Indirect Production of X-Ray Line Radiation*

S. I. SALEM AND D. G. ZARLINGO†
Department of Physics-Astronomy, California State College, Long Beach, California
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It has been shown experimentally by Webster and by Hansen and Stoddard that there is a constant ratio between the number of K electrons ejected directly by cathode rays and the number ejected by the resulting bremsstrahlung radiation. Their work on thick anodes of silver and palladium extended over a wide range of energy. In the present work, a new technique has been used to reexamine the problem. It was found that a copper anode operating at low energies gives the same results, thus showing, as one might expect from theory, that the said ratio is independent of the anode material. Although indirect ionization is a contributing factor in thick targets, it has been observed that the line emission from extremely thin anodes is caused by direct collisions.

INTRODUCTION

SOON after the speculation of Barkla,[1] Beatty,[2] and Balderston[3] as to the origin of line radiation, several experiments were performed to determine what fraction of the line is produced by the direct collision of cathode-ray electrons with the anode atomic electrons and what fraction is the result of the indirect process, where the bombarding electron interacts with the nuclear Coulomb field, producing a photon which, in turn, ionizes one of the target atoms.

The process known as internal photoelectric effect, where the absorption of the photon occurs in the same atom in which it is produced (an Auger-type interaction), is considered part of the direct process.[4] Neither this nor previous works could differentiate between the internal flourescence and the strictly direct process.

Webster,[5] and Hansen and Stoddard[6] used a compound-anode technique where a base target of atomic number Z is covered with a foil of an element $Z-1$, thick enough to stop all incident electrons. Thus, all line radiation from the base target is indirectly produced. Their results indicated that, for silver and palladium, the line radiation observed normally to the target face is some 33% indirectly produced and 67% directly produced. These figures were reported as voltage-independent.

Hanson and Cowan,[7] working with a bare copper anode at 11.8 keV, reported that only 10% of the line radiation is indirectly produced. This was done by comparing the number of x-ray quanta in the K line spectrum to the number of quanta in the continuous spectrum having energies greater than the critical value for K excitation.

EXPERIMENTAL TECHNIQUES

Copper ingots, 99.98% pure, were evaporated on smooth, flat beryllium blocks, and used as anodes in a continuously pumped, water-cooled x-ray tube. During evaporation, the pressure was maintained at better than 10^{-5} mm of Hg and the process was carried out slowly to avoid spattering. The beryllium block was suspended with its flat surface down some 10 cm above the evaporating boat. The complete experimental setup is described elsewhere.[8] The thickness of the copper film was determined by measuring the attenuation of the copper $K\alpha_1$ line by the bare beryllium block and by the composite anode, respectively, and is calculated from the relation

$$r_{\mathrm{Cu}}=\frac{1}{(\mu\rho)_{\mathrm{Cu}}}\ln\left(\frac{I'I_0''}{I_0'I''}\right), \tag{1}$$

where the double prime indicates intensity measurements for the composite anode and the prime for the bare beryllium block. The other symbols have their conventional meanings. The mass absorption coefficient of copper, μ_{Cu}, for the Cu $K\alpha_1$ line is 50.9 cm^2/g; this value is reported with an accuracy of better than 1%.[9]

The intensity $I_{K\alpha_1}$ of the $K\alpha_1$ line from the thin anodes is plotted as a function of the applied voltage, Fig. 1. It was found that the empirical relation[10]

$$I_{K\alpha_1}=C(E-E_k)^{n(r)} \tag{2}$$

is obeyed up to a certain electron energy, after which the line intensity declines from its initial rate of rise. The function $n(r)$ varies rather rapidly with r for small r and becomes almost a constant for large r. The points of departure from the exponential rise correspond, within experimental error, to the maximum range of electrons at the indicated energies and are in good agreement with the relation $r\propto E^2$.

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† Present address: Department of Physics, University of California, Riverside, California.

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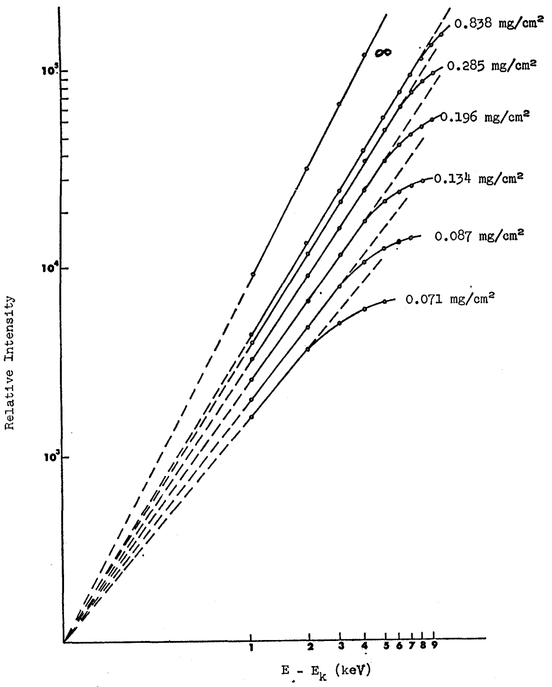

