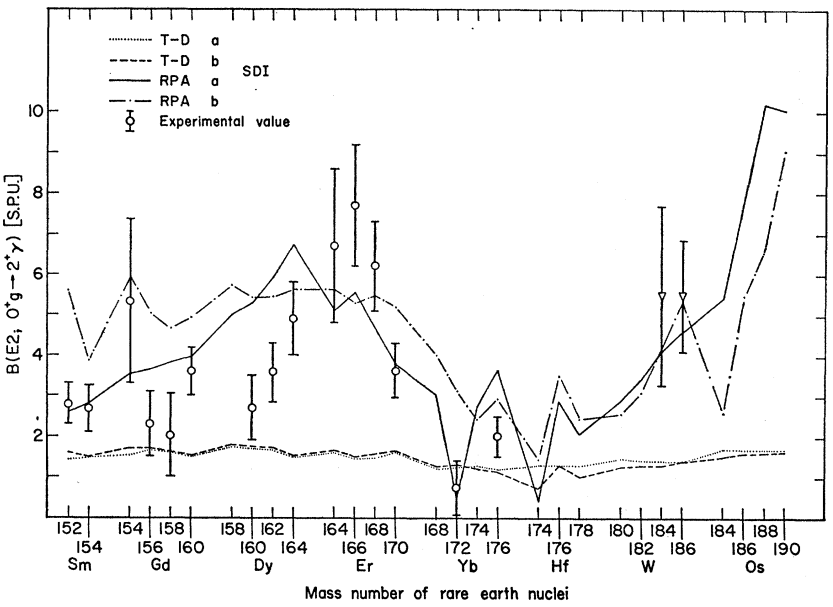

FIG. 1. The reduced electric quadrupole transition probability from the ground state into the γ-vibrational band head is shown for the rare-earth region. The SDI has been utilized as the residual interaction. The solid and the dashed-dotted line are calculated utilizing the QRPA. For the two other lines the QTDA has been employed. Case (a) is calculated with the parameters of Table I. For case (b) the coupling parameter has been adapted by fitting for every nucleus the energy of the γ-vibrational band head. For the effective charge the value $e_{\text{eff}}=Z/A\simeq 0.4$ has been taken. The experimental values are from Yoshizawa *et al.* (Ref. 9) and from McGowan and Stelson (Ref. 9). All values are given in single-particle units.

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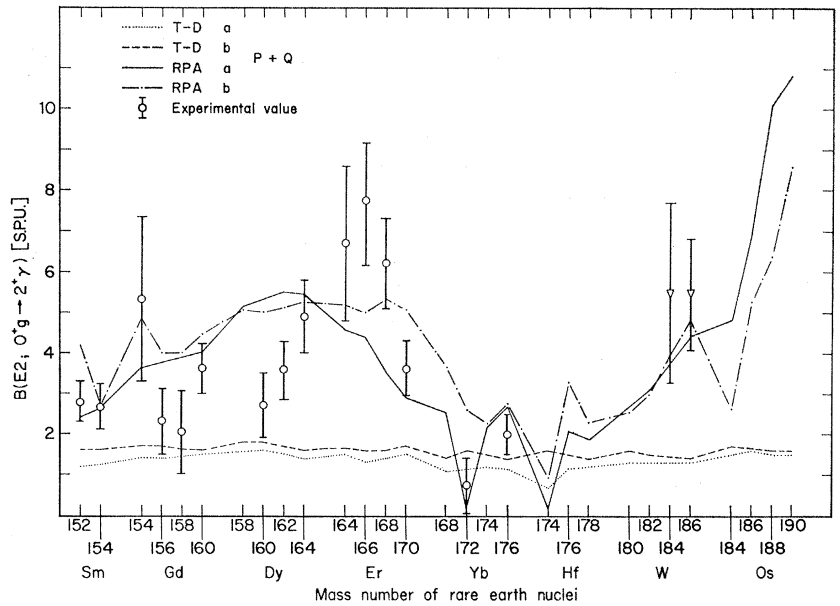

FIG. 2. The reduced electric quadrupole transition probability is given in single-particle units for the rare-earth region. The residual interaction is the PQF. Further details are the same as in Fig. 1.

In method (b) the coupling parameter has been adapted to reproduce for every nucleus the experimental γ-vibrational energy.

