Second-Sound Attenuation in Rotating Helium II Close to the Lambda Point

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The attenuation of second sound in rotating resonant cavities containing helium II has been measured in the temperature range $10^{-1} > T_\lambda - T > 6\times10^{-4}$ °K, and values of Hall and Vinen's parameter B have been calculated from these data. The results show that B is only slightly temperature dependent, in qualitative agreement with the predictions of Pellam, but in disagreement with the theories of Hall and Vinen and of Lifshitz and Pitaevskii.

INTRODUCTION

SOME time ago, Hall and Vinen[1] measured the excess attenuation of second sound in rotating liquid helium in the temperature range 1.25 to 2.10°K. They thus determined the temperature dependence of the parameter B, which appears in their equations of motion of helium II.[2] Their results agree with their kinetic theory based on the interaction of independent particle excitations (rotons) with vortex lines,[3,4] and also with the two-fluid theory of Pellam.[5] In the region above 2.1°K, according to Hall and Vinen, the behavior of B depends strongly on whether the vortex lines are assumed to experience a Magnus force and move relative to the superfluid, or obey Kelvin's theorem for free vortex filaments and move with the superfluid. These two alternatives require B to vanish or go to infinity, respectively. However, as the temperature approaches the λ point ($T_\lambda = 2.1720$°K), the independent-particle approximation is not valid, and so the Hall-Vinen theory cannot be expected to give good predictions in this temperature range. Pellam's phenomenological theory only requires the two-fluid model to hold. It predicts that if the flow pattern is roughly temperature-independent, B should be temperature-insensitive and of the order unity.

The purpose of the present experiment was to measure B between 2.1°K and the λ point and to see whether any of the above theories has a basis in experiment. Our results suggest that B approaches a finite value at the λ point, in conflict with the theory of Hall and Vinen, but not inconsistent with that of Pellam.

EXPERIMENTAL TECHNIQUE

The total-attenuation coefficient α, of second sound was measured by observation of the rate of decay of a resonant excitation in a cavity, when the input signal was turned off.

The difference between the time decay constants λ [such that the amplitude is proportional to $\exp(-\lambda t)$] with the resonator rotating and nonrotating was measured, from which the parameter B was obtained. If the decay is exponential, we can set $\lambda = \alpha u_2$, and then[1]

$$\begin{aligned} B &= (2u_2/\omega)[\alpha(\text{rotating}) - \alpha(\text{nonrotating})] \\ &= (2/\omega)[\lambda(\text{rotating}) - \lambda(\text{nonrotating})] \\ &= (2/\omega)(\lambda' - \lambda)\,, \end{aligned}$$

where u_2 is the second-sound velocity and ω is the angular rotation speed.

The apparatus was developed from that designed and used in a preliminary form by Hall while on a short visit to this department. Second sound was generated by passing an alternating current through an Aquadag film on a cavity wall, and was detected by observing the varying resistance of a similar film, painted on the opposite wall. The films employed had a temperature sensitivity $(1/R)dR/dT$, of approximately 0.25 deg^{-1}.

The principal cavity $C(1)$ consisted of a Perspex cylinder symmetric about its axis of rotation with films on both ends and on the curved wall. Its length and radius were both 2.5 cm. Its ends were perforated by six 1-mm-diam holes (outside the Aquadag film) to ensure good thermal contact between the inside and the external helium bath, and it was contained within a brass bucket which rotated with it. Two other resonators of different shape from $C(1)$ were also used and are discussed below. The signal from the second-sound detector was amplified by about 10^4, filtered, and displayed on an oscilloscope. Displays of the decaying signal from the cavity were photographed to yield λ or λ'. A correction had to be made for the frequency response of the filter which depended on the resonant frequency.

In order to maintain a constant temperature in the cavity during the decay of a resonance near the λ point, it was found necessary to turn on a dc heat supply in the resonator at the instant the signal was turned off. The rate of the dc heat supply was chosen so that, when the signal was turned on again after the elapse of a few decay times, it had minimum rise time. This proved to be at a power level close to that of the ac input, which was about 5 mW. For u_2 greater than

[1] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. (London) **A238**, 204; **A238**, 215 (1956).
[2] H. E. Hall, in *Proceedings of the International School of Physics, "Enrico Fermi" Course XXI* (Academic Press Inc., New York, 1963).
[3] L. Onsager, Nuovo Cimento **6**, Suppl. 2, 249 (1949).
[4] R. P. Feynman, *Progress in Low-Temperature Physics* (North-Holland Publishing Company, Amsterdam, 1955), Vol. 1, Chap. II.
[5] J. R. Pellam, Phys. Rev. Letters **9**, 281 (1962).