FIG. 1. The relative intensity of the Cu $K\alpha_1$ line is plotted as a function of $E-E_k$ for different sample thicknesses.

The mean range is given, in terms of the maximum range, by the relation[11]

$$\langle \bar{r}^2\rangle^{1/2}=\left[\int_0^{r_m} 4\pi r^4\rho(r)dr \Big/ \int_0^{r_m} 4\pi r^2\rho(r)dr\right]^{1/2}. \quad (3)$$

We assumed for $\rho(r)$ the simple linear form[12]

$$\rho(r)=k(r_m-r), \quad 0\leqslant r\leqslant r_m. \quad (4)$$

Our values of the mean range, thus calculated, are some 5% smaller than the values most quoted in literature.[12] Thus, it became logical to assume that, at the points of departure from the exponential rise, electrons with energies equal to or greater than E_k for copper, i.e., that can potentially contribute to the line intensity, began to leak through the copper film into the beryllium block.

Two anodes of thickness r and $r+dr$, respectively, exhibit different values of line intensity at all voltages except at $E=E_k$. The difference in the intensities at the point where the line intensity from the thin anode falls off its initial exponential rise gives the number of quanta produced indirectly in the layer dr at an approximate depth $r+\frac{1}{2}dr$. Observations from similar pairs of anodes give the indirectly produced line intensity in a layer of unit thickness as a function of the layer depth, Fig. 2. A power series of the form,

$$I^i_{K\alpha_1}=\sum_{j=0}^{\infty} C_j r^j \quad (5)$$

was adjusted by computer analysis to give the best fit to the experimental points. The values of the first ten constants are given in Table 1. Equation (5), when integrated between the limits shown in Fig. 2, gives the total indirectly produced line radiation observed (uncorrected for self-absorption) at a mean applied voltage of about 12.7 keV. The total (direct plus indirect) emergent $K\alpha_1$ line radiation is experimentally determined (Fig. 1). Therefore, the fraction of the $K\alpha_1$ line radiation directly produced by electron impact is

$$P/(P+1)=0.664\pm0.005, \quad (6)$$

where P is defined as

$$P\equiv\frac{\text{directly produced line radiation}}{\text{indirectly produced line radiation}}, \quad (7)$$

$$P=1.98\pm0.04. \quad (8)$$

The errors in (6) and (8) are estimated experimental deviations. These results are in excellent agreement with those reported in Refs. 5 and 6, thus demonstrating

TABLE I. Values of the constants in Eq. (5).

$C_0=+11.2492$	$C_5=+5.0005\times10^7$
$C_1=+8.16120\times10^3$	$C_6=-1.01722\times10^8$
$C_2=-3.72196\times10^4$	$C_7=+1.15078\times10^8$
$C_3=+1.43903\times10^6$	$C_8=-6.86354\times10^7$
$C_4=-1.30046\times10^7$	$C_9=+1.68658\times10^7$

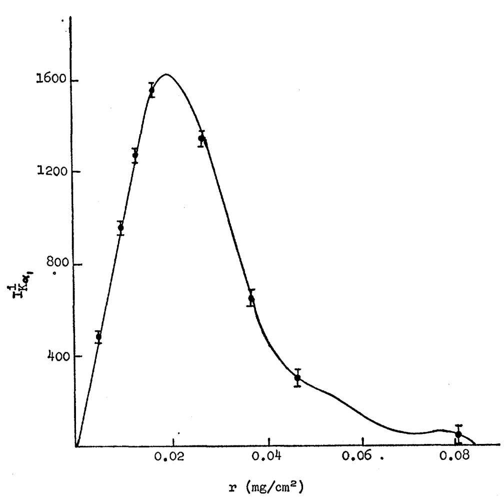

FIG. 2. The indirectly produced line intensity from an increment of thickness plotted as a function of the mean increment depth. The flagged points are experimental and the solid curve is from Eq. (5).

[11] W. E. Burcham, *Nuclear Physics/An Introduction* (McGraw-Hill Book Company, Inc., New York, 1963), p. 405.

