Behavior of Thin-Film Superconducting Bridges in a Microwave Field

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Quantitative measurements are presented on the behavior of thin-film superconducting bridges in the presence of an applied microwave field. Classical rectification is expected when an rf signal is applied to the sample because of its nonlinear I–V characteristic. This is verified experimentally at 10 kc/sec. Deviation from classical rectification starts to occur at frequencies as low as 10 Mc/sec, and increases with increasing frequency up to $\sim$2 Gc/sec, where the critical current starts to increase with the applied microwave field. The increase of the critical current with the field is more pronounced at higher frequencies and at temperatures closer to the transition temperature. Dependence of this behavior on frequency, power, and temperature, as well as on the width of the bridge, was studied in detail. The possibility of "microwave-induced condensation" is briefly discussed. Additional measurements were carried out on the other effects of the microwave field, namely, the production of constant-voltage steps in the I-V characteristic. It was found that the size of the steps increases smoothly over three decades of microwave power and then drops rapidly to zero. A brief comparison between bridge samples and Josephson tunnel junctions is included.

I. INTRODUCTION

IN a previous publication[1] we discussed the effect of a microwave field on the behavior of a small superconducting bridge between two large superconducting reservoirs. The resistive effects in such a configuration were attributed to the flow of quantized vortices across the bridge region. The constant voltage steps in the I-V characteristic which appear in the presence of a microwave field were attributed to the synchronization of the vortex flow by the applied frequency in accordance with the well-known Josephson condition $mhf=2neV$. Similar effects were also observed[2] in superfluid helium flow through a small orifice connecting two reservoirs.

We also observed a baffling effect, namely, the increase of the critical supercurrent of the bridge with the applied microwave field for frequencies larger than 2 Gs/sec. Since no satisfactory explanation could be found, we carried on an extensive experimental study of the influence of relevant parameters (e. g., frequency, temperature, etc.) on the behavior of the critical current in a microwave field. We found that the effect becomes most pronounced at temperatures about 0.001°K below the critical temperature T_c. Therefore, Sec. III of this paper contains measurements of the transition from the normal to the superconducting state of a bridge sample as well as the dependence of the critical current on temperature in the range $0<T_c<0.03$°K. This is compared with theory and with the similar behavior of a stripe of uniform width. We start Sec. IV with a discussion of classical rectification, expected when an rf signal is applied to the nonlinear I-V characteristic and its experimental verification at 10 kc/sec. We then show how deviation from classical rectification starts to occur at low frequencies $\sim$10 Mc/sec and increases with increasing frequency up to $\sim$2 Gc/sec where the critical current starts to increase with the applied microwave field. Dependence of the behavior on frequency, power, and temperature is also discussed. The critical dependence of the effect on bridge width is described in Sec. V, while Sec. VI contains a comparison with Josephson tunnel junctions. Synchronization effects of the applied microwave are discussed in Sec. VII together with the variation of the first three steps with power.

II. EXPERIMENTAL

The samples used in our experiments were designed to represent two superconducting reservoirs weakly coupled by a short, narrow bridge. That the bridge should be extremely short was discovered at an early stage of our study when we tested several rectangular bridges similar to those used by Parks and Mochel.[3] These samples were prepared in two steps. A 7-μ tungsten wire was laid across the substrate in one evaporation and a special mask with a 4-μ-wide stripe perpendicular to the direction of the wire in the subsequent evaporation. We obtained several samples with perfect 4×7-μ rectangular bridges. One sample, however, was "bad"; a speck of dust on the mask produced a constriction of about $2\times0.1\,\mu$ in the bridge. All samples were tested and none but the "bad" sample produced the desired behavior.

The mask used in preparing all subsequent samples consisted of two pointed V-shape strips mounted on a special holder. To produce the required short, narrow bridges it was found necessary to reduce the size of the point in the V-shaped strips to about 0.1 μ (see Fig. 1). This was achieved by extensive lapping of all three planes forming the point. The holder of the V-shaped strips was designed to permit motion of the strips in three perpendicular dimensions to obtain nearly perfect alignment. Thus the bridges we used were such that the narrowest part of the bridge, to be denoted by NP, was essentially a straight line of almost zero

[1] P. W. Anderson and A. H. Dayem, Phys. Rev. Letters **13**, 195 (1964).
[2] P. L. Richards and P. W. Anderson, Phys. Rev. Letters **14**, 540 (1965).
[3] R. D. Parks and J. M. Mochel, Rev. Mod. Phys. **36**, 284 (1964); Phys. Rev. Letters **13**, 331 (1964).

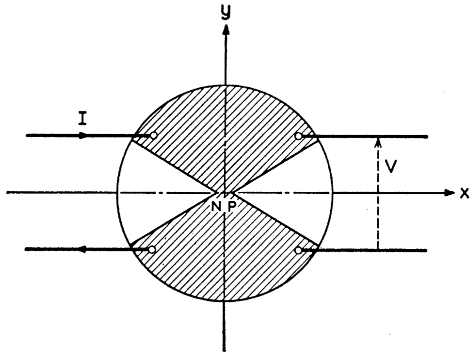

FIG. 1. Sample configuration. NP is the narrowest part in the bridge and is typically 2–6 μ wide. Currents passing NP flow in the direction of the y axis.

length (x axis in Fig. 1) perpendicular to the direction of current flow. Materials used are Sn, In, and Al. Since all samples behaved in the same fashion, most of the results discussed here are for Sn samples. The experiments were carried out in the same fashion described previously.[1] The only refinement was the use of a nanovoltmeter for measuring the vlotage produced across the bridge. This resulted in a reduction of the noise by about one order of magnitude.

