Transition Radiation from Thin Foils due to Non-Normally Incident Electrons*

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The results of an exact treatment of the transition radiation from a foil of arbitrary thickness bombarded by electrons at an arbitrary angle of incidence are presented. Several distributions of the radiation as calculated from the theory are given for Al and for Ag, as well as a comparison of theory with experimental results for the case of Ag.

I. INTRODUCTION

A DISCUSSION of transition radiation and references to the important works in this field may be found in a recent review paper by Frank[1] so we will mention here only works relevant to our specific problem. The transition radiation generated by electrons incident at an arbitrary angle on a plane interface between two dielectric media ("thick" foil) has been discussed by Garibyan[2] (see also references cited therein), and more recently Howe and Ritchie[3] have calculated the transition radiation *and* bremsstrahlung to be expected for this geometry.

Experimental work on thin aluminum foils prompted an approximate calculation of transition radiation from thin foils bombarded with non-normally incident electrons to explain maxima and minima in photon yield as a function of angle of incidence of the electrons.[4] The approximate calculation, valid for the electric susceptibility of the foil small compared to unity, could account for the experimental findings as due to constructive and destructive interference of radiation from the thin foil.

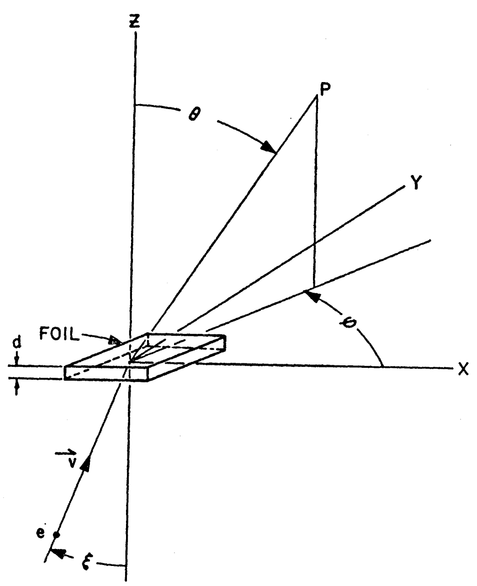

FIG. 1. Geometry for the transition radiation calculation. The foil has faces parallel to the x-y plane and extends from $z=0$ to $z=d$. The angle ξ is measured from the $-z$ axis in the x-z plane.

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[1] I. M. Frank, Usp. Fiz. Nauk **87**, 189 (1965) [English transl.: Soviet Phys.—Usp. **8**, 729 (1966)].

[2] G. M. Garibyan, Zh. Eksperim. i Teor. Fiz. **38**, 1814 (1960) [English transl.: Soviet Phys.—JETP **11**, 1306 (1960)].

[3] H. J. Howe and R. H. Ritchie, Oak Ridge National Laboratory Report No. ORNL-TM-1105 (unpublished).

II. THEORY

In this paper we present the results of a more exact treatment of the transition radiation from a thin foil due to non-normally incident electrons using Maxwell's equations and appropriate boundary conditions. The geometry is indicated in Fig. 1. The film specified by the dielectric constant $\epsilon(\omega)$ has faces parallel to the x-y plane and extends from $z=0$ to $z=d$. An electron is incident in the x-z plane at an angle ξ to the $-z$ axis.

It is convenient to work with the Hertz vector $\boldsymbol{\pi}(\mathbf{r},t)$ and its transform $\boldsymbol{\pi}_{\mathbf{K},\omega}(z)$ defined by

$$\boldsymbol{\pi}(\mathbf{r},t)=\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\int_{-\infty}^{\infty}d\omega \times e^{i(xk_x+yk_y-\omega t)}\boldsymbol{\pi}_{K,\omega}(z)\,, \tag{1}$$

$$\boldsymbol{\pi}_{K,\omega}(z)=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_{-\infty}^{\infty}dt\, e^{-i(xk_x+yk_y-\omega t)}\boldsymbol{\pi}(\mathbf{r},t)\,,$$

where $\mathbf{K}=k_x\mathbf{i}+k_y j$.

