Surface Impedance in the Surface Superconducting State

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To enable a simple calculation of the microwave surface resistance, the surface superconducting state is approximated by a model in which a layer of *uniform* order parameter is considered superposed on the normal bulk. Appropriate values for the order parameter and thickness of this layer were chosen with the use of the numerical solutions of the Ginsburg-Landau equations by Fink and Kessinger. A Gorter-Casimir temperature dependence was used for the order parameter and the Maki temperature dependence for κ. Our experiments on Pb-In alloys at frequencies of 9.5, 23, and 55 Gc/sec are in good agreement with the calculations.

I. INTRODUCTION

It was originally pointed out by Saint-James and de Gennes[1] that while the superconductivity of an infinite type-II superconductor disappears at a magnetic field H_{c2}, if a surface exists parallel to the applied magnetic field a thin layer at the surface will remain superconducting to H_{c3} where $H_{c3}=1.69H_{c2}$. Since that time, this surface superconductivity has been extensively studied both theoretically[2–4] and experimentally.[5] A theoretical treatment of the effects of the surface sheath on the surface impedance has not yet been given. Maki's calculation[6] of the surface impedance near H_{c2} did not include the surface sheath. A simplified model of the surface superconducting state in which the superconducting state is considered to have a constant order parameter over its thickness enables us to readily calculate the surface impedance. Fink and Kessinger[4] have solved the Ginzburg-Landau equations to determine the order parameter in the surface sheath as a function of distance from the surface for various values of the Ginzburg-Landau parameter κ. We have used their numerical results for the thickness and order parameter of the surface layer. We then calculated the surface impedance as a function of field, frequency, temperature, and κ. The temperature dependence was inserted by assuming a Gorter-Casimir[7] temperature dependence for the square of the order parameter and the Maki[8] temperature dependence for κ. The experimental results on Pb-In alloys which we report here are in good agreement with our calculations.

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II. THEORY

The expression for the surface impedance in terms of the electric field is given by

$$Z(\omega)=\frac{4\pi i\omega}{c^2}\frac{E(z)}{\partial E(z)/\partial z}\bigg|_{z=0}, \tag{1}$$

where ω is the frequency, and the surface is the plane $z=0$. We assume the uniform superconducting sheath to be of thickness δ on an infinite bulk material. In the sheath both an ongoing and a reflected wave exist, while in the bulk only the evanescent wave exists. For plane polarized waves (assume that the static magnetic field and the electric field are both in the x direction) incident normally in the z direction we can write for the sheath

$$E_s=A\exp[i(K_sz-\omega t)]+B\exp[-i(K_sz+\omega t)], \tag{2}$$

and for the bulk

$$E_b=C\exp[i(K_bz-\omega t)]. \tag{3}$$

From the continuity of E and H at $z=\delta$, we can solve Eqs. (2) and (3) to obtain

$$Z(\omega)=\frac{4\pi\omega}{c^2K_s}\left[\frac{K_s(1+\alpha)+K_b(1-\alpha)}{K_s(1-\alpha)+K_b(1+\alpha)}\right], \tag{4}$$

where $\alpha=\exp[2iK_s\delta]$.

Using Ohm's law and Maxwell's equations, the propagation constant K is given by

$$K=\left(\frac{4\pi\omega}{c^2}\right)^{1/2}(i\sigma_1-\sigma_2)^{1/2}, \tag{5}$$

where σ_1 is the real, and σ_2 the imaginary part of the complex conductivity. We are treating the case of "dirty" superconductors (assuming no nonlocal effects), and in the normal state our equations reduce to the classical skin-effect results.

We approximate σ_1 and σ_2 in the sheath by a two-fluid model. In the sheath, the order parameter at the surface $F(0)$ gives a measure of the number of superconducting electrons in the sheath. We can write for

σ_1 and σ_2[9]

$$\sigma_1 = a\sigma_N = [1 - F^2(0)]\sigma_N\,, \tag{6}$$

$$\sigma_2 = b\sigma_N = F^2(0)\pi(\Delta_{00}/\hbar\omega)\sigma_N\,. \tag{7}$$

For the temperature dependence of F, we assume $F^2(0,T) = F^2(0,0)(1-t^4)$.