III. THE I-V CHARACTERISTIC

A typical I-V characteristic is shown in Fig. 2. It consists of a region of supercurrent oc where all the sample is in the pure superconducting state (PS-state). At c, a sudden voltage jump occurs and the sample enters the so-called resistive-superconducting state (RS state). In the PS state the phase of the order parameter is stationary in time and the supercurrent may be expressed in the form[4]

$$I=\text{const}\times(\varphi_1-\varphi_2),$$

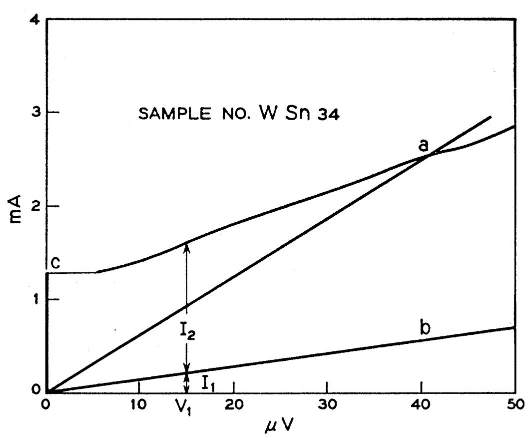

FIG. 2. Typical I-V characteristic; oca at $T=3.779$°K; ob at 3.85°K; $T_c=3.837$°K.

where φ_1 and φ_2 are the phases of order parameter on either side of the bridge at points sufficiently far from NP. The onset of the RS-state may be attributed to the formation of supercurrent vortices in the narrowest part of the bridge, each vortex supporting one quantum of flux. These vortices are driven along the x axis by the dc current. The time-average rate is given by the Josephson relation,[4,5]

$$\left\langle\frac{dn}{dt}\right\rangle_t=\frac{2eV}{h},$$

where V is the difference in potential between the two reservoirs again measured between points sufficiently removed from NP. Since the core of each vortex is in the normal state, there is an energy barrier to the formation of a vortex in the bridge region. The formation of vortices will, therefore, be subject to thermal fluctuation and bears the randomness inherent in all thermally activated processes. The passage of each vortex from vacuum to vacuum across the bridge will cause the phase of one reservoir to "slip" past the phase of the other by 2π. This phase-slippage phenomenon and its importance to our understanding of the superconducting state was discussed in detail by Anderson.[4]

The I-V curve in the RS state is nonlinear but smooth up to a point denoted by "a" in Fig. 2. At point "a" a sudden change in the slope, and often a voltage jump, takes place. It was found experimentally that well-defined interaction and synchronization effects produced by the microwave field are confined to the region above the straight line oa in Fig. 2. Two consequences follow from this experimental fact. The first is that point "a" defines the end of the pure RS state, in the sense that vortex flow is confined to the narrowest part of the bridge. As the current is increased beyond point "a" the region of vortex flow gradually spreads on either side of NP until the whole sample enters the mixed state characteristic to type-II superconductors. The second consequence concerns the nature of the current flowing in the bridge in the RS state. Obviously, this current is composed of Cooper pairs as well as normal electrons driven by the dc voltage. However, one can not safely assume that at a voltage V_1 in Fig. 2, the normal component of the current is given by I_1 and supercomponent is given by I_2. If this were the case one would expect to find a well-defined interaction with the microwave field well below the oa line.

The value of the current at the end of the S state will be denoted by I_c. The most interesting feature of the interaction to be described in this paper is the increase of I_c with the applied microwave field in the temperature range $T_c-T<0.03$°K. It is of interest, therefore, to describe the experimental behavior in the transition from the normal to the superconducting state and the variation of I_c with temperature both for a bridge and for a stripe and to compare the experi-

[4] P. W. Anderson, in *Lectures on the Many Body Problem*, edited by E. R. Caianiello (Academic Press Inc., New York, 1964), Vol. 2; Rev. Mod. Phys. (to be published).

[5] B. D. Josephson, Rev. Mod. Phys. **36**, 216 (1964).

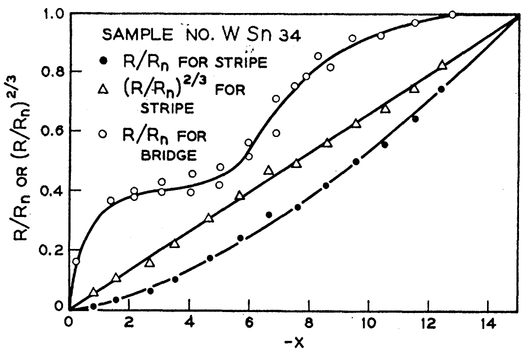

FIG. 3. Normalized resistance in the transition from the normal to the superconducting state for a stripe and a bridge of equal thickness=2100 Å. Stripe is 99 μ wide and 0.376 cm long while the bridge is 2.12 μ wide.

mental results with theory. Two tin samples, one a bridge and the other a stripe, were prepared under identical conditions (substrate preparation, evaporation rate, background pressure, and film thickness). The experimental results are shown in Figs. 3 and 4. Here and in all subsequent figures we use x to denote the quantity

$$x=[P_c-P)\times 10^3/P_c],$$

where P is the pressure inside the Dewar and $x=1$ corresponds to $T_c-T\cong 0.001$°K for the tin samples. In the transition region we determine graphically the slope of the I-V curve at $I=V=0$ to obtain the sample resistance R. We find that the stripe transition is

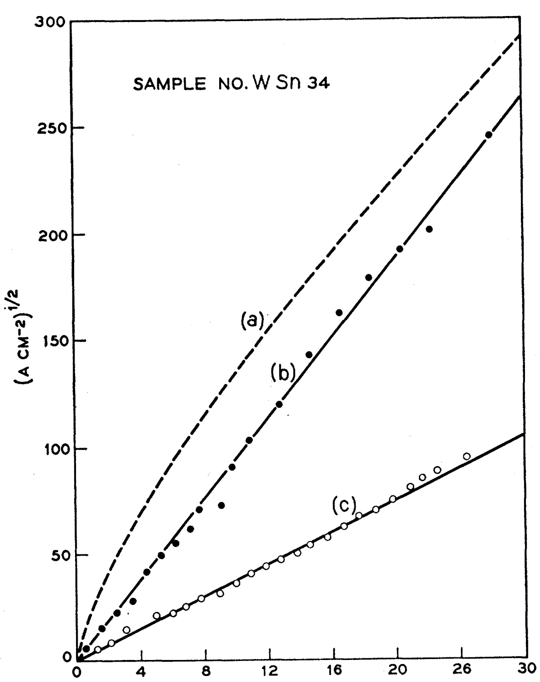

FIG. 4. Variation of critical current with temperature: (a) theoretical curve, (b) bridge, and (c) stripe. Samples are the same as in Fig. 3.

smooth, though rather wide ($\Delta T\sim 0.015$°K), and can be represented by the relation

$$R/R_n\propto x^{3/2}$$

rather well. On the other hand the transition of the bridge sample is rather complicated and looks like two successive transitions with the "wrong" curvature when compared with the stripe transition. This, undoubtedly, is a result of the complex geometry and may indicate that the bridge region reaches the superconducting state at a temperature lower than the rest of the sample.