The electric and magnetic fields are obtained from $\boldsymbol{\pi}(\mathbf{r},t)$ by the relations

$$\mathbf{E}=\nabla(\nabla\cdot\boldsymbol{\pi})-\frac{\epsilon}{c^2}\frac{\partial^2\boldsymbol{\pi}}{\partial t^2},\qquad \mathbf{H}=\frac{\epsilon}{c}\frac{\partial}{\partial t}(\nabla\times\boldsymbol{\pi})\,. \tag{2}$$

We work in Gaussian units, and the magnetic permeability is set equal to one. The wave equation is given by

$$\left\{\frac{d^2}{dz^2}-K^2+\frac{\epsilon\omega^2}{c^2}\right\}\boldsymbol{\pi}_{K,\omega}(z)=-\frac{4\pi i}{\omega\epsilon}\mathbf{J}_{K,\omega}(z)\,. \tag{3}$$

The transform of the current density, $\mathbf{J}_{K,\omega}(z)$, is ob-

[4] J. C. Ashley, L. S. Cram, and E. T. Arakawa (to be published).

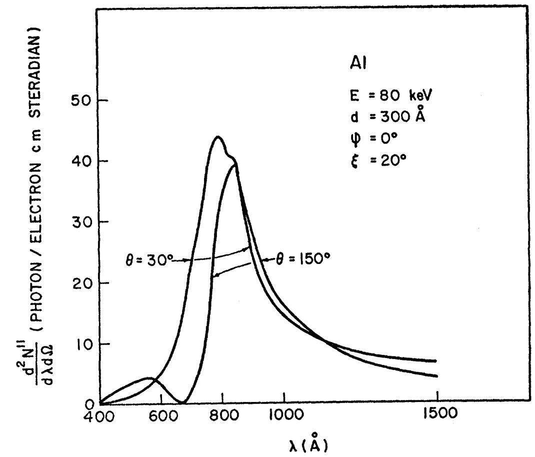

FIG. 2. Parallel component of transition radiation as a function of wavelength for a 300 Å aluminum foil.

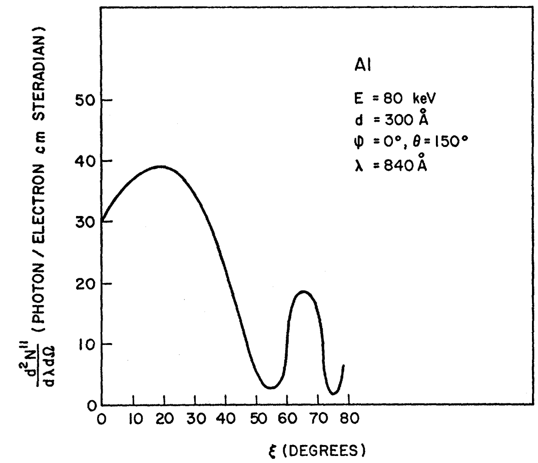

FIG. 3. Theoretical photon intensity as a function of angle of electron incidence for a 300 Å aluminum foil.

tained from

$$\mathbf{J}(\mathbf{r}.t)=e\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t) \quad \text{with} \quad \mathbf{v}=\mathbf{i}v\sin\xi+\mathbf{k}v\cos\xi,$$

and is given by

$$\mathbf{J}_{\mathbf{K},\omega}(z)=e(\mathbf{i}\tan\xi+\mathbf{k})\exp[i(\omega/v-k_x\sin\xi)z/\cos\xi]. \quad (4)$$

The Hertz vector thus has only x and z components. Using (4) in the wave equation (3), the components of $\boldsymbol{\pi}_{\mathbf{K},\omega}(z)$ are calculated in the three regions $z<0$, $0<z<d$, and $z>d$. The six resulting equations contain eight arbitrary amplitudes which are specified by application of the boundary conditions on the fields at the interfaces $z=0$ and $z=d$. Since we are interested in the far-zone fields, only one of the eight amplitudes must be calculated explicitly. The Hertz vector in the far zone, $\boldsymbol{\pi}(\mathbf{r},t)$, is obtained using the stationary phase approximation, and the fields calculated. Dividing the energy radiated into a solid angle $d\Omega$ and in a frequency interval ω to $\omega+d\omega$ by the energy per photon, $\hbar\omega$, we can express our results in terms of number of photons per electron per frequency interval $d\omega$ and solid angle $d\Omega$, $d^2N/d\omega d\Omega$. If we define parallel ($\|$) and perpendicular ($\perp$) polarization as $\mathbf{E}$, respectively, parallel and perpendicular to the plane formed by the normal to the foil and the radius vector to the point of observation of the photons, we find that while in the case of normally incident electrons there occurred only parallel polarization of the photons, for non-normally incident electrons we find both polarizations are present. The transition radiation is thus specified by the two distributions, $d^2N^{\|}/d\Omega d\omega$ and $d^2N^{\perp}/d\Omega d\omega$. The results of a lengthy calculation give for the distributions in the region $z\gg d$:

$$\begin{aligned}\frac{d^2N^{\|}}{d\Omega d\omega}=\frac{\alpha\beta^2\mu^2\tau^2}{\pi^2\omega}\left|\frac{\epsilon-1}{\gamma_+\gamma_-}\right|^2\Bigg|\frac{\sin\theta e^{it\rho/\beta\tau}}{d_1\delta_+\delta_-}&[m_+\gamma_+(\rho^2-\beta\sigma\tau\rho-\beta^2\tau^2)e^{-it\sigma}+m_-\gamma_-(\rho^2+\beta\sigma\tau\rho-\beta^2\tau^2)e^{it\sigma}\\
&-2\sigma\delta_+(\rho^2-\beta\mu\tau\rho-\epsilon\beta^2\tau^2)e^{-it\rho/\beta\tau}]+\frac{\beta(1-\mu^2)\cos\varphi\sin\xi}{d_1d_2}\bigg\{\frac{e^{it\rho/\beta\tau}}{\delta_+}[n_-^2m_-\gamma_-e^{2it\sigma}-n_+^2m_+\gamma_+e^{-2it\sigma}\\
&+n_-^2m_+\gamma_--n_+^2m_-\gamma_++4\sigma^2\beta\tau(\epsilon-1)]+(2\sigma/\delta_1)\bigg\{\bigg[\beta\tau m_+n_-+\frac{n_+}{\delta_+}(\rho^2-\mu\sigma\beta^2\tau^2)\bigg]n_+e^{-it\sigma}\\
&+[\beta\tau m_-n_+-(n_-/\delta_+)(\rho^2+\mu\sigma\beta^2\tau^2)]n_-e^{it\sigma}-2\sigma\beta\tau\rho(\epsilon-1)/\delta_+\bigg\}\bigg\}\\
&+\frac{\beta^2\tau\mu\sin\xi\cos\varphi e^{it\rho/\beta\tau}}{d_2\delta_+\delta_-}(n_-\gamma_-e^{it\sigma}+n_+\gamma_+e^{-it\sigma}-2\epsilon\sigma\delta_+e^{-it\rho/\beta\tau})\Bigg|^2 \quad (5)\end{aligned}$$

$$\frac{d^2N^{\perp}}{d\Omega d\omega}=\frac{\alpha\beta^6\mu^2\tau^4(1-\tau^2)\sin^2\varphi}{\pi^2\omega}\left|\frac{\epsilon-1}{d_2\delta_+\delta_-\gamma_+\gamma_-}\right|^2|n_-\gamma_-e^{it\sigma}+n_+\gamma_+e^{-it\sigma}-2\epsilon\sigma\delta_+e^{-it\rho/\beta\tau}|^2, \quad (6)$$

where $\alpha=e^2/\hbar c$, $\beta=v/c$, $\mu=\cos\theta$, $\tau=\cos\xi$, $\rho=1-\beta\cos\varphi\sin\theta\sin\xi$, $t=\omega d/c$, $\sigma^2=\epsilon-1+\mu^2(\mathrm{Im}\sigma>0)$, $d_1=(\mu\epsilon-\sigma)^2e^{it\sigma}-(\mu\epsilon+\sigma)^2e^{-it\sigma}$, $d_2=(\mu-\sigma)^2e^{it\sigma}-(\mu+\sigma)^2e^{-it\sigma}$, $\delta_\pm=\rho\pm\beta\mu\tau$, $\gamma_\pm=\rho\pm\beta\sigma\tau$, $m_\pm=\sigma\pm\mu\epsilon$, $n_\pm=\sigma\pm\mu$. These results derived for $0\leq\theta\leq\frac{1}{2}\pi$ (i.e., $z>d$) may be transformed to the other hemisphere of direction, i.e., $\frac{1}{2}\pi\leq\theta\leq\pi$, by the prescription $\mu\rightarrow|\mu|$, $\beta\rightarrow-\beta$, and $\cos\varphi\rightarrow-\cos\varphi$.