To calculate α we consider the combination $(\sigma_N 4\pi\omega/c^2)^{1/2}\xi$, where ξ is the coherence length. Using the relations

$$H_{c2} = \varphi_0/2\pi\xi^2(T) = \sqrt{2}\kappa(T)H_c(T) \tag{8}$$

to define $\kappa(T)$ and $\xi(T)$, and the result for "dirty" superconductors[10] near T_c, that $\kappa(T_c) = 0.75\lambda_L(0)/l$, where l is the mean free path and $\lambda_L(0)$ the London penetration depth $[\lambda_L(0) = (3c^2/8\pi N(0)V_F{}^2e^2)^{1/2}]$, we can write

$$\left(\sigma_N\frac{4\pi\omega}{c^2}\right)^{1/2}\xi = \frac{0.466}{\kappa(T_c)}\left(\frac{\hbar\omega}{\Delta_{00}}\right)^{1/2}\left(\frac{\kappa(T_c)H_c(0)}{\kappa(T)H_c(T)}\right)^{1/2}. \tag{9}$$

We have used the relations[10] $\sigma_N = \frac{2}{3}N(0)e^2V_Fl$ and $H_c(0) = [4\pi N(0)]^{1/2}\Delta_{00}$ in the calculation. For the temperature dependence of H_c, we use $(1-t^2)$ while for $\kappa(T)$ we use an empirical expression $\kappa(T) = 1.20\kappa(T_c)(1-0.30t^2+0.13t^4)$ which fits Maki's[8] result quite well over the entire range. The expression for α is thus

$$\alpha = \exp\left[i\frac{0.85}{\kappa(T_c)}\left(\frac{\hbar\omega}{\Delta_{00}}\right)^{1/2}\frac{(ia-b)^{1/2}(\delta/\xi)}{[(1-t^2)(1-0.3t^2+0.13t^4)]^{1/2}}\right]. \tag{10}$$

In the numerical calculations we have taken the values of $F(0)$ and (δ/ξ) from the graphs of Fink and Kessinger[4] for the appropriate values of κ. For the detailed comparison with the data our calculations were done on a digital computer.

III. EXPERIMENTS

The measurements were made with a standard microwave bridge with the sample forming one wall of an undercoupled TE_{011} rectangular cavity. The actual measured quantity was the microwave power reflected from the sample cavity as a function of temperature and magnetic field. For small changes, the variation in the reflected power is proportional to the change in the Q of the cavity. This is in turn proportional to the change in the surface resistance R of the sample if that is the only material in the cavity whose R varies significantly with T and H. Care was taken that these conditions were met. It was always the change in R/R_n which was determined. We assume that R $(H=0)=0$ at our lowest temperature (1.7°K). Since the changes are large fractions of R_n, this is an excellent approximation. The experimental equipment is similar to that previously described.[11]

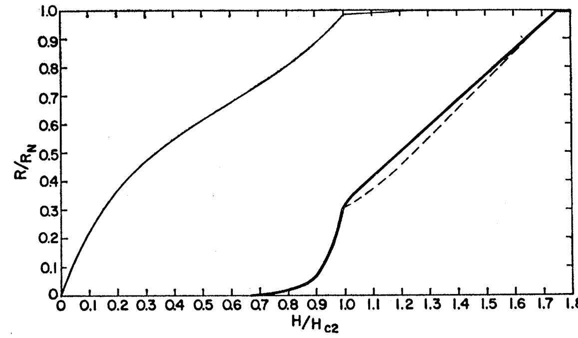

FIG. 1. R/R_n at 23 Gc/sec and 1.73°K versus applied magnetic field. The light line is the experimental curve for the magnetic field normal to the surface. The bold line is for magnetic field parallel to both the surface and the microwave current. The dashed line is the theoretical curve and lies on the bold line near H_{c3}.

The samples were alloys of high-purity lead and indium. The alloy was solidified rapidly while under vacuum and while being agitated. The alloy was then pressed into plates 1-cm across and about $\frac{1}{2}$-mm thick. This pressing was done with highly polished metal blocks and the resulting sample surface was mirror-like. The samples were then annealed in vacuum at 200°C for a day or longer. The resulting recrystallization slightly dulled the mirror surface.

