However, Harrison's criterion of transition from type-B to type-C kinetics is found to be too restrictive by about 10^2. Analysis of self-diffusion data (under type-B conditions) using Fisher's formula provided similar activation energies as those obtained from type-C kinetics. Moreover, the two kinetics together provided a method of determining the effective disordered region around a grain-boundary or dislocation pipe. The experimental values of this parameter, obtained for the first time in any system, is about 1.5×10^{-5} cm. This is about two orders of magnitude higher than usually assumed in grain-boundary diffusion studies (about three lattice parameters).

2000-ppma indium impurity (which is expected to affect point-defect concentration as aluminum) has been found ineffective in enhancing self-diffusivity along the grain-boundary high-diffusivity path. Thus, it is concluded that the high-diffusivity paths are always saturated with vacancies and the activation energy of self-diffusion along these paths could be less than but not greater than the activation energy of motion of vacancies in tellurium lattice (in corresponding crystallographic directions). From these observed data and from the behavior observed in ionic materials,[30] it can be estimated that the activation energy of motion of vacancies in tellurium is about 0.7 eV along [0001] and about 1 eV along $\langle 10\bar{1}0\rangle$ or $\langle 11\bar{2}0\rangle$.

ACKNOWLEDGMENTS

The author wishes to thank R. C. Keezer of this laboratory for supplying the main bulk of single crystals used in this work, and also to thank many of his colleagues for helpful discussions and suggestions.

PHYSICAL REVIEW VOLUME 155, NUMBER 3 15 MARCH 1967

Resistance and Magnetoresistance of Thin Indium Wires*

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(Received 19 September 1966)

The resistances of indium wires of diameter ranging between 0.642 and 0.0156 mm were measured at regular temperature intervals between 1.2 and 4.2°K and in transverse magnetic fields up to 18 kG. The bulk resistivity at 4.2°K, $\rho_b(4.2)=(0.93\pm0.03)\times10^{-9}$ Ω cm, and bulk mean free path, $l_b(4.2)=(1.61\pm0.08)\times10^{-2}$ cm, deduced from our data agree with other recent measurements, as does the average Fermi momentum, $p_F=(1.0\pm0.15)\times10^{-19}$ g cm/sec, determined from observations of the MacDonald-Sarginson effect. Size-dependent deviations from Kohler's rule suggest that a new magnetoresistive mechanism may be effective in wires of very small diameter ($d<0.08$ mm). Comparison of the product $\rho_b l_b$ at 4.2 and 0°K shows evidence of the size- and temperature-dependent resistivity contribution observed previously in indium and a number of other metals.

I. INTRODUCTION

Transport measurements on thin metallic wires and films are of considerable interest because the results provide a variety of information on both bulk and surface properties. The character of surface scattering (diffuse or specular), the bulk mean free path, and the average Fermi momentum of conduction electrons can be extracted from resistance and magnetoresistance data[1,2]; measurements of thermoelectric power as a function of sample size yield information on the dependence of the electronic mean free path on energy and also shed some light on phonon mean free paths and phonon-surface scattering.[3]

We report here resistance and transverse magnetoresistance results for very thin indium wires. Although several previous studies on size effects in indium wires[4,5] and foils[6,7] have already appeared in the literature, our measurements exhibit new effects that manifest themselves only in very thin wires ($d<0.08$ mm) and were not observed in earlier work[4,5] wherein only wires of larger diameter were employed.

An exact treatment of scattering of electrons at external surfaces requires a careful analysis of the character of the surface and should include effects of localized surface states.[8] In the theoretical discussion of transport[1,2] in thin samples, this difficulty is generally circumvented by assuming that a fraction p of the electrons suffer specular reflection upon striking the

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[1] E. H. Sondheimer, Advan. Phys. **1**, 1 (1952).
[2] J. L. Olsen, *Electron Transport in Metals* (Interscience Publishers, Inc., New York, 1962), Chap. 4.
[3] R. Huebener, Phys. Rev. **140**, A1834 (1965); **136**, A1740 (1964).
[4] J. L. Olsen, Helv. Phys. Acta **31**, 713 (1958).
[5] P. Wyder, Physik Kondensierten Materie **3**, 263 (1965).
[6] K. Forsvoll and I. Holwech, Phil. Mag. **10**, 181 (1964).
[7] P. Cotti, J. L. Olsen, J. G. Daunt, and M. Kreitman, Cryogenics **4**, 45 (1964).
[8] R. F. Greene, Phys. Rev. **141**, 687 (1966).

surface, and the remaining fraction, $1-p$, experiences diffuse reflections. The problem then reduces to the solution of the Boltzmann transport equation subject to appropriate boundary conditions. This procedure was followed by Fuchs[9] and Dingle,[10] who derived expressions for the resistivity of thin films and wires, respectively.

In the absence of a magnetic field the solution of the Boltzmann equation obtained by Dingle yields an expression for the resistivity as a function of l_b/d, where l_b is the bulk mean free path and d is the wire diameter, which closely approximates the result deduced by Nordheim[11] on the basis of an elementary mean-free-path argument

$$\rho_w=\rho_b\left(1+\frac{(1-p)l_b}{d}\right), \qquad (1)$$

where ρ_w is the resistivity of the wire and ρ_b that of a bulk sample. Previous data on indium[3–5] are consistent with perfectly diffuse scattering, and we shall hereafter set $p=0$.