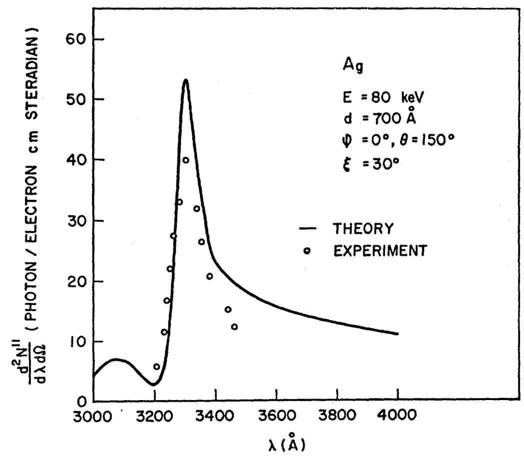

FIG. 4. Comparison of experimental and theoretical photon distributions for a 700 Å silver foil.

It may readily be verified that these results reduce to the results of Ritchie and Eldridge[5] for normal electron incidence ($\xi=0$). In the limit of thick foils ($t\rightarrow\infty$) these reduce to the transition radiation terms calculated by Howe and Ritchie as described in Ref. 3.

III. THEORETICAL DISTRIBUTIONS AND COMPARISON WITH EXPERIMENT

A computer program has been written to evaluate the two photon distributions, Eqs. 5 and 6. The calculated distributions for Al are based on the optical constants of Hunter[6] and for Ag on the work of Huebner *et al.*[7]

Figure 2 gives the distribution of photons of parallel polarization as a function of wavelength for a 300 Å thick Al foil with 80-keV electrons incident at an angle $\xi=20°$. This example is for $\varphi=0$, (the perpendicular component of radiation is zero) and for $\theta=30°$ and 150°. Figure 3 exhibits, for the same electron energy and film thickness, the parallel photon distribution as a function of the electron angle of incidence for $\varphi=0$ and $\theta=150°$ at $\lambda=840$ Å. Note the maxima and minima which result from constructive and destructive interference.

A representative distribution for Ag is shown in Fig. 4. Here electrons of 80 keV are incident on a 700 Å foil at $\xi=30°$. The photons are observed at $\varphi=0°$, $\theta=150°$. The distribution $d^2N^{\parallel}/d\Omega d\lambda$ versus λ for various values of ξ have closely the same shape for $\xi=0°$ to $\xi=65°$ with the peak at about 3300 Å, first increasing in magnitude by about 20% with increasing ξ to a maximum at $\xi\approx30°$, then decreasing to about $\frac{1}{3}$ of the maximum value at $\xi=60°$. Experimental data[8] from a 700 Å Ag foil is included in Fig. 4 which indicates reasonably good agreement with the theory. For angles of electron incidence greater than about 50° the bremsstrahlung component becomes more important, and this theory is no longer applicable.

IV. CONCLUSIONS

We have presented the results of a calculation of the transition radiation to be expected from a foil of arbitrary thickness bombarded by electrons at an arbitrary angle of incidence. Comparison with experimental data on Ag indicates reasonable agreement with theory for angles of electron incidence not too far from $\xi=0°$. For ξ near 90° the main contribution to the observed emission appears to be from bremsstrahlung (see Ref. 3), at least for very thick foils. A theoretical description of the transition radiation *and* bremsstrahlung from a thin foil is needed for further investigations along these lines.

ACKNOWLEDGMENTS

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[5] R. H. Ritchie and H. B. Eldrigde, Phys. Rev. **126**, 1935 (1962).
[6] W. R. Hunter, J. Opt. Soc. Am. **54**, 208 (1964).
[7] R. H. Huebner, E. T. Arakawa, R. A. MacRae, and R. N. Hamm, J. Opt. Soc. Am. **54**, 1434 (1964).
[8] L. S. Cram (private communication).