Model for Low-Energy Meson-Baryon Scattering

GARY R. GOLDSTEIN*
Department of Physics, University of Chicago, Chicago, Illinois
and
Argonne National Laboratory, Argonne, Illinois

AND

KAMESHWAR C. WALI†
Argonne National Laboratory, Argonne, Illinois
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A model for low-energy meson-baryon scattering within the framework of $U(6,6)$ symmetry is studied. The effective $U(6,6)$-symmetric terms are treated as background terms representing distant singularities. These are supplemented by the appropriate one-particle exchange contributions. The model is used to calculate the real parts of the S, P, and D partial-wave amplitudes in the low-energy region of πN and KN elastic scattering. It is found that the results are in reasonably good agreement with the experimental information in $T=\frac{3}{2}$ and $T=\frac{1}{2}$ πN and in $T=1$ KN scattering. However, the model predicts large S-wave scattering in the $T=0$ KN channel.

I. INTRODUCTION

In dispersion theoretic treatments of scattering processes, it has been customary to treat the long-range forces (or, equivalently, singularities near the physical region) by appropriate one-particle exchange graphs. The short-range forces (or, equivalently, distant singularities) are approximated by low-order polynomials in the usual Mandelstam variables s and t. The resulting representation for the scattering amplitude is known as the Cini-Fubini approximation[1] and has proved successful in semiphenomenological analyses of πN and KN elastic scattering in a restricted energy region.[2,3]

The present paper is concerned with the investigation of a specific model for meson-baryon scattering, which leads to Cini-Fubini–type amplitudes in a very natural way. The model is based on a symmetry scheme which is presently known as $U(6,6)$ symmetry.[4–6] The main ideas leading to the model were briefly discussed in I. As regards the development of $U(6,6)$ symmetry, its successes and failures, the interested reader is referred to a number of recent review articles.[7] What is relevant for our present purposes is the consideration that $U(6,6)$ symmetry provides a satisfactory description of three-point functions.[8] This implies in particular that the trilinear meson-baryon coupling constants, where the mesons and baryons belong respectively to the 35-dimensional (octet of pseudoscalar and nonet of vector mesons) and 56-dimensional (octet of $J^P=\frac{1}{2}^+$ and decuplet of $J^P=\frac{3}{2}^+$ baryons) representations of $SU(6)$, are determined in terms of the πN and ρ-$\pi\pi$ coupling constants. Thus, all the one-particle exchange contributions involving these particles are determined in terms of these two coupling constants.

In addition to the one-particle exchange contributions, there are "effective" $U(6,6)$ symmetric terms which are polynomials in the variables s and t.[9,10] These terms may be considered as arising from more complicated diagrams. Taken by themselves as representing the full scattering amplitude, they lead to some consequences which are in total disagreement with experimental results.[9,10] Our viewpoint is to regard these terms, supplemented by the one-particle exchange contributions, as input Born terms for dynamical calculations based on dispersion theoretic methods. To see whether such an approach is meaningful, we consider in this paper pseudoscalar meson-baryon scattering in partial wave amplitudes f_l, where $l=0, 1, 2$. As no unitarity corrections are taken into account, the scattering amplitudes are real and the model is applicable only to small phase shifts in the low-energy region.

In Sec. II, we analyze the structure of the $U(6,6)$ symmetric terms. Section III is devoted to the discussion of free-particle propagators and the calculation of one-particle exchange contributions. The general features and the comparison of the numerical results

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* Associated Midwest Universities—Argonne National Laboratory Predoctoral Fellow.
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[7] R. Delbourgo, M. A. Rashid, A. Salam, and J. Strathdee, in *Proceedings oj the Seminar Conference on High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965); A. Pais, Rev. Mod. Phys. **38**, 215 (1966); B. Sakita, Advances in High-Energy Physics (to be published).
[8] It should be noted that only the collinear subgroup of $U(6,6)$, $SU(6)_W$ is relevant for the three-point functions. H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965).
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of the model in the case of low-energy πN and KN scattering are presented in Sec. IV. The final section contains the summary and the discussion of the results.

II. $U(6,6)$-SYMMETRIC FOUR-POINT FUNCTIONS

In the $U(6,6)$ symmetry scheme, one can construct effective interaction terms that are formally $U(6,6)$ invariant. However, the kinetic terms or the free Lagrangian parts break this symmetry. This was the main reason that led Bég and Pais[11] to suggest that such invariant terms represent effective S-matrix elements. As remarked in the Introduction, we interpret the four-point interaction terms only as approximations to distant singularities in two-body scattering.

For meson-baryon scattering there are four independent $U(6,6)$ symmetric amplitudes and, in momentum space, the effective scattering matrix element is given by

$$\begin{aligned} M_S=&\alpha\bar{\Psi}^{ABC}(p')\Psi_{ABC}(p)\bar{\Phi}_E{}^D(-k')\Phi_D{}^E(k)\\ &+\beta\bar{\Psi}^{ABC}(p')\Psi_{ABD}(p)\bar{\Phi}_E{}^D(-k')\Phi_C{}^E(k)\\ &+\tilde{\beta}\bar{\Psi}^{ABC}(p')\Psi_{ABD}\Phi_E{}^D(k)\bar{\Phi}_C{}^E(-k')\\ &+\gamma\bar{\Psi}^{ABC}(p')\Psi_{ADE}(p)\bar{\Phi}_B{}^D(-k')\Phi_C{}^E(k)\,, \end{aligned} \tag{2.1}$$

where Ψ_{ABC} and $\Phi_A{}^B$ are, respectively, the totally symmetric third-rank and mixed second-rank tensors of $