Figure 4 shows the dependence of the critical current on temperature in the range $0\leq T_c-T<0.03$°K. The dashed curve represents the prediction of the theory[6] and was calculated using Eq. (5.19) of Ref. 6 and taking $T_c=3.8$°K, $H_0=310$ G, and $\sigma=3.58\times 10^6\Omega^{-1}$ cm^{-1}. (The value of σ was determined experimentally from the sample resistance at $T\simeq 3.9$°K.) In both the bridge and the stripe we find that the best fit is given by $I_c\propto x^2$ rather than $x^{3/2}$.[7] We also find that the critical current/unit width obtained in the bridge sample is about 6.2 times that obtained in the stripe, assuming linear dependence of critical current on width. The value of I_c for the bridge is only $\frac{1}{2}$ the theoretical value at $x=10$. It has been found[8] that the measured critical current is appreciably smaller than the theoretical value because of the effect of the magnetic field on the current distribution. Since the measured values of I_c for the stripe compare favorably with previously published results,[8] it is apparent that the bridge critical current is not subject to the same limitatons as that of the stripe.

IV. VARIATION OF I_c WITH INDUCED MICROWAVE CURRENTS

Given the nonlinear I-V characteristic shown in Fig. 2 one expects that the superposition of an alternating current on the direct current will result in the familiar rectification process. Let i denote the amplitude of the superposed alternating current. Voltage will start appearing across the sample when the direct current reaches a value J_c given by

$$J_c=I_c-i. \quad (1)$$

If the direct current I is increased beyond J_c the direct voltage across the sample will be given by the time-average value

$$\bar{V}=\frac{m}{\pi}[(I-I_0)\theta+i\sin\theta], \quad (2)$$

[6] J. Bardeen, Rev. Mod. Phys. 34, 667 (1962).

[7] In all samples where the measurements were extended to temperatures such that $(T_c-T)>0.03$°K, we found the best fit is $I_c\propto x^{3/2}$ in this temperature range but not for $T_c-T<0.03$°K (see Fig. 11).

[8] V. L. Newhouse, *Applied Superconductivity* (John Wiley & Sons, Inc. New York, 1964), p. 217.

where

$$\cos\theta = (I_c - I)/i.$$

In deriving Eq. (2) we have assumed that the RS state characteristic is given by a straight line of slope m and intercept I_0. Equation (1) was checked experimentally when a 10-kc/sec signal was directly applied to the terminals of the bridge. The alternating current flowing in the bridge was measured by amplifying the voltage appearing across a resistance in series with the bridge. The critical currents J_c and I_c were obtained from plots of the I-V curves with and without i, respectively. As expected, the experimental results were in exact agreement with Eq. (1).

At higher frequencies the alternating currents are not directly applied to the terminals of the bridge but rather induced in the sample by a loop terminating a coaxial line. Equation (1) will be replaced by

$$1 - R = (P/P_c)^{1/2}, \quad R = J_c/I_c, \tag{3}$$

where P is the incident power and $P_c = P$ when $J_c = 0$.

In Fig. 5 we show experimental rectification curves obtained by an induced 17 Mc/sec current. The circles in this figure are points calculated from Eq. (2) where i was determined from the measured values of J_c and I_c using Eq. (1). The calculated points fit the experimental curves quite well. In Fig. 6 the experimental values of $(1 - J_c/I_c)$ are plotted versus the incident power level in dBm. One obtains a straight line of slope 0.6 instead of 0.5 predicted by Eq. (3). Similar plots at higher frequencies show a good fit to a straight line; however, the slope seems to increase with frequency, e. g., a slope of 0.8 was obtained at 941 Mc/sec. This points out the fact that deviation from classical rectification may be responsible for an exponent larger than 0.5 in Eq. (3).

Drastic deviation from classical rectification[9] obtains at frequencies in the vicinity of 2 Gc/sec and higher. One finds that J_c is larger than I_c and increases with increasing microwave power. This increase is measurable over at least two decades of power level. J_c attains a maximum value J_m and then drops rather rapidly to zero with further increase of microwave power. Typical behavoir at successive power levels is depicted in Figs. 7 and 8. We summarize the various phases of the interaction.

(a) The initial rise of $(R-1)$ is approximately proportional to the microwave power. The rate of rise gradually decreases as R approaches its maximum value $R_m = J_m/I_c$ reminiscent of a saturation phenomena.

(b) The maximum value R_m becomes larger as one approaches the critical temperature. This aspect of the interaction will be discussed in more detail.

(c) The drop from the maximum value to zero is rather fast. The rate of drop is larger the closer one gets to the critical temperature and the higher the applied frequency. In fact R drops from R_m to zero over a small fraction of a dB at frequencies larger than 8 Gc/sec.

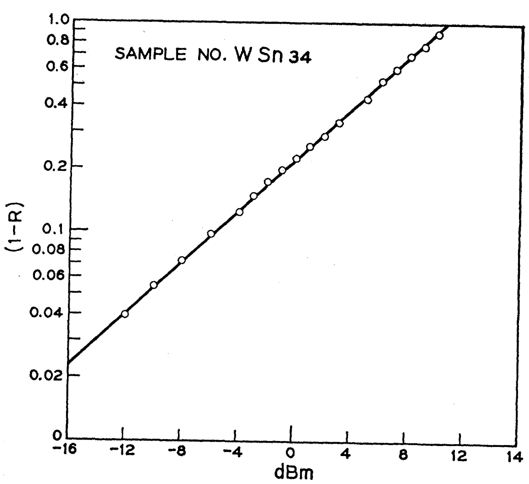

FIG. 6. Plot of $1-R$ versus incident power level at 17 Mc/sec.

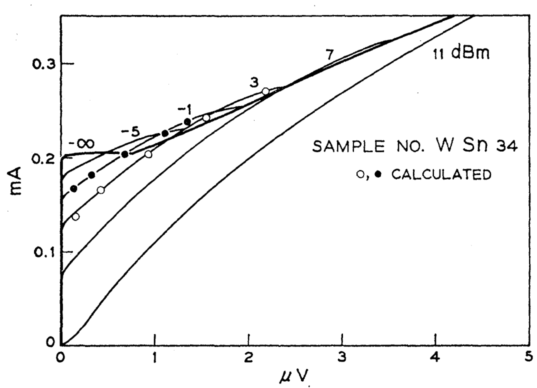

FIG. 5. Classical rectification at 17 Mc/sec for different incident power levels indicated in dBm.