If one neglects the two-quasiparticle states with $|\Omega_1+\Omega_2|=2$ for the γ-vibrational band, the coupling constant F is renormalized. One needs a value of about $A\times F=40$ MeV (instead $A\times F=27.5$ MeV) to reproduce the experimental energies and the transition probabilities. This means that, in fitting the strength of the SDI, first using odd-even mass differences and then reproducing γ-vibrational energies, one finds that those two values do not agree with each other. Therefore, the above-mentioned special approximation cannot be used for an SDI.

TABLE I. The parameters for the pairing-plus-quadrupole (PQF) or -octupole (POF) force model and the surface delta interaction (SDI) are listed for the random-phase approximation (RPA) and the Tamm-Dancoff approximation (TDA). The parameters are adapted independently by the odd-even mass differences for protons and neutrons (subscripts p and n), by the γ-vibrational band heads (subscript 2), and by the $K=0$ octupole vibrational energies (subscript 3). The coupling constants of the quadrupole (k_2) and octupole force (k_3) are given as in Soloviev (Ref. 1). In the PQF or POF model one gets three different parameters. For the SDI the RPA yields within $\pm 20\%$ the same value $A\times F=27.5$ MeV.

		RPA	TDA
PQF, POF	$A\times G_p$ [MeV]	28–29	28–29
PQF, POF	A$\times G_n$ [MeV]	26–27	26–27
PQF	k_2 [dimensionless], $A<178$	8.57	13.46
PQF	k_2 [dimensionless], $A\geq 178$	7.65	13.46
POF	k_2 [dimensionless]	1.04	1.84
SDI	$A\times F_p$ [MeV]	28–29	28–29
SDI	$A\times F_n$ [MeV]	26–27	26–27
SDI	$A\times F_2$ [MeV], $A<178$	27.5	43.0
SDI	$A\times F_2$ [MeV], $A\geq 178$	22.5	43.0
SDI	$A\times F_3$ [MeV]	34.0	58.0

The experimental values (circles) are taken from Yoshizawa *et al.*[9] The two values for the W isotopes are measured by McGowan and Stelson.[9] The QRPA yields values which agree nicely with the experimental results. The QTDA gives too small values. This will be discussed later on. The effective charge is $e_{\text{eff}}=Z/A \simeq 0.4$ for the electric quadrupole transitions. Mottelson[10] has shown, utilizing a self-consistency argument, that one expects, far away from closed shells, this effective charge. Tamura and Udagawa[11] have fitted the effective charge in adjacent spherical nuclei: The effective charge is equal to 1 in the Sn isotopes and decreases to 0.4 in the Nd isotopes. A similar diminution is found if one goes from the Hg to the Pt isotopes.

The electric octupole transitions $B(E3; 0^+g \rightarrow 3^-_{K=0})$ into the 3^- state of the $K=0$ octupole vibrational band are shown in Figs. 3 and 4. The coupling constants have been chosen according to Table I. But the value corresponding to the mass number 152 was utilized for the whole rare-earth region.

For the $E3$-transition probabilities, no theoretical argument exists for the effective charge. We chose the same value as for the quadrupole transitions ($e_{\text{eff}} = Z/A \simeq 0.4$). The QRPA gives good results for the SDI and POF model. The QTDA yields, as in the case of the quadrupole transitions, too small values. The experimental values are taken from Ref. 12.

The unsatisfactory results of the QTDA come from the poorer quality of this approximation compared with

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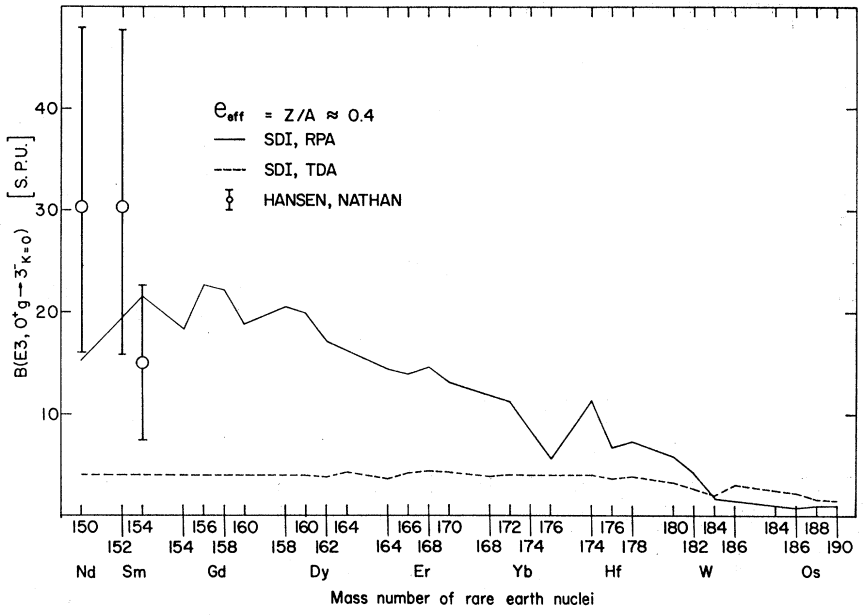