The bulk resistivity, in this approximation, is given by

$$\rho_b=\frac{m^*}{ne^2\tau}=\frac{p_F}{ne^2l_b}, \qquad (2)$$

where p_F is the average Fermi momentum of the conduction electrons.

Since Eq. (1) agrees[1] within 5% with the more exact treatment of Dingle over the entire range $0<l_b/d<\infty$, we shall follow current practice[5,12] and interpret our data in terms of Eq. (1). It is important to recognize, however, that Eq. (1) and also Dingle's result fail to explain certain experimental facts.

Nordheim's expression, Eq. (1), was derived on the assumption that Matthiessen's rule is valid, and predicts unequivocally that the temperature-dependent part of the resistivity of a thin wire will be independent of the wire diameter. In many metals, however, the temperature-dependent resistivity of thin wires increases, at low temperatures, as the wire diameter diminishes.[4,12–14] This deviation from Matthiessen's rule has been explained as follows[4]:

In the free-electron approximation, the maximum angle through which an electron can be deflected through normal electron-phonon scattering is

$$\theta_{\max}\simeq\frac{\text{momentum of phonon}}{\text{momentum of electron}}\simeq\left(\frac{T}{\Theta_D}\right)\left(\frac{q_D}{k_F}\right), \qquad (3)$$

where q_D is the wave vector of a phonon of energy $k\Theta_D$, Θ_D is the Debye temperature, and k_F is the Fermi wave vector. For metals $q_D/k_F\sim1$, so that at low temperatures, $T\ll\Theta_D$, only small-angle scattering events can occur. Consequently, in the bulk material many phonon collisions are required to randomize an electron distribution with a net linear momentum, and electron-phonon scattering at low temperatures is thus a rather ineffective resistive mechanism. In a thin wire, however, even a small deflection due to phonon scattering may suffice to bring the electron to the surface where it suffers diffuse scattering. Thus, with diminishing wire diameter small-angle electron-phonon scattering takes on an ever increasing importance.

This mechanism has been studied by Blatt and Satz[15] who derived the following analytic expression for this additional size-and-temperature-dependent contribution to the resistivity of a thin wire:

$$\rho_{ps}=(2\rho_bl_bT/\Theta_D)^{2/3}(2\pi g\rho_b)^{1/3}d^{-2/3}, \qquad (4)$$

where g is the fraction of electron-phonon events in the bulk that are of normal type, and ρ_b in Eq. (4) is the ideal bulk resistivity.

The problems of transport in thin samples are further complicated by application of a magnetic field. The magnetoresistance of thin films has been examined theoretically by Sondheimer[16] and by MacDonald and Sarginson[17] for longitudinal and transverse orientations of the magnetic field. The longitudinal magnetoresistance of thin wires was treated by Chambers.[18] However, the problem posed by the cylindrical symmetry in a transverse field has not been solved as yet. The presence of a nonuniform Hall field in the sample makes the formulation of an exact theory extremely difficult; even in the case of a thin film, MacDonald and Sarginson had to content themselves with an approximate solution.

Provided an isotropic relaxation time can be defined one can show,[19] on very general grounds, that the magnetoresistance must obey Kohler's rule

$$\frac{\Delta\rho(H,T)}{\rho(0,T)}=f\left[\frac{H}{\rho(0,T)}\right], \qquad (5)$$

where f is a characteristic function for a given metal. In bulk samples Eq. (5) is generally obeyed fairly closely. This relation cannot, however, be applied blindly to thin wires. Under the influence of a longitudinal magnetic field, electrons traverse helical orbits whose axes are parallel to that of the wire and whose

[9] K. Fuchs, Proc. Cambridge Phil. Soc. **35**, 100 (1938).
[10] R. B. Dingle, Proc. Roy. Soc. (London) **A201**, 545 (1950).
[11] L. Nordheim, Act. Sci. Ind., No. 131 (Hermann & Cie, Paris, 1934).
[12] M. Yaqub and J. F. Cochran, Phys. Rev. **137**, A1182 (1965).
[13] E. R. Andrew, Proc. Phys. Soc. (London) **A62**, 77 (1949).
[14] B. N. Alexandrov, Zh. Eksperim. i Teor. Fiz. **43**, 399 (1962) [English transl.: Soviet Phys.—JETP **16**, 286 (1963)].
[15] F. J. Blatt and H. G. Satz, Helv. Phys. Acta **33**, 1007 (1960). [See also B. Luthi and P. Wyder, Helv. Phys. Acta **33**, 667 (1960); M. Ya Azbel' and R. N. Gurzhi, Zh. Eksperim. i Teor. Fiz. **42**, 1632 (1962) [English transl.: Soviet Phys.—JETP **15**, 1133 (1962)].]
[16] E. H. Sondheimer, Phys. Rev. **80**, 401 (1950).
[17] D. K. C. MacDonald and K. Sarginson, Proc. Roy. Soc. (London) **A203**, 223 (1950).
[18] R. G. Chambers, Proc. Roy. Soc. (London) **A202**, 378 (1950).
[19] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, London, 1960).