To gain an idea of how this deviation progresses with increasing frequency we refer reader to Fig. 9 which depicts measured values of R as a function of P/P_c at different frequencies. The lowest curve is a plot of Eq. (3) and represents measurement results at 10 kc/sec. Long before J_c becomes larger than I_c deviation from rectification starts, probably at a frequency as low as 10 Mc/sec. This initial deviation is such that

$$1 > R > 1 - (P/P_c)^{1/2}.$$

There is a steady rise of R with frequency until R finally becomes larger than unity at some frequency in the vicinity of 2 Gc/sec. It is to be noted that P_c is determined experimentally from the power level at which $J_c = 0$. Although we know that $i = I_c$ when $P = P_c$ at very low frequencies, it is doubtful that the

[9] In experimenting with stripes of uniform width we never observed an increase in the critical current even when the stripe was 7 μ long and 4 μ wide.

amplitude i of the induced alternating current satisfies the same relationship at higher frequencies. Since J_c attains values many times larger than I_c, one has the tendency to assume that i should be equal to or even greater than $(J_c - I_c)$. It is a serious drawback in the present experiments that the exact value of i cannot be determined. The complex geometry of the samples and the extremely small size of the bridges render it impossible to determine i by any of the microwave techniques known to us.

We turn now to a study of the dependence of R_m on the frequency at fixed temperature. The results are shown in Fig. 10. We notice that at any temperature

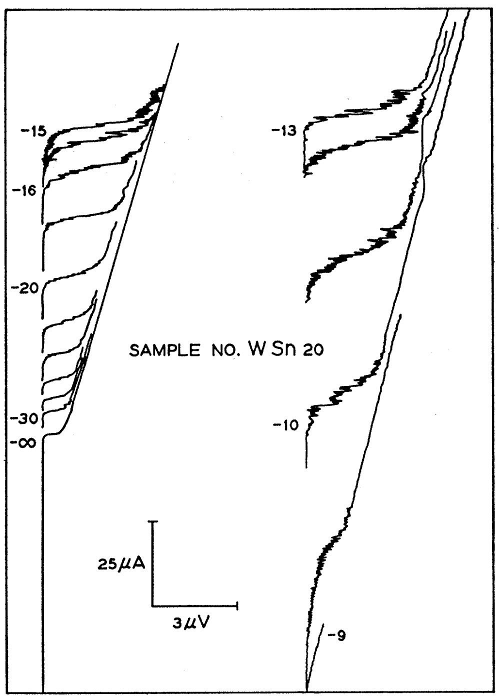

FIG. 7. *I-V* characteristics at different microwave power levels showing increase of the critical current at 4.15 Gc/sec and $x = 9.7$.

R_m first rises rather rapidly with the frequency and then saturates. One may extrapolate the curves down to a "cutoff" frequency f_c below which R remains less than unity. Furthermore, R_m attains a maximum at some "resonance" frequency f_r which lies within the given frequency band at small values of x. Both f_c and f_r increase with decreasing temperature which seems to indicate a dependence on the energy gap. The size of the gap expressed in Gc/sec together with the corresponding value of x are also shown in Fig. 10. One finds easily that there is no correlation between the size of the gap and either f_c or f_r. The introduction of a "cutoff" frequency may be misleading, since the deviation from rectification which finally culminates in an

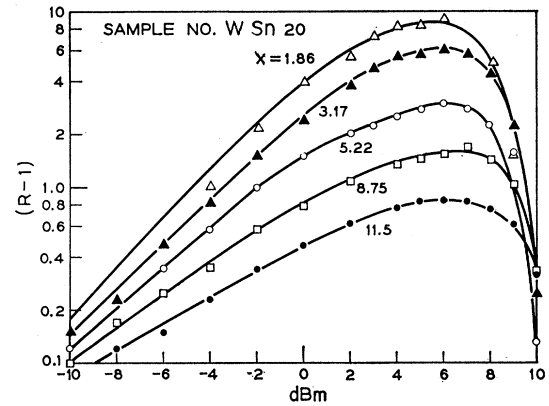

FIG. 8. Variation of critical current with incident power at 4.15 Gc/sec with temperature as parameter.

$R > 1$ actually starts at very low frequencies as shown in Fig. 9.

Probably the most important characteristic of the interaction is the dependence of R_m on temperature at fixed frequency. Of particular interest is the value R_m approaches as $x \to 0$. We encountered appreciable difficulties in making measurements at temperatures which lie within 0.001°K below the critical temperature. This is partly due to the temperature control unit which

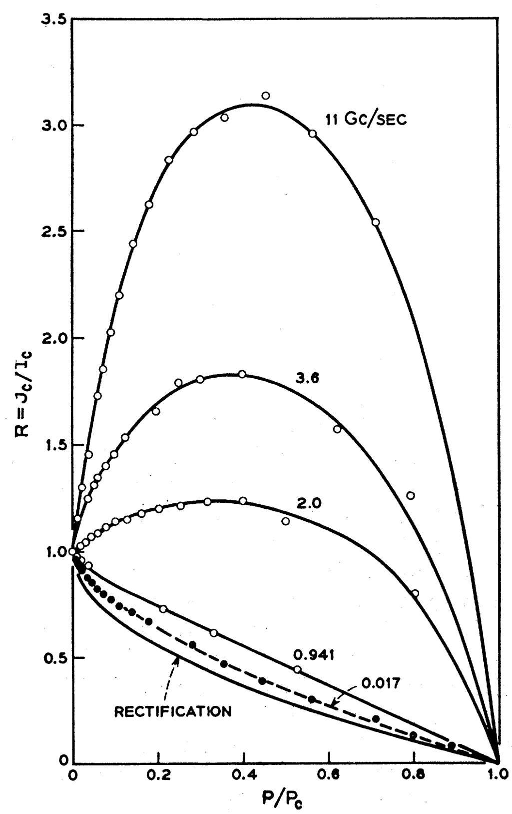

FIG. 9. Deviation from classical rectification with frequency as parameter.