FIG. 3. The reduced electric octupole transition probability from the ground state into the 3^- $K=0$ state of the octupole vibrational band is given, for the rare-earth nuclei, in single-particle units. The residual interaction is the SDI. The solid line is calculated in the QRPA. For the dashed line the QTDA has been employed. The effective charge is again $e_{eff}=Z/A$. The experimental values are measured by Hansen and Nathan (Ref. 12).

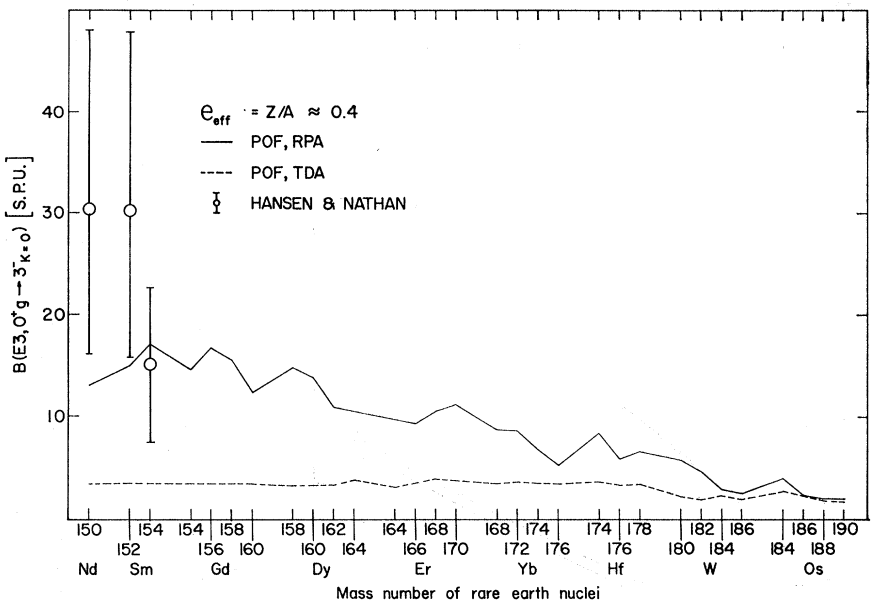

FIG. 4. The reduced electric octupole transitions from the ground state into the 3^- state of the $K=0$ octupole vibrational band are shown in single-particle units for the rare-earth nuclei. The residual interaction is the POF. Further details are the same as in Fig. 3.

the QRPA.[13] The difference between both approximations can be discussed analytically for the PQF and the POF in the degenerate model (see, for example, Brown[14]). In the degenerate case, one takes all the two-quasiparticle energies equal to one value E. Then one finds in the QTDA for the PQF or POF model

$$B_{\mathrm{TDA}}(E\lambda) \propto \sum_{\mu\geq\nu;\,\mu>0} Q_{\mu\nu}{}^{\lambda K} \tag{6}$$

and in the QRPA

$$B_{\mathrm{RPA}}(E\lambda) = (E/\omega) B_{\mathrm{TDA}}(E\lambda). \tag{7}$$

Here we have put all charges equal to 1 in order to simplify the expression. For the QTDA one finds, therefore, in the degenerate case, a constant value for the reduced transition probability. Figures 1 to 4 show that this is also approximately true for the correct

[13] It should be noted that the special approximations that one is forced to make because of the involved numerical calculations required in this region of the periodic table (Ref. 3), make the TDA a particularly poor approximation. As a matter of fact, when separability is not a condition one must enforce (Ref. 3), the TDA gives results which are not so poor in comparison to those yielded by the RPA for a surface delta interaction.

[14] G. E. Brown, *Unified Theory of Nuclear Models* (North-Holland Publishing Company, Amsterdam, 1964).