radii are inversely proportional to H. Thus with increasing magnetic field an ever larger fraction of the conduction electrons will be prevented from striking the surface and the influence of surface scattering is thereby reduced. Ultimately, the resistivity $\rho_w(H\rightarrow\infty)$ must equal $\rho_b(H\rightarrow\infty)$. Since for $l_b/d\gg 1$ surface scattering is the dominant resistive mechanism when $H=0$, this purely geometric magnetic effect leads to a *negative* longitudinal magnetoresistance in fine wires, even though the bulk magnetoresistance may be positive. The geometric effect is most readily observed in metals whose bulk magnetoresistance is very small[18,20] or saturates.[4] In the latter instance it is necessary to use wires sufficiently fine that the cyclotron radius of electrons at the saturation field exceeds that of the wire. At low fields, such that the cyclotron radius r_c is much larger than d, where

$$r_c = p_F c/eH\,, \tag{6}$$

the usual bulk magnetoresistivity will manifest itself also in a fine wire and give rise to an increase in resistance with magnetic field.

Although it is difficult to devise similar simple physical arguments in the case of a transverse magnetic field, the magnetoresistance of a fine wire in this configuration also approaches the bulk value at high fields.[12,20,21] Consequently, at a sufficiently high transverse magnetic field the resistivity of a very thin wire, whose bulk magnetoresistance is small or saturates for $r_c > d/2$, should attain a maximum and decrease to the bulk value. A maximum in the transverse magnetoresistance of thin sodium wires was observed by MacDonald and Sarginson.[17] The origin of this transverse magnetomorphic effect is again the reduction of surface scattering when the cyclotron orbit of the conduction electrons becomes smaller than the radius of the wire. The position of the magnetoresistance maximum is expected to occur when $r_c \simeq d/2$. Thus the location of the magnetoresistance maximum in a wire of known diameter is a measure of the Fermi momentum p_F.

Finally, Sondheimer[16] predicted that the magnetoresistance of a thin film in a magnetic field normal to the plane of the film should be an oscillatory function of the applied field. This effect has recently been observed by Forsvoll and Holwech, who studied films of aluminum[22] and indium,[6] and also by Babiskin and Siebenmann[23] and by Cotti.[24]

Evidently, analysis of the magnetoresistance of thin wires is encumbered, on the one hand, by lack of an adequate theory and, on the other, by a diversity of geometrical effects. At low fields, when $r_c \gg d/2$, it is reasonable to apply the Kohler relation in the form

$$\frac{\Delta\rho(H,T)}{\rho_w(0,T)} = f\left(\frac{H}{\rho_w(0,T)}\right), \tag{7}$$

where a single function $f(x)$ should suffice for all wires and temperatures. However, as H is increased and bulk conditions are approached, the bulk resistivity should be used as the reference in Kohler's rule; thus at high fields

$$\frac{\Delta\rho(H,T)}{\rho_b(0,T)} = f\left(\frac{H}{\rho_b(0,T)}\right) \tag{8}$$

should be the more appropriate relation. In other words, Kohler's rule also assumes a size dependence, and should be written[4]

$$\frac{\rho_w(H,T)-\rho_x}{\rho_x} = f\left(\frac{H}{\rho_x}\right), \tag{9}$$

where $\rho_x \rightarrow \rho_b(0,T)$ for strong fields ($r_c \ll d$) and $\rho_x \rightarrow \rho_w(0,T)$ for weak fields ($r_c \gg d$).

In his calculation of the longitudinal magnetoresistance of thin wires, Chambers[18] computed ρ_x for various values of l_b/d and l_b/r_c. Since a satisfactory theory of the transverse magnetoresistance is not at hand a corresponding tabulation of ρ_x for this configuration is, of course, also unavailable. We, therefore, follow the practice adopted earlier[4,5] and present Kohler plots based on Eq. (7). We must, then, anticipate size-dependent departures from Kohler's rule at high fields where (7) is no longer valid and (8) may be a better approximation.

II. EXPERIMENTAL PROCEDURE

Wire samples were prepared by extrusion through diamond dies. The raw material was $\frac{1}{4}$-in.-diam indium rod, 99.999% pure, supplied by ASARCO. For die sizes smaller than 0.003 in. smooth wire surfaces were obtained only when the extrusion assembly, including the indium rod, was heated to about 120°C. Higher temperatures resulted in wires with a beaded appearance, while lower temperatures demanded pressures that placed a severe stress on the dies. Even so, examination of the dies subsequent to extrusion revealed that some had been damaged by excessive pressures. In every case the first few feet of extruded wire were discarded to reduce the probability of contamination.