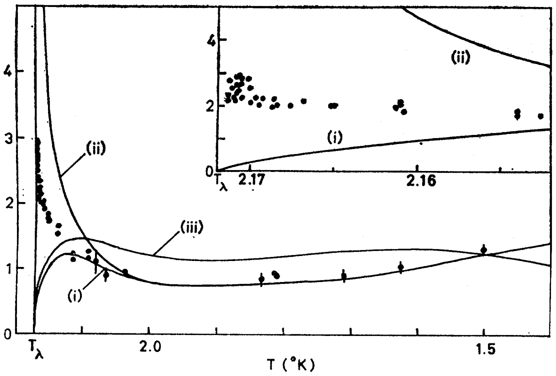

FIG. 1. Comparison of experimental and theoretical values of B in the temperature range investigated by Hall and Vinen (Ref. 1) and by the authors. Experimental values: solid circle with error bars, Hall and Vinen; solid circle data from $C(1)$. Solid line, theoretical values calculated from the equations of (a) Hall and Vinen (Ref. 1) (i) including the Magnus force, and (ii) ignoring this force, and (b) Lifshitz and Pitaevskii, (Ref. 9) (iii) assuming that ρ_s/ρ varies as $(T_\lambda - T)^{0.666}$.

4 m/sec its only apparent effect was to improve the shape of the decays without altering λ or λ' appreciably. For u_2 less than 4 m/sec it not only made the decays more closely exponential, but it also decreased the observed values of λ and λ'. However, in this temperature region it is possible that one would obtain still lower decay constants if the dc heat supply were not used and temperature constancy was maintained by some other means. (In this connection, Vinen[6] has observed increased attenuation with heat flow perpendicular to the second-sound wave vector, which he attributes to the onset of superfluid turbulence.)

The temperature of the helium bath was controlled by a Cartesian manostat[7] and was measured with an ordinary U-tube oil manometer. However, the pressure over the helium bath is not an accurate measure of the temperature inside the resonator near the λ point, and therefore, in a subsequent experiment to be published separately, the second-sound velocity as a function of temperature near the λ point was determined using similar resonators to those described in this paper. These results were used for the linear temperature scale of Fig. 1 whereas the measurements of second-sound velocity are used on Fig. 2.

OBSERVATIONS

Transient effects were investigated in $C(1)$ at about 5 mdeg below T_λ. On suddenly starting rotation at 1.7 rad/sec the signal amplitude dropped to about a third of its former value in the first 80 sec and did not change appreciably after this. When this steady state had been reached, sudden cessation of rotation caused the signal amplitude to decrease by approximately 20% in the first 10 sec and then rise to its former value, nonrotating, as $1-\exp(-t/\tau)$, with τ approximately 1 min. Thus it took considerably longer to establish the equilibrium amplitude after the rotation was stopped, as is to be expected since the nonrotating state has a higher Q. It was found that the value of λ observed 10 min after rotation had stopped did not differ appreciably from the value observed when there was no previous rotation. Accordingly, all data were taken at least 10 min after any change in the rotation state.

Also, at 5 mdeg below the λ point, the attenuation was measured as a function of rotation speed ω. B was found to be independent of ω to within $\pm 10\%$ for ω between 0.5 and 2.5 rad/sec in agreement with Snyder.[8] Thus measurements were made at a value of ω sufficient to make $\lambda' - \lambda$ greater than λ but low enough for the second-sound amplitude to be considerably greater than the noise level.

The effect of heater power input was examined in $C(1)$ at this temperature. Below about 8 mW, B was power-independent but above this value it decreased steadily, having dropped by 30% at a power of 16 mW. All measurements of B were made at 4.7 mW, which was the lowest power convenient.

The results for B and λ for $C(1)$ are plotted against u_2 on a logarithmic scale in Fig. 2(a). The absolute values of λ are mainly governed by geometric imperfections as indicated by their cavity dependence and their dependence on slight structural imperfections in the resonators. The curve in Fig. 2(a) was typical. Values of λ could generally be measured to $\pm 5\%$ and B values to about $\pm 10\%$ below 2.17°K and to about $\pm 20\%$ above 2.17°K.

To determine whether the measured value of B had any dependence on the geometry of the resonator employed, measurements were made using two other cavities of different shapes. Of these, $C(2)$ was an elongated perspex cylinder closed at both ends, which

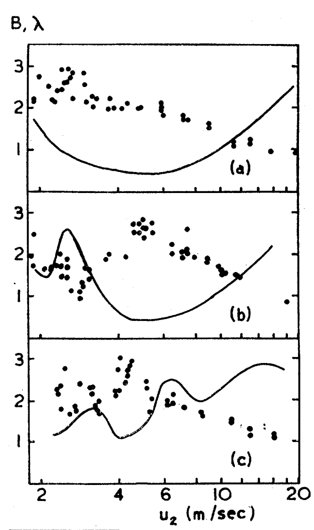

FIG. 2. Comparison of B and λ values from three resonators: (a): $C(1)$; (b): $C(2)$; and (c): $C(3)$. Solid line typical curve for $\lambda(T)$; solid circle, individual determinations of B.