In Fig. 1, R/R_n is plotted for the entire range of magnetic field for which superconductivity is exhibited. The region from H_{c2} to H_{c3} (here $H_{c3} = 1.75H_{c2}$) is the surface superconducting state with which we are presently concerned. When the magnetic field is normal to the metal surface, there is no surface superconductivity (light curve). The large surface resistance at low fields in this geometry is due to the viscous motion of the flux tubes in the mixed state when a component of the microwave current is perpendicular to the applied magnetic field.[12,13] When the magnetic field is parallel to the microwave current (bold curve), there is no absorption due to flux tube motion[14] since the Lorentz force on the flux tubes is zero. Just below H_{c2} there is an additional absorption mechanism in both geometries: the onset of gaplessness. This will be discussed in a forthcoming paper.[13]

[9] Equations (6) and (7) are generalizations of the two-fluid picture. See M. Tinkham, in *Low-Temperature Physics*, edited by G. DeWitt, B. Dreyfus, and P. G. de Gennes (Gordon and Breach Science Publishers, Inc., New York, 1962), p. 149. See also E. A. Lynton, *Superconductivity* (John Wiley & Sons, Inc., New York, 1962), p. 104.

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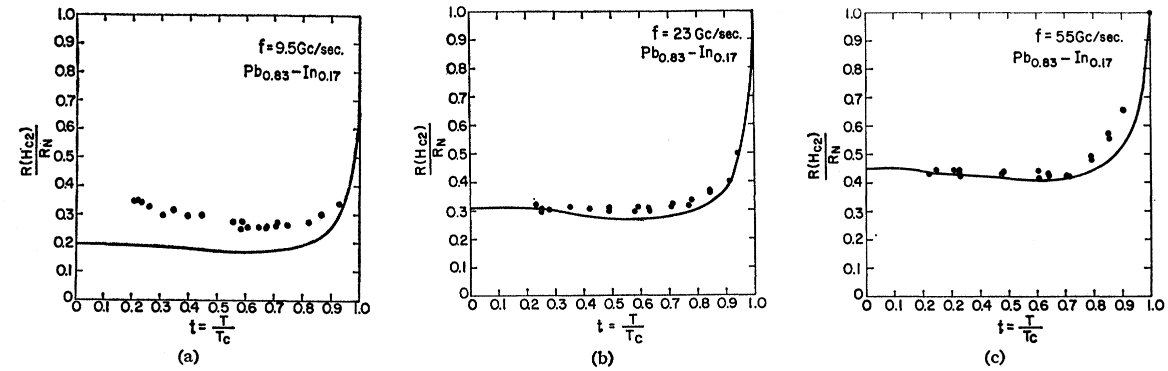

FIG. 2. Temperature variation of R/R_n at H_{c2}. The curves are theoretical and the points experimental; (a) at 9.5 Gc/sec, (b) at 23 Gc/sec, (c) at 55 Gc/sec. The 9.5-Gc/sec data were provided by Dr. G. Fischer.

The dashed line in Fig. 1 is the theoretically calculated surface resistance in the surface superconducting state. There are no adjustable parameters; the only place where any fit with experiment is forced in the model is $R/R_n=1$ at H_{c3}. The value of κ was taken to be 3.7 by using the value of H_{c2} determined by the data of Fig. 1 and assuming that the thermodynamic critical field is that of pure lead. This assumption is in agreement with the magnetization data of Druyvesteyn and Volger.[15] We obtained a value of Δ_{00} from the relation[16] $2\Delta_{00}=4.3kT_c$, where $T_c=7.1°$K. Since the data show $H_{c3}/H_{c2}=1.75$ while the theoretical ratio is 1.695, to facilitate comparison with the experimental curve the theoretical scale was expanded by shifting the calculated points by $0.08(H/H_{c2}-1)$. The curvature seen in the theoretical results is somewhat more pronounced for X band (9.5 Gc/sec) while for V band (55 Gc/sec) the results nearly form a straight line. For very low κ material (i.e., $\kappa\sim0.5$), such curvature is readily seen.[17]

In Fig. 2, we plot the temperature variation of the surface resistance at H_{c2}. At this field the bulk of the sample is normal but the surface sheath has its maximum order parameter. The curves are theoretical (with no adjustable parameters) and the points experimental. The presence of the minimum stems from the fact that the sheath's thickness increases while its order parameter decreases with temperature; initially the reduced absorption in the bulk exceeds the increased absorption in the sheath. The agreement with the theoretical curves is quite good except in two respects. First, the 9.5-Gc/sec data generally show more absorption than predicted. It is possible that this is due to irregularities in the sample surface. Second, at 55 Gc/sec, the absorption at higher reduced temperatures is greater than that given by the theoretical curve. This is not surprising since the temperature dependence put into the theory was the Gorter-Casimir $(1-t^4)$. At these higher frequencies direct transitions (excitation of quasiparticles) across the "optical" gap would be expected to play a significant role in the absorption process even though the surface superconducting state is "gapless."[13]

In Fig. 3, we plot $R(H_{c2})/R_n$ versus κ. The experimental points were determined from a series of Pb-rich Pb-In alloys. At present we have no explanation of why the data for $\kappa<2$ depart from the theoretical curve. Our assumption of locality will, of course, break down as the material is made purer and κ is reduced.