The wire surfaces were examined microscopically and only wires of regular shape and reasonably uniform diameter (variations less than 10%) were retained. The wires were allowed to anneal at room temperature for periods ranging from 1 to 300 days. We did not observe any aging effects of the kind reported elsewhere.[25]

[20] D. K. C. MacDonald, Nature **163**, 637 (1949).
[21] J. H. Condon, Bull. Am. Phys. Soc. **9**, 239 (1964); A. Goldstein and S. Foner, *ibid.* **9**, 239 (1966).
[22] K. Forsvoll and I. Holwech, Phil. Mag. **9**, 435 (1964).
[23] J. Babiskin and P. G. Siebenmann, Phys. Rev. **107**, 1249 (1957).
[24] P. Cotti, in Proceedings of the International Conference on High Magnetic Fields (unpublished).
[25] R. T. Bate, B. Martin, and P. F. Hille, Phys. Rev. **131**, 1482 (1963).

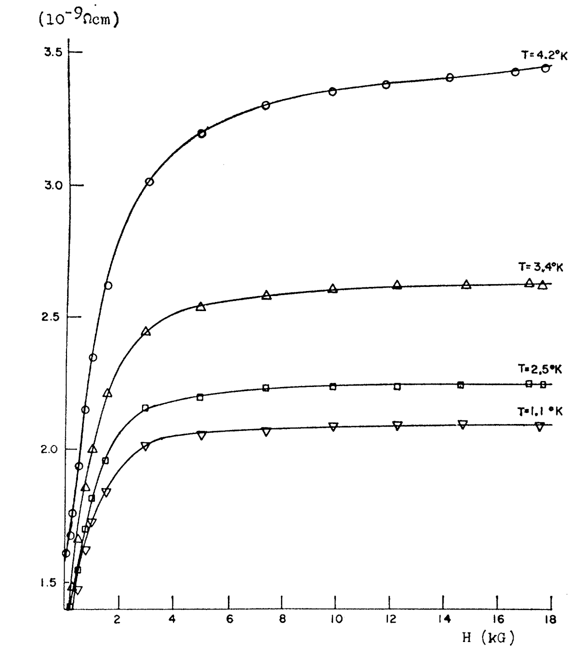

FIG. 1. Resistivity versus magnetic field. Sample III–1, $d=0.248$ mm.

Wire diameters were measured both optically at a number of points along the wire and also calculated from the measured room-temperature resistance, using for the resistivity of indium at 296°K the value 8.8×10^{-6} Ω-cm.[26] During the resistance measurements the wires were submerged in a kerosene bath maintained at 23°C. The length of the wire between potential contacts was measured with a traveling microscope. Diameters determined optically and resistively agreed within 3%. The diameters d used to calculate resistivities were those obtained from resistance measurements since they represent a more reliable average value than those based on optical measurements at a number of randomly selected points along the wire.

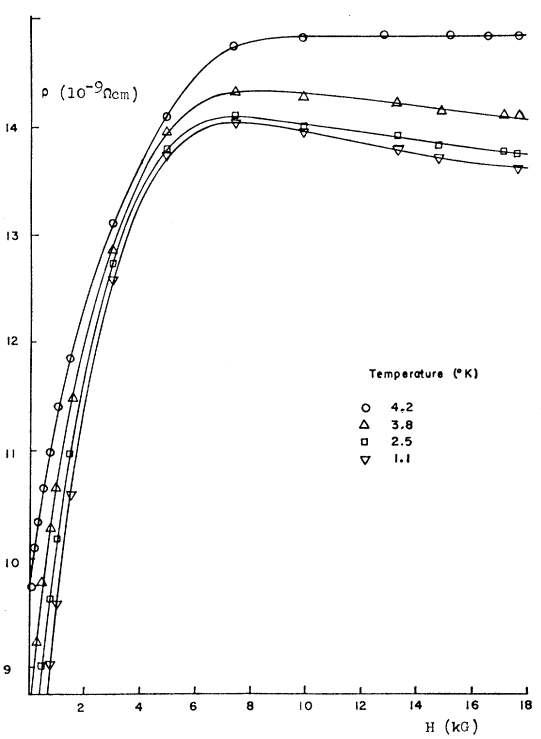

FIG. 2. Resistivity versus magnetic field. Sample III–6, $d=0.0189$ mm.

The wires were placed in a sample holder which was fashioned from a 1-in.-diam, 10-in.-long glass-filled epoxy rod, whose axial coefficient of linear expansion closely matched that of indium. Six groves were cut along the length of the rod and after the wires had been placed in position they were secured by filling the grooves with a warm glycerin-soap solution which jelled at room temperature. If this procedure was not used, the Lorentz force during magnetoresistance measurements deformed the samples resulting in an irreversible resistivity increase.

Current and potential contacts at each end were made to thin platinum strips. The uncertainty in the location of potential contacts was ±0.05 cm, the average distance between potential contacts 20.8 cm.

Electrical leads were fed through the stainless steel tube which supported the sample holder, then passed through a wax seal in the Dewar cover plate and terminated at an 18-pin connector. This connector was surrounded by glass wool and placed inside a brass shield to minimize thermal emf. Current and potential leads were brought from this connector to the potentiometers through a grounded $\frac{1}{4}$-in.-diam copper tube. Currents and voltages were measured by standard potentiometric methods, using a Leeds-Northrup type K-3 and a Rubicon six-dial potentiometer, respectively. The current, supplied from a large lead battery and adjusted by means of a constant impedance (π-network) attenuator, was monitored continuously and kept constant to one part in 10^5. The null detector for the Rubicon potentiometer was a photo-electric galvanometer; noise, stability, and drift limited the resolution to 2×10^{-8} V.