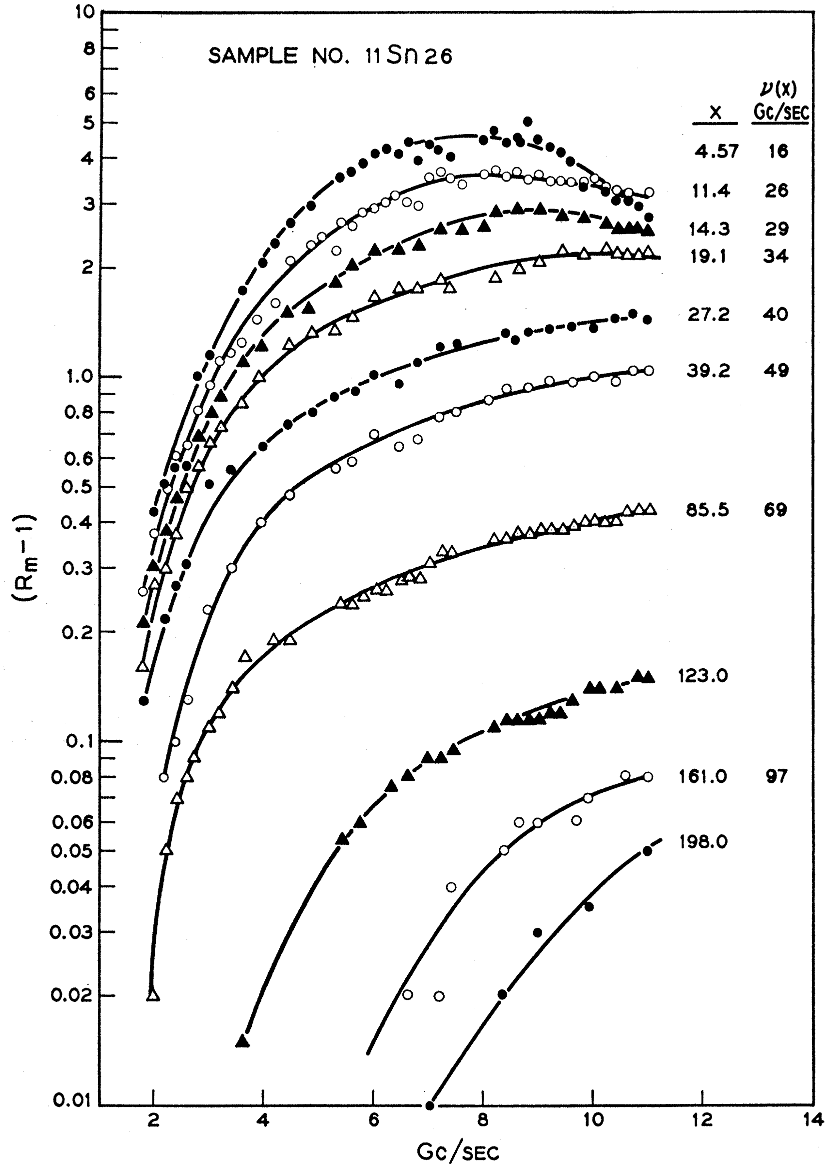

FIG. 10. Dependence of critical current on frequency for fixed values of temperature; $\nu(x)$ is the corresponding gap expressed in Gc/sec.

is capable of stabilizing the bath temperature to within 10^{-4}°K only, and partly due to the extremely small currents and voltages which one obtains near $x=0$. We found it necessary to rely upon graphical extrapolation to determine the behavior of R_m as $x \rightarrow 0$. This method entails a certain degree of arbitrariness which will prove to be rather serious as we shall see later on. We start with Fig. 11 where $I_c^{2/3}$ as well as $J_m^{2/3}$ are plotted versus temperature. As mentioned previously, the characteristics are practically linear as required by theory but this linearity is limited to temperature below ($T_c-0.03$°K). Closer to the transition temperature Fig. 12 shows that $I_c^{\frac{1}{2}}$ and J_m behave linearly with the temperature. From plots similar to Fig. 12 we determine accurately T_c where $I_c=0$. We always find, however, if we define T_c' as the temperature at which $J_m=0$ that $T_c<T_c'$. For the particular case presented in Fig. 12, $T_c-T_c'\simeq 0.0007$°K. To explain this difference in T_c we argue that either (a) a certain degree of order should be present before the microwave interaction can make itself noticeable, i. e., a well-established superconducting state is a prerequisite for the production of an $R_m>1$; or (b) because of its presence in a microwave field the sample is at a slightly higher temperature than the bath. If the latter point of view is adopted, the most plausible correction is to shift the straight line representing J_m in Fig. 12 parallel to itself to pass through T_c. This will lead to an R_m which varies like $1/x$ in the immediate vicinity of T_c. Measured values of R_m are shown in Fig. 13 over the range $0\leq x\leq 100$. Near $x=0$ the solid line represents the case where R_m varies like $1/x$ while the dashed line corresponds to the