[12] R. D. Evans, *The Atomic Nucleus* (McGraw-Hill Book Company, Inc., New York, 1955), pp. 623–624.

that these ratios are independent of the anode atomic number.

THEORETICAL DISCUSSION

At low energy, the cross section for electron-K-electron interaction may be derived from the Bethe expression[13]

$$-\frac{dT}{dr}=\frac{2\pi e^4}{T}(2n)\ln\left(\frac{dT}{I}\right), \tag{9}$$

where n is the number density or the number of atoms per cubic centimeter, I is the ionization potential, and dT/dr, the stopping power of the medium to the traversing electron beam, is related to the cross section in the following fashion.[7]

$$-dT/dr=nT\sigma_d. \tag{10}$$

This yields the cross section for the direct process in the form

$$\sigma_d=\frac{4\pi e^4}{T^2}\ln\left(\frac{2T}{I}\right)\text{ cm}^2/(K\text{ electron}). \tag{11}$$

The simplifying assumptions introduced in obtaining Eq. (9) alter the magnitude of the cross-section equation (11) by some 3% and have very little bearing on the present discussion.

The bremsstrahlung cross section, sometimes referred to as the radiative cross section σ_{rad}, is given in the low-energy case, $T\ll m_0c^2$, by the Racah[14] expression

$$\sigma_{\text{rad}}=\frac{16}{3}\left(\frac{e^2}{m_0c^2}\right)^2\frac{z^2}{137}\frac{\text{cm}^2}{\text{nucleus}}. \tag{12}$$

Under similar energy conditions and ignoring quadrupole and octapole terms, the cross section for the photoelectric process acquires the form[15]

$$\sigma_{\gamma,e}=128\pi\sigma_0\frac{137^3}{Z^2}\left(\frac{I}{k}\right)^4\frac{\exp(-4\xi\cot^{-1}\xi)}{1-\exp(-2\pi\xi)}, \tag{13}$$

where

$$\xi=\left(\frac{I}{(k-I)}\right)^{1/2}, \tag{14}$$

and where k, under the present experimental conditions, is the weighted mean energy of that portion of the continuum spectrum whose frequency is equal to or greater than the frequency of the K-absorption edge of the anode material.

The ratio of directly to indirectly produced line radiation, P, is proportional to the ratio of the cross sections.

$$P\propto\frac{\sigma_d}{\sigma_{\text{rad}}\sigma_{\gamma,e}}\varphi(\mu,r), \tag{15}$$

$$P=B(r)\frac{3(m_0c^2/T)^2\ln(2T/I)}{512(137)^2\sigma_0(I/k)^4\exp(-4\xi\cot^{-1}\xi)/(1-e^{-2\pi\xi})}\times\phi(\mu,r). \tag{16}$$

Here, $B(r)$ is a function of the effective thickness of the sample under bombardments; its value approaches unity as $r\rightarrow 0$.

At low incident energy, that is, as $k\rightarrow I$, one may write

$$\frac{\exp(-4\xi\cot^{-1}\xi)}{1-e^{-2\pi\xi}}\simeq\frac{1}{16\pi}\left(\frac{k}{I}\right)^{1/2}. \tag{17}$$

The term $\phi(\mu,r)$, which corrects for absorption in the anode material, because the directly and indirectly produced characteristic lines have different mean depths of formation, is given by

$$\phi(\mu,r)=\exp[-\mu\rho(\bar{r}_d-\bar{r}_i)\csc\psi], \tag{18}$$

where $\bar{r}_d$ and $\bar{r}_i$ are, respectively, the mean depth of formation of directly and indirectly produced characteristic lines and ψ is the take-off angle. Under the present geometrical conditions, expression (18) differs from unity by less than 1% and is a very slowly varying function of the energy. Therefore,

$$P\simeq B(r)\frac{48\pi(m_0c^2/T)^2\ln(2T/I)}{512(137)^2\sigma_0(I/k)^{7/2}}. \tag{19}$$

This expression shows that, from a "thin" anode $B(r)\rightarrow 1$, all the characteristic line radiation is directly produced in accordance with Fig. 2, and that the ratio P is independent of Z. It is true that $I\propto(Z-\sigma)^2$, but for low energy, $T\simeq I$. The fraction I/k is independent of the atomic number.

Expression (19), to be independent of the electron incident energy as the data of Hansen and Stoddard indicate, requires that $k\propto T^{4/7}$. Although there is no available relation between the cathode-ray energy and k, such proportionality does not seem unreasonable.

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