The cryostat consisted of a conventional glass double Dewar with a narrow tail section. Temperatures below 4.2°K were attained by pumping on the helium bath in which the sample holder was submerged. Temperature stability was achieved by pumping through a $1\frac{1}{2}$-in.-diameter Walker regulator[27] which maintained pressure constant to one part in 10^4 between 760 mm and 6 mm Hg. The vapor pressure was measured with mercury and oil manometers and a McLeod gauge. Short-term temperature fluctuations were monitored with a carbon resistance thermometer, which formed one arm of a conventional bridge, and was attached to the sample

[26] G. K. White and S. B. Woods, Rev. Sci. Instr. **28**, 638 (1957).
[27] E. J. Walker, Rev. Sci. Instr. **30**, 834 (1959).

TABLE I. Description of samples.

Designation	Diameter, d (mm)	$1/d$ (mm^{-1})	Age (days)
III–1	0.248	4.03	21
III–3	0.0795	12.6	1
III–4	0.0603	16.6	1
III–6	0.0189	53.0	2
IV–4[a]	0.033	30.3	4
IV–5[a]	0.0256	39.1	3
IV–6	0.0156	64.1	50
V–1	0.642	1.56	37
V–2[b]	0.249	4.02	300
VI–3	0.159	6.29	4

[a] Damaged dies.
[b] This sample was taken from the same wire as III–1.

holder. During the course of an experiment observation of bridge unbalance indicated that the bath temperature remained constant to within 0.5 mdeg.

Transverse magnetic fields were applied by means of a 15-in. Harvey-Wells electromagnet. The sample lengths were less than 80% of the pole force diameter, and carefully centered. Measurements with an NMR probe showed a field uniformity of ±0.5% over the sample length. Magnetic fields were measured with a Rawson-type 720 fluxmeter.

Measurements on nine samples, listed in Table I, are reported here. However, complete magnetoresistance data at eight different temperatures were not performed on all of these.

III. RESULTS AND DISCUSSION

A. Magnetoresistance

Plots of resistivity versus magnetic field of several samples at four different temperatures are shown in Figs. 1–3. Figure 1 displays the expected saturation, with the knees sharper and the plateau flatter the lower the temperature. The smaller diameter samples show the MacDonald-Sarginson effect, i.e., a decrease in resistivity above saturation. This effect is also temperature-dependent, the decrease being more pro-

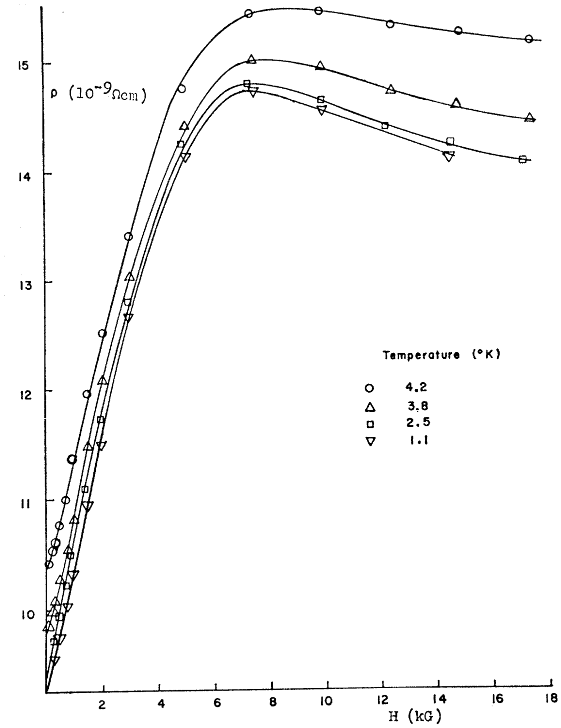

FIG. 3. Resistivity versus magnetic field. Sample IV–6, d=0.0156 mm.

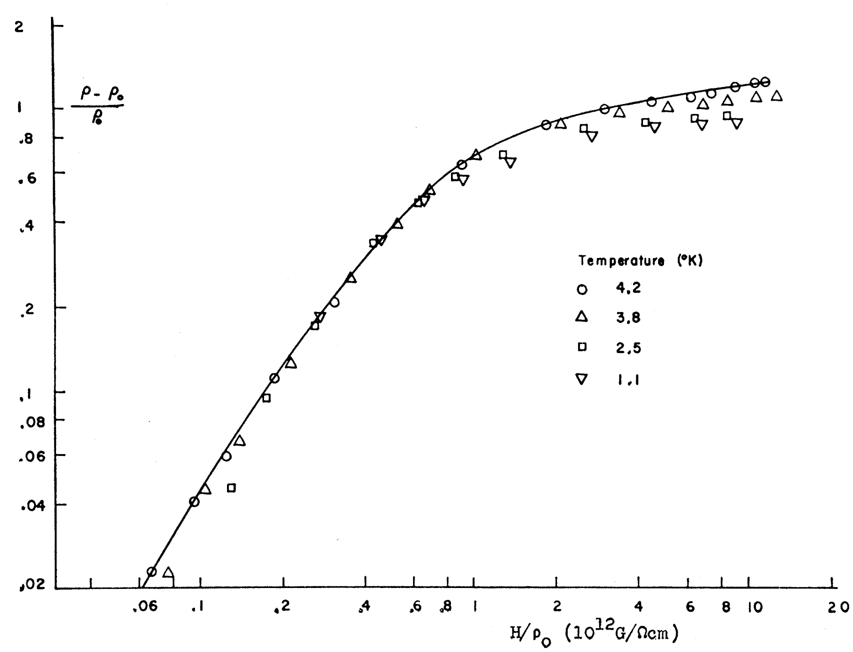

FIG. 4. Kohler plot for sample III–1.