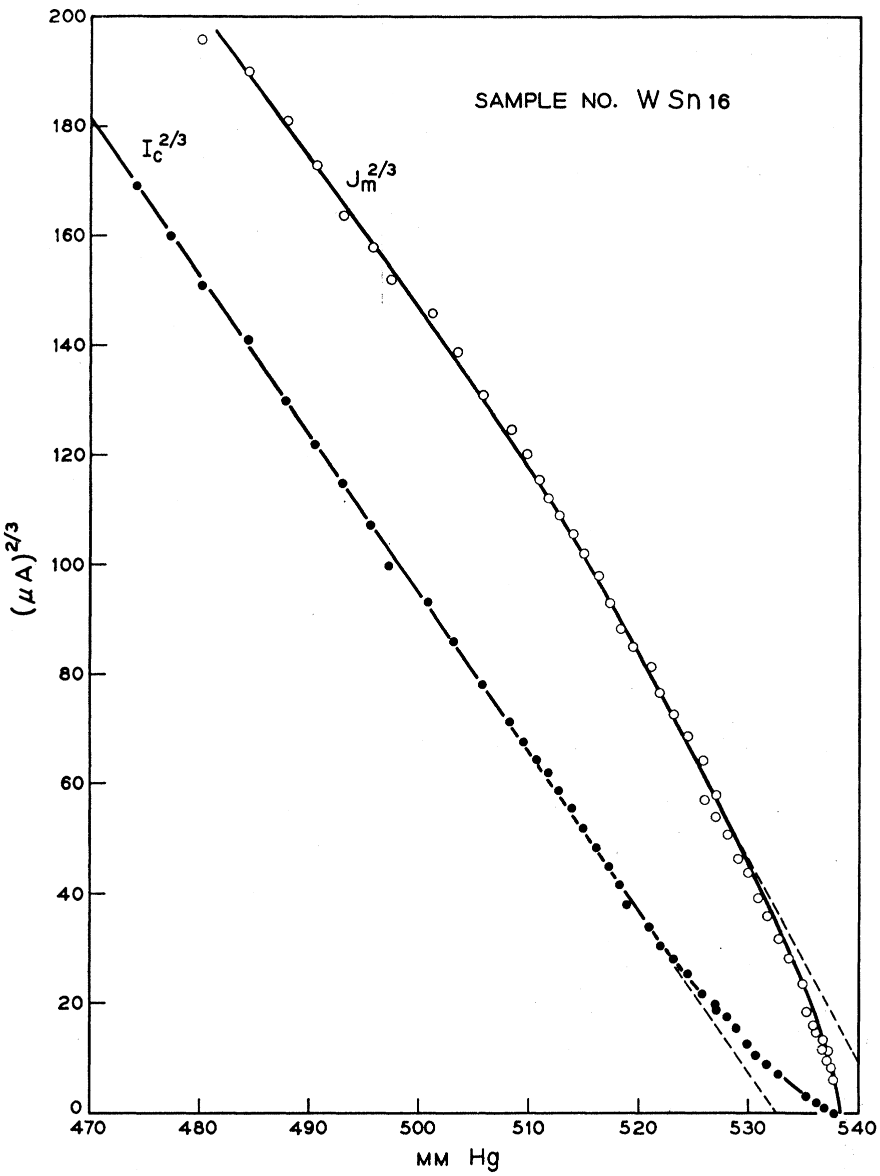

FIG. 11. Dependence of I_c and J_m on temperature. Both are approximately proportional to $(T_c-T)^{3/2}$ for $(T_c-T)>0.03$°K but deviate appreciably at $(T_c-T)<0.03$°K.

measured data without modification where one assumes $T_c'<T_c$ as given by extrapolation. Either assumption leads to essentially the same $R_m(x)$ away from $x=0$. From the dashed curve in Fig. 13 R_m reaches a maximum at $x=2[T_c-T\simeq 0.002$°K, $\nu(x)=10$ Gc/sec as compared to applied frequency$=8.95$ Gc/sec], and then drops to zero at $x=0.2$.

In attempts to extend the measurements to come as closely as possible to the transition temperature T_c, we often applied the microwave field when the temperature was slightly above T_c. In this region the I-V curve in the absence of the microwave field, though quite nonlinear, does not show evidence of any supercurrent and in fact the slope of the curve at the origin is finite, i.e., the sample is in the transition region above T_c. The application of a microwave field resulted in a finite supercurrent with well-defined J_c which increased with increasing applied power and then dropped to zero. Removal of the microwave field and immediate plot of the I-V curve gave exactly the same original curve. These results are shown in Fig. 14 and they correspond to the point indicated by the arrow in Fig. 12. Such observations seem to point out the possibility of a "microwave-induced condensation" where unpaired electrons condense into superconducting pairs via a weak interaction with the electromagnetic field. In fact one may, in a plausible manner, summarize all the above results as properties of such an interaction: (1) The contribution of the microwave field to the condensation should vary like $1/x$, i.e., roughly proportionately to the number of unpaired electrons (Fig. 13), and should vary linearly with the number of

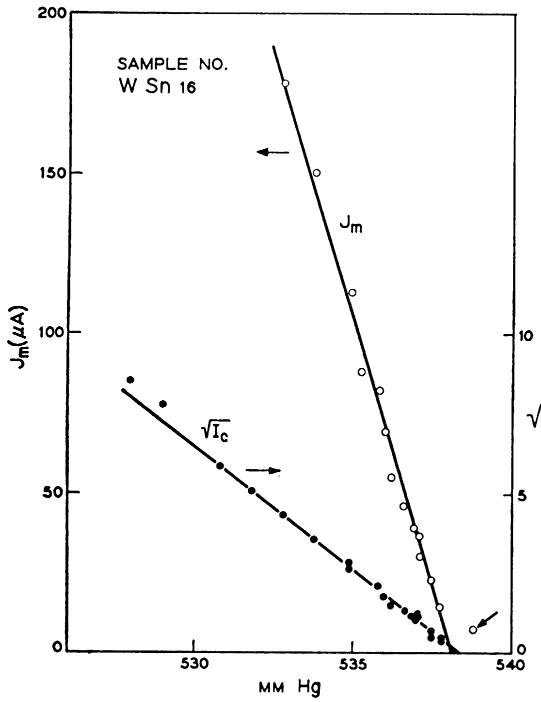

FIG. 12. Linear dependence of $\sqrt{I_c}$ and J_m on temperature for $(T_c-T)<0.02°K$. Applied frequency is 8.95 Gc/sec.

photons, i.e., power (Fig. 8). (2) At a given temperature there is an optimum photon energy at which maximum pairing is obtained (Fig. 10). (3) It should manifest itself only in samples having very narrow bridges.

V. DEPENDENCE ON BRIDGE WIDTH AND FILM THICKNESS

The value of R_m decreases rapidly when the width of the bridge is increased as shown in Fig. 15. From the values of R_m at $x=10$ one finds that $R_mw=20$, 15.6, and 17.6 for three different widths $w=2$, 4.6, and 8 μ, respectively. Thus R_m is approximately proportional to $1/w$. It is obvious that in the given bridge geometry the critical region will propagate with increasing current at a rate which is also inversely proportional to the bridge width.

We also made measurements on samples of equal bridge width and different film thickness. In general R_m decreases with increasing thickness. Because of imperfections in the mask used in preparing these samples, the evaporated film did not have a sharp edge. The film thickness tapered away from the edge and covered a width almost equal to the width of the bridge used. The visible taper also increased with film thickness. Consequently the conducting portion of the bridge was appreciably wider for thicker films. The observed decrease in R_m may therefore be due to the increased width rather than thickness. We have not attempted to repeat these measurements with the improved masks.