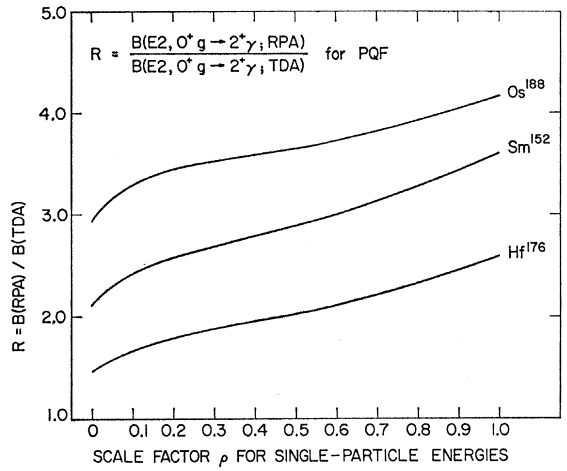

FIG. 5. The ratio R of the reduced electric quadrupole transition into the γ-vibrational band head of the RPA over that of the TDA [see formula (8)] is shown for three rare-earth nuclei. The residual interaction is the PQF. This ratio R is plotted against a scale factor ρ with which the single-particle energies are multiplied. If ρ varies from one to zero, one goes from the correct single-particle energies to the degenerate case. At the same time the pairing constants G_p and G_n are reduced to maintain the experimental energy gap.

two-quasiparticle energies. Equation (7) shows that the transition probability in the QRPA is proportional to the reciprocal of the collective energy. This again is also approximately the case for the correct solution.

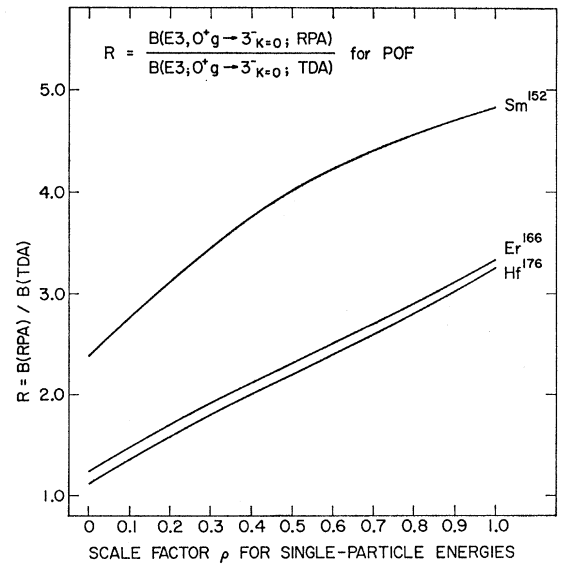

FIG. 6. The ratio R is given for electric octupole transitions into the 3^- state of the $K=0$ octupole vibrational band in three nuclei. The resudual interaction is the POF. Further details are the same as in Fig. 5.

Figures 5 and 6 show the ratio

$$R=B_{\mathrm{RPA}}(E\lambda)/B_{\mathrm{TDA}}(E\lambda) \tag{8}$$

obtained when one goes from the correct to the degenerate solution, for several nuclei in the rare-earth region. The single-particle energies are multiplied with a scale factor ρ, which varies between 1 and (the degenerate case) 0. At the same time the pairing constants G_p and G_n are also reduced to reproduce always the experimental energy gap. The figures show two things which one expects according to Eqs. (6) and (7): The ratio R decreases as ρ gets smaller, because the average two-quasiparticle energy is reduced together with ρ. The ratio R is larger for nuclei with a low vibrational energy ω.

3. CONCLUSION

Electric quadrupole and octupole transition probabilities from the ground state into the γ band and $K=0$ octupole vibrational band have been calculated for the rare-earth nuclei. The residual interaction is the surface delta interaction (SDI).

This force has been already successfully applied to calculate energies in different types of nuclei.[2–4,15] Here for the first time transition probabilities have been calculated with this force. We have selected the heavy deformed even mass nuclei. Because of their high level density, it is not numerically feasible today to use here more realistic forces, like the Yale potential, in calculating excited states. In these nuclei only the pairing plus quadrupole (PQF) or octupole (POF) have been employed to calculate low-lying vibrational states. Compared to this model the SDI is an improvement: It could be qualitatively understood from the free nucleon-nucleon scattering[16] and it has only one parameter compared with three for the PQF and POF model.

ACKNOWLEDGMENTS

We wish to express our gratitude to Professor S. A. Moszkowski for suggesting this problem and for his constant interest and support.

We are very indebted to the UCLA Computing Facility for making computer time available to us, and are very grateful also to Dr. D. Saxon for his hospitality in the UCLA Physics Department.

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