TABLE II. Zero-field resistivities, $\rho(0,T)$.

		Temperature (°K)								
		4.2	3.8	3.4	3.0	2.5	2.0	1.6	1.1	0 (extr.)
Sample	d (mm)	Resistivity (Ω-cm$\times 10^9$)								
III–1	0.248	1.582	1.413	1.277	1.190	1.145	1.105	1.096	1.089	1.085
III–3	0.0795	2.669	2.476	2.34	2.23	2.17	2.13	2.08	2.058	2.05
III–4	0.0603	3.584	3.353	3.18	3.04	2.93	2.84	2.80	2.77	2.76
III–6	0.0189	9.410	8.98	8.70	8.42	8.18	7.91	7.80	7.71	7.67
IV–4[a]	0.033	6.727	6.39	6.13	5.93	5.72	5.60	5.54	5.51	5.48
IV–5[a]	0.0256	8.340	8.025	7.83	7.67	7.48	7.32	7.21	7.16	7.15
IV–6	0.0156	10.388	10.097	9.87	9.70	9.52	9.41	9.36	9.31	9.30
V–1	0.642	1.2067								
V–2	0.249	1.505								
VI–3	0.159	2.013	1.836				1.53		1.52	1.51

[a] Damaged dies.

nounced the lower the temperature. This temperature dependence of the MacDonald-Sarginson effect arises because the lower the temperature the lower is the bulk resistivity and, hence, the saturation field. In the larger sample the MacDonald-Sarginson effect is entirely obscured since the maximum would occur in a region of the magnetic field where the bulk magnetoresistance has not yet attained saturation.

If we use Eq. (6) and assume that the magnetoresistance maximum appears when $r_c \simeq d/2$, we obtain the average value of $p_F = (1.0 \pm 0.15) \times 10^{-19}$ g cm/sec, in fair agreement with the result reported by Forsvoll and Holwech,[6] 1.3×10^{-19} g cm/sec, and with that deduced by Rayne[28] from magnetoacoustic measurements, 0.83×10^{-19} g cm/sec.

To construct Kohler plots one needs $\rho_w(0,T)$. Since indium is superconducting below $T_c = 3.4$°K, it is necessary to obtain the zero-field resistivities below T_c by indirect methods. The procedure which we adopted was based on application of Kohler's rule in the low-field region (see Figs. 4–6). For $T > 3.4$°K, plots of $\Delta\rho(H,T)/\rho_w(0,T)$ versus $H/\rho_w(0,T)$ were obtained directly from measured values of $\rho_w(H,T)$ and $\rho_w(0,T)$ for each wire. These curves were used to define the function $f(H/\rho_w(0,T))$ in the low-field region, i.e., for $r_c > d/2$. For $T \leq 3.4$°K, the low-field magnetoresistance data for $H > H_c$ was then fitted to this function by adjusting $\rho_w(0,T)$. The values of $\rho_w(0,T)$ determined in this manner are given in Table II, and they are probably not in error by more than 2% for $T > 2$°K and 6% for $T \leq 2$°K.

As anticipated, Kohler plots for a given wire at various temperatures show departures in the saturation region. For the larger wires—Fig. 4 is a typical example—the saturation value of $\Delta\rho/\rho(0)$ diminishes with decreasing temperature. This is the same behavior as observed previously by Olsen[4] and Wyder[5] and can be understood from the following argument.

As the temperature is decreased, the knee of the Kohler plot appears at progressively lower values of H since $\rho_w(0,T)$ also decreases. However, the magnetic field at which the magnetoresistance is effectively that

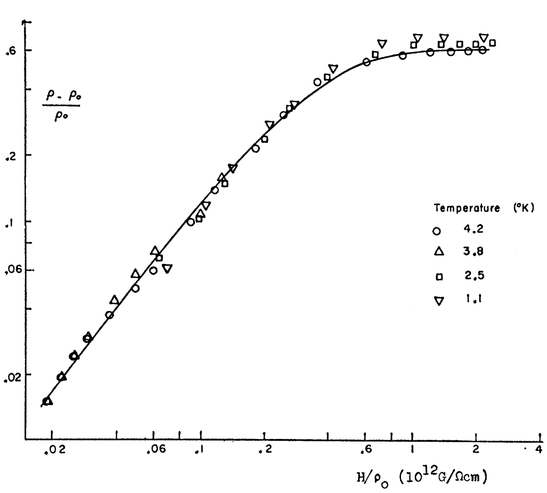

FIG. 5. Kohler plot for sample IV–5.