VI. COMPARISON WITH JOSEPHSON TUNNEL JUNCTIONS

Scalapino[10] suggested that our experimental results may be explained by Riedel's[11] theoretical calculation of the effect of a microwave field on tunneling currents in a Josephson junction. Additional theoretical work has since been published by Werthamer.[12] We carried out a large number of experiments on low resistance Sn-Sn junctions ranging in width from 0.064 in. to 0.001 in. The samples were tested at X-band frequencies at various temperatures ranging from T_c to 1°K. No increase in Josephson direct current was ever observed under the influence of a microwave field. Although one can present some arguments showing that the behavior

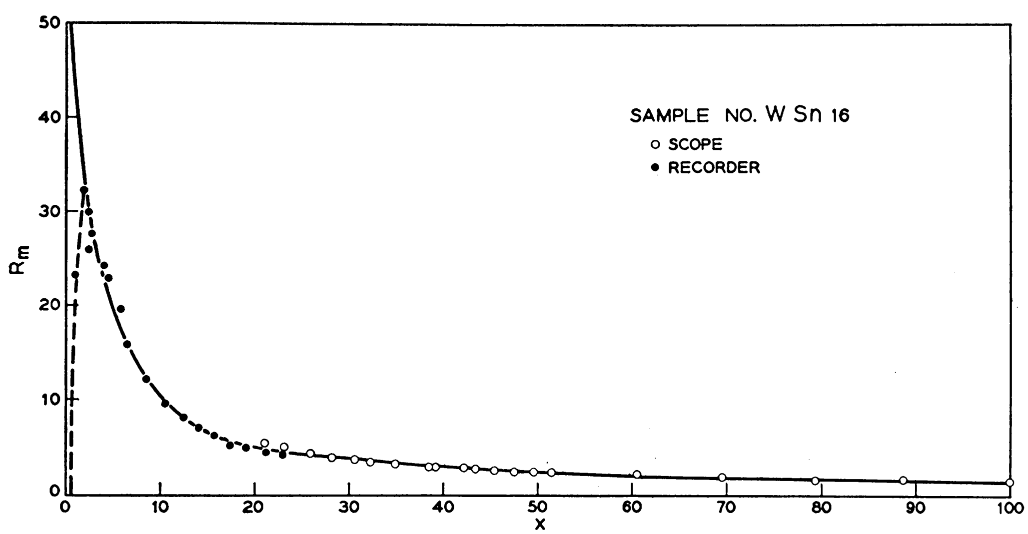

FIG. 13. Dependence of R_m on x at an applied frequency of 8.95 Gc/sec.

[10] D. J. Scalapino (private communication).
[11] E. Riedel, Z. Naturforsch 19A, 1634 (1964).
[12] N. R. Werthamer, Phys. Rev. 147, 253 (1966).

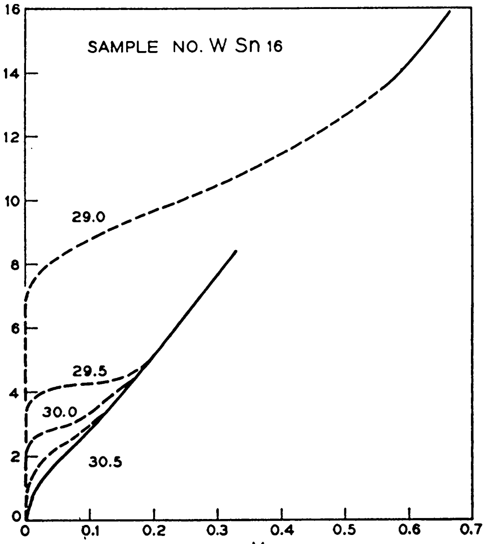

FIG. 14. Microwave-induced supercurrent at $x=-0.74$. Applied frequency is 8.95 Gc/sec. The dashed lines are regions of instability. Relative power level is shown in dB.

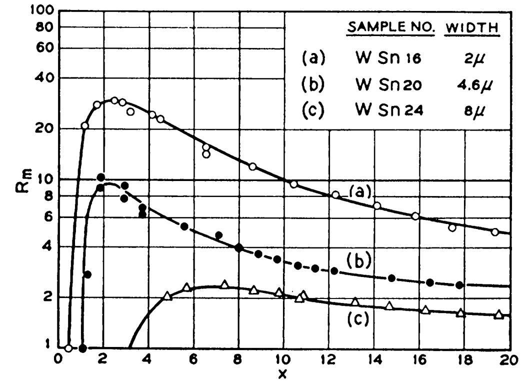

FIG. 15. Effect of the bridge width on the behavior of R_m with temperature. Applied frequency is 8.95 Gc/sec.

of a bridge should be similar to that of a tunnel junction our experience shows that they are quite different. The microwave effect on the supercurrent of a bridge does not resemble in any way the effect on the Josephson direct current in a tunnel junction. Furthermore the steps produced by microwaves in the I-V curve of a