Data taken with the electric field perpendicular to the static magnetic field which is still parallel to the surface show similar behavior except that R/R_n is shifted upward at H_{c2} by 0.1 to 0.2. The existence of this anisotropy might be attributable to the same mechanism[18] which is thought to cause anisotropy in pure materials, i.e., an effective reduction of the gap due to a drift velocity of the pairs near the surface. Since this represents an added complication, we have confined our attention to the case in which the microwave electric field and the static magnetic field are parallel.

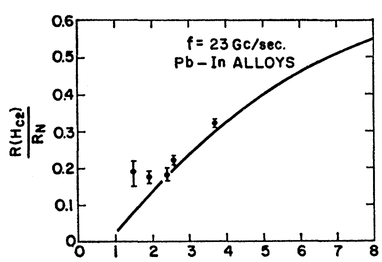

FIG. 3. Kappa variation of R/R_n at $H_{c2}(0)$ and 23 Gc/sec. The curve is theoretical and the points are experimental.

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IV. CONCLUSIONS

The good agreement between our calculations and our data indicates that the model used contains the essential features of the problem. The fact that results from the solution of the linearized Ginzburg-Landau equations can be extended to the full temperature range[4] so readily is due to the fact that we are dealing with local or "dirty" alloys, for which an equation of the Ginsburg-Landau type exists at all temperatures.[10] The discrepancies which exist between the data and the calculated results for the higher κ material may be attributed to an inadequacy of the constant thickness of the sheath approximation, and to deviations from the simple two-fluid model temperature dependence.[19]

ACKNOWLEDGMENTS

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Density of States of a Short-Mean-Free-Path Superconductor in a Magnetic Field by Electron Tunneling

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Measurements of the density of states of a thin-film, short-mean-free-path superconductor in a magnetic field are reported. The measurements were made on indium-tin films by a conventional tunneling technique. Temperatures near 0.5°K were used, in order to minimize thermal smearing. The magnetic field was in the plane of the film, and the film thickness was such that the order parameter was expected to be substantially independent of position. The results are in excellent agreement with theoretical calculations of Maki, except very close to the critical field, where small discrepancies occur. Maki's calculations predict gapless superconductivity close to the critical field, but thermal smearing prevented a direct verification. An attempt to fit the data to a BCS density of states with a field-dependent energy gap was completely unsuccessful except at zero field, where the two theories agree.

I. INTRODUCTION

SEVERAL years ago, Abrikosov and Gor'kov[1] pointed out that the addition of magnetic impurities to a superconductor should have a profound effect on its excitation spectrum. Their most startling prediction was the possibility of gapless superconductivity, that is, the existence of superconductivity without a gap in the excitation spectrum. Measurements by Woolf and Reif[2] appear to have confirmed their prediction. More recently, Maki,[3] and Maki and Fulde[4] have treated the effect of a magnetic field and/or transport current on a superconductor containing nonmagnetic impurities and have found essentially identical results. In this paper, we report on measurements made on superconducting films with a magnetic field in the plane of the film, and with the film thickness and mean free path chosen in such a way as to satisfy the conditions implicit in Maki's paper. The density of states in these films was measured by a conventional electron-tunneling technique[5] and the data were found to be in rather good agreement with calculations based on Maki's theory. In contrast, an attempt to fit the data to a BCS-type[6] density of states with field-dependent energy gap was completely unsuccessful, except at zero field.

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II. THEORY

A. Density of States

The geometry of a tunnel junction of the type used in our experiment is shown in Fig. 1. Here, metal 1 is a normal metal, the "insulator" is a mixture of oxide and adsorbed gases having a thickness of ≈ 20 Å, and metal

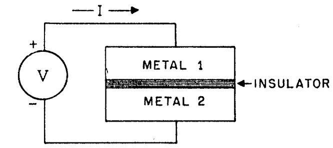

FIG. 1. Schematic representation of the tunneling experiment.

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