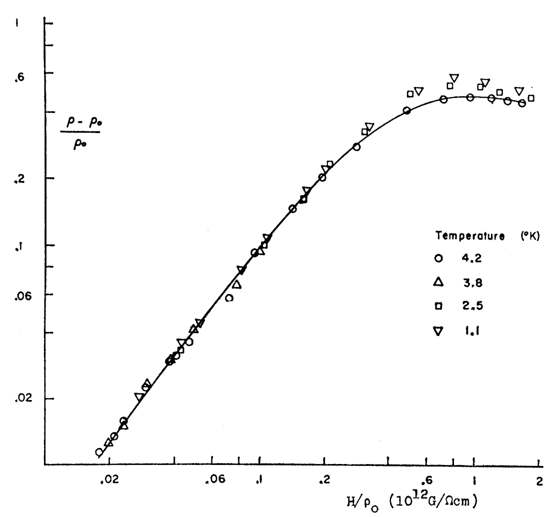

FIG. 6. Kohler plot for sample IV–6.

[28] J. A. Rayne, Phys. Rev. **129**, 652 (1963).

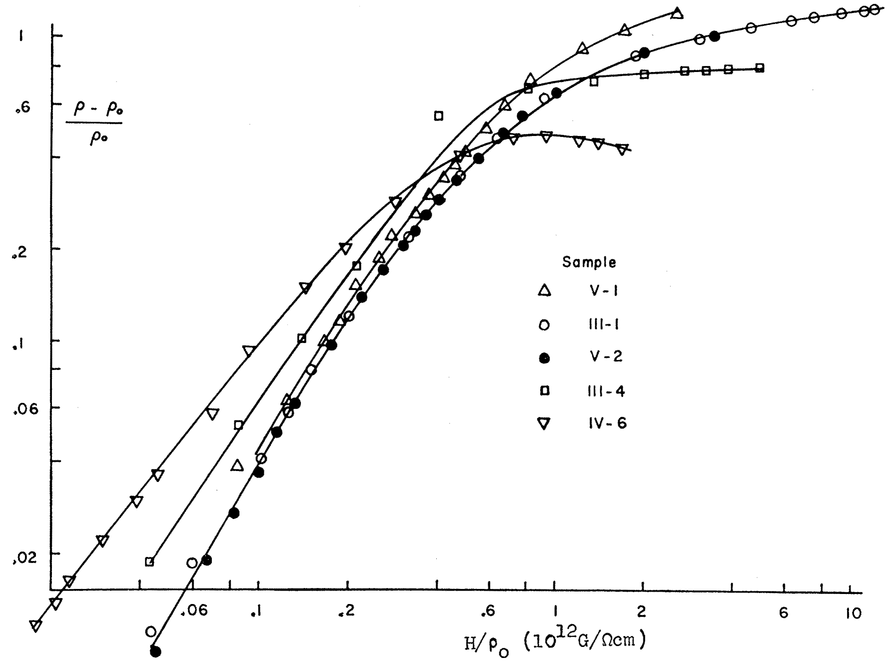

FIG. 7. Composite of Kohler plots for five samples at 4.2°K.

of a bulk sample, and which occurs below saturation in the larger samples, is independent of temperature since the transition to the bulk effect is a purely geometric property. Thus on a Kohler plot which uses a universal function $f(H/\rho_w(0,T))$, and not $f(H/\rho_b(0,T))$, the saturation knees would be shifted to higher values of the abscissa. Moreover, as we remarked earlier, we should also on the ordinate of the Kohler plot use $\rho_b(0,T)$ as our reference in the high-field region. Since surface scattering is almost temperature-independent, the lower the temperature the greater is the ratio $\rho_w(0,T)/\rho_b(0,T)$. Hence the continued use of $\rho_w(0,T)$ in the denominator of the ordinate will artificially reduce the ratio $\Delta\rho(H,T)/\rho(0,T)$ in the high-field region, and this error becomes more pronounced as the temperature is reduced.

In the smallest samples the deviation of the Kohler plot at high fields is, however, in the opposite sense. This behavior is a manifestation of the MacDonald-Sarginson effect which is more pronounced at lower temperatures and has not been observed previously since wires of sufficiently small diameter were not used in earlier work.

Another interesting and new feature is the dependence of the slope of the Kohler plots in the low-field region on sample size. As can be seen from Figs. 4–6, and more dramatically in Fig. 7, the slope in the low-field region decreases monotonically with decreasing diameter, and, moreover, the magnetoresistance itself, that is $\Delta\rho/\rho_w(0)$, is greater in the low-field region the smaller the diameter of the wire. These results suggest that in the smaller wires a new magnetoresistive mechanism which obeys a power law less rapid than H^2 makes a substantial contribution. This additional contribution may possibly arise in the following way:

In a two-band, or two-carrier, conductor such as indium the Hall field cannot exactly compensate the Lorentz force on both groups of carriers and, hence, both groups of carriers will follow circular trajectories under the influence of the applied transverse magnetic field. In a fine wire, the magnetic field will therefore deflect toward the surface carriers which, for $H=0$, would move along the wire axis and by virtue of their relatively long mean free paths carry a substantial fraction of the total current. Thus in a very thin wire, application of a relatively weak transverse magnetic field can enhance the contribution of surface scattering significantly and bring about a substantial reduction of the effective mean free path. Simple geometric arguments suggest that this size-dependent magnetoresistive contribution should be proportional to $l_b^2/r_c d$, i.e., to H rather than H^2 for $r_c > l_b > d$.