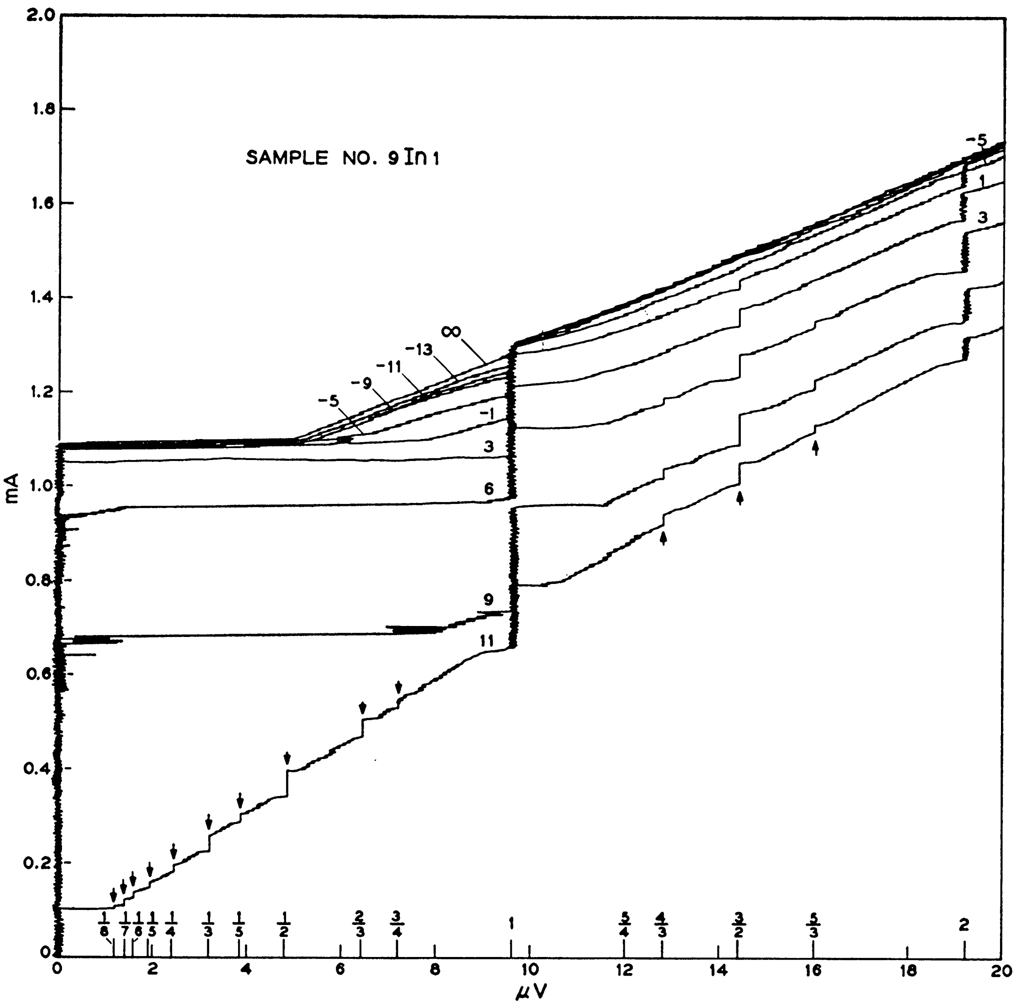

FIG. 16. Constant-voltage steps satisfying Eq. (4) at different power levels. Applied frequency is 4.26 Gc/sec.

bridge occur at voltages given by $mhf/2en$, i.e., at subharmonics as well as harmonics of the applied frequency while in a Josephson junction they occur only at the harmonics ($V=mhf/2e$).

VII. VORTEX-FLOW SYNCHRONIZATION BY MICROWAVE FIELD

We have ascribed the voltage appearing in the pure RS state to the flow of quantized vortices across the bridge. Since the bridge is a nonresonant structure, the I-V curve in the RS state is smooth and the rate of vortex flow at any voltage is given by

$$\nu=2eV/h.$$

Since the vortices are thermally activated, ν represents an average rate of flow. In general, at an applied direct voltage V one expects a Gaussian frequency spectrum centered around ν.

The application of a microwave field of frequency f will synchronize the vortex flow whenever the voltage-frequency relation satisfies the equation

$$m\times 2eV=n\times hf, \quad (4)$$

$$m\nu=nf,$$

where m and n are integers. At $m=1$, ν is a harmonic of f and when $m\neq 1$ and $m/n\neq$ integer then ν is a subharmonic of f. If the sample is carrying a direct current $I_1<I_c$ in the S state and a microwave field is applied the voltage will jump at a given power level to some harmonic or subharmonic, depending on temperature and applied frequency. This behavior is illustrated in Fig. 16 for various levels of microwave power. Of particular interest in this figure is the large number of subharmonics which are well resolved below as well as above the fundamental $n=m=1$. Some n/m values are shown on the abscissa and those harmonics which are resolved beyond doubt are indicated by arrows. At a subharmonic the synchronization occurs due to the flow of exactly one vortex every m/n cycles. The existence of subharmonics is a characteristic of the bridge but not of Josephson tunnel junctions in which only harmonics have been observed.

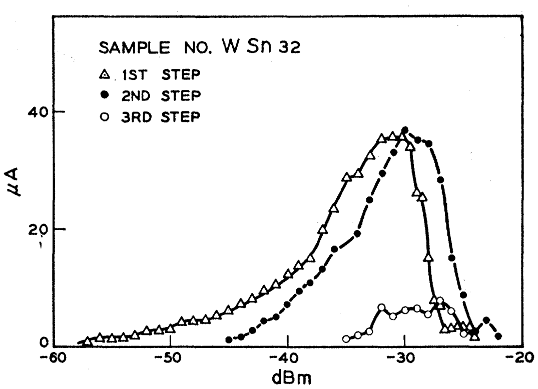

FIG. 17. Variation of the size of the constant-voltage steps with incident power level at 8.95 Gc/sec. The ($m=n=1$) step was measured at $T=3.837$°K, while the ($m=0$, $n=1$) and ($m=0$, $n=2$) steps were measured at $T=3.825$°K.

Also noticeable in Fig. 16 is the pulling effect of the applied field. A careful study of the I-V curve shows that the major steps are preceded and followed by a well-defined jump. Thus, as the current is increased to a given value, a voltage jump takes place to a state of sychronism with a certain n/m. This state is maintained over a certain current region at the end of which the system jumps out of synchronism, usually into another with a different value of n/m. One expects the I-V curve to be composed of a series of jumps between successive states of synchronism in accordance with (4). This often happens at low frequencies where the jumps occur between harmonics ($m=1$). For high values of m, however, the stability of synchronism is small, the successive steps are smeared and a rather uniform I-V curve is obtained until a strong synchronization is encountered belonging to a small m.

It is obvious that if the voltage jump at $I=I_c$ is larger than the voltage at which the fundamental step occurs ($m=n=1$), this step will not appear at small microwave power levels. Thus in order to study the variation of the size of the step with power we choose a temperature at which the voltage jump to the RS state is just smaller than that of the fundamental. The variation with power of the fundamental step as well as the second and third harmonics are shown in Fig. 17. For the sample used to obtain the results depicted in Fig. 17 we found that the ratio of the maximum size of a step to I_c was approximately 1/4, 1/7, and 1/17 for $n=1$, 2, and 3, respectively. This shows that the maximum size attained by a harmonic varies with its order like $1/(n+1)^2$.

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