[6] W. F. Vinen, Proc. Roy. Soc. (London) **A240**, 114 (1957); **A240**, 128 (1957); **A242**, 493 (1957); **A243**, 400 (1957).

[7] G. K. White, *Experimental Techniques in Low-Temperature Physics* (Oxford University Press, London, 1959), p. 204.

[8] H. A. Snyder, Phys. Fluids 6, 755 (1963).

was 5.0 cm long and 1.6 cm in diameter. This cylindrical resonator was used in two ways: With graphite films on its curved walls, it was rotated about the axis of the cylinder, and with graphite films on its end walls, it was rotated about an axis through the centroid of the cylinder and perpendicular to its axis. $C(3)$ was a $3\times4\times4$ cm^3 rectangular cavity, rotated about the normal through the centers of two opposite faces. Three of the faces were of Perspex and three of fired Lavite as used by Snyder,[8] (through which liquid helium, but no second sound, may pass) to allow good thermal contact between the inside and outside. The dimensions of the cavities were chosen so as to avoid mode degeneracies. B and λ as measured using $C(2)$ and $C(3)$ are shown in Figs. 2(b) and 2(c). The results for $C(2)$ did not appear to depend on the manner of rotating it.

In Fig. 2 the data for B are taken from a number of runs, whereas the curves for λ are from single runs only. The results for B from the three resonators are in fair agreement except for a dip in B for $C(2)$ at $u_2=2.9$ m/sec. This minimum is associated with a pronounced peak in λ at $u_2=2.6$ m/sec. λ was found to be strongly power-dependent near the peak, even at the lowest power inputs used (2 mW). This peak is thought to be spurious and due to an accidental resonant absorption; it occurred with both rotational arrangements. Values of B from $C(1)$ are expected to be most pertinent to any theoretical analysis that does not take into account the effect of the shape of the resonator on the steady-state flow pattern of the helium inside it. Also, in this cavity the results were not complicated by power effects at least down to $u_2=5$ m/sec.

CONCLUSIONS

Data from cavity $C(1)$ are plotted with a linear temperature scale in Fig. 1. It may be seen that B increases slowly as T tends to T_λ and seems to approach a value of 2.5 ± 0.5. However, the data are not inconsistent with a very weak singularity at the λ point. [A previously published value of $B=5\pm1$ at 2.165°K (Snyder[8]) may be too high since the temperature of the helium was not well controlled (Snyder[9]).]

Hall and Vinen's theory, if applicable, predicts that close to the λ point, B is proportional either to the normalized superfluid density ρ_s/ρ or to $(\rho_s/\rho)^{-1}$, with or without the Magnus force, respectively, and the same is true of the modified theory of Lifshitz and Pitaevskii.[10] The theoretical curves of Fig. 1 were derived using recent measurements of ρ_s/ρ close to the λ point (Tyson and Douglass,[11] Clow and Reppy[12]). For the Hall and Vinen curves, the transverse scattering term D' was taken to be zero as in Ref. 1, and for the Lifshitz-Pitaevskii curve, their "strong interaction" term was neglected, since this has inappreciable effect on B near the λ point. There is little better agreement between these theoretical curves and the experimental data if a temperature-dependent vortex core radius of 4 $(T_\lambda-T)^{-1/2}$ Å, as suggested by Ginzberg and Pitaevskii,[13] is used instead of the constant 1 Å of Hall and Vinen. Pellam has shown that B should be of the order unity as long as there exist regions with curl $v_s=0$, where v_s is the local superfluid velocity in the rotating bucket with no second-sound field. However the exact magnitude and temperature dependence of B depends on the detailed nature of the flow pattern assumed. It appears then that our results are not inconsistent with the ideas put forward by Pellam, and they show that Hall and Vinen's theory is not valid near the λ point.

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[9] H. A. Snyder (private communication).

[10] E. M. Lifshitz and L. P. Pitaevskii, Zh. Eksperim. i Teor. Fiz. **33**, 335 (1957) [English transl.: Soviet Phys.—JETP **6**, 418 (1957)].

[11] J. A. Tyson and D. H. Douglass, Phys. Rev. Letters **17**, 472 (1966).

[12] J. R. Clow and J. D. Reppy, Phys. Rev. Letters **16**, 887 (1966).

[13] V. L. Ginzburg and L. P. Pitaevskii, Zh. Eksperim. i Teor. Fiz. **34**, 1240 (1958) [English transl.: Soviet Phys.—JETP **7**, 858 (1958)].