FIG. 8. The Nordheim relation. Zero-field resistivity versus $1/d$ at 4.2 and 0°K. The dashed lines show the results of Wyder (Ref. 5).

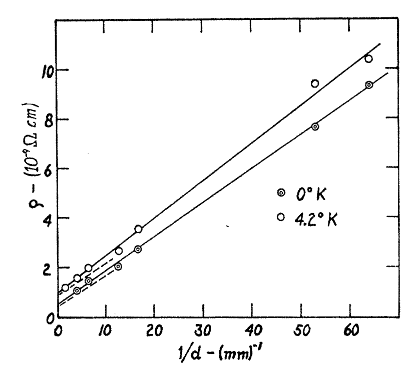

TABLE III. Values of ρ_b and $\rho_b l_b$ for indium at 4.2 and 0°K.

	4.2°K	0°K
ρ_b—Ω-cm	$(0.93\pm0.03)\times10^{-9}$	$(0.5\ \pm0.04)\times10^{-9}$
$\rho_b l_b$—Ω-cm^2	$(1.50\pm0.06)\times10^{-11}$	$(1.35\pm0.04)\times10^{-11}$

B. Zero-Field Resistivity

According to Eq. (1) a plot of ρ_w versus $1/d$ should yield a straight line whose intercept is ρ_b and whose slope is $\rho_b l_b$. Thin-wire measurements thus provide a relatively direct method for determining the bulk mean free path. Moreover, since $\rho_b l_b = p_F/ne^2$ the slope of the straight line should be temperature independent.

Plots of ρ_w versus $1/d$ at 4.2 and 0°K are shown in Fig. 8. The corresponding lines obtained by Wyder[5] are shown dashed. From the intercepts and slopes of the best straight line through the experimental points we obtain the values of ρ_b and $\rho_b l_b$ shown in Table III. These results compare well with $\rho_b(4.2)=0.7\times10^{-9}$ Ω-cm and $\rho_b l_b(4.2)=1.4\times10^{-11}$ Ω-cm^2 (Forsvoll and Holwech),[6] and $\rho_b(4.2)=0.87\times10^{-9}$ Ω-cm and $\rho_b l_b(4.2) =1.3\times10^{-11}$ Ω-cm^2 (Wyder).[5]

Despite the fair amount of scatter of the experimental points and the uncertainty in the zero-field resistivity at low temperature ($T<T_c$), it is apparent that the slope of the ρ_w versus $1/d$ line at 0°K is less than that of the 4.2°K line. Presumably the greater slope at 4.2°K is due to the additional size-and-temperature-dependent resistivity given by Eq. (4). [An alternative explanation, suggested by Wyder[5] and based on Pippard's argument[29] that the strength of the electron-phonon coupling is enhanced through a reduction of the electronic mean free path, cannot be ruled out.] This additional resistivity ρ_{ps} is of the same magnitude as that found by Olsen.[4] Similar size-dependent deviations from Matthiessen's rule in thin wires have also been observed in Al, Sn, Cd, Hg, and Ga.[12–14] Wyder,[5] on the other hand, draws lines of identical slope through his 4.2 and 0°K points. However, the range in $1/d$ (0 to 10 mm^{-1}) and the scatter of his points are such that a change in slope of the magnitude found by us is consistent also with his data. Moreover, the corresponding plots of Wyder[5] for the thermal resistivity, where the scatter of points is rather less, show a definite increase in slope of the line through the 4.2°K points over that drawn through the 0°K points.

[29] A. B. Pippard, Phil. Mag. **46**, 1104 (1955).

IV. CONCLUSION

We have extended measurements of the resistivity and transverse magnetoresistance of fine indium wires to diameters significantly smaller than those employed in previous studies. Measurements on the smaller wires clearly display the MacDonald-Sarginson magnetomorphic effect from which an average value of the Fermi momentum of the carriers may be deduced. The resulting p_F value of $(1.0\pm0.15)\times10^{-19}$ g-cm/sec is in good agreement with values quoted in the literature. Size-dependent deviations from Kohler's rule are observed on the larger, as well as the smaller, wires. However, the sense of these deviations depends upon the wire diameter and agrees with previous results only for the larger samples. Moreover, in the smaller wires the dependence of the resistance on magnetic field at low fields differs substantially from that observed using larger samples. A qualitative explanation of this phenomenon is suggested. Finally, comparison of $\rho_b l_b$ at 0 and 4.2°K shows evidence of the size-and-temperature-dependent resistivity treated theoretically by Blatt and Satz and observed earlier by Olsen in indium wires and more recently in other metals as well.

ACKNOWLEDGMENTS

We wish to thank Dr. M. Garber for valuable assistance during the course of the experiments. One of us (F.J.B.) would like to thank the Physics Division of the Aspen Institute for Humanistic Studies for their hospitality while this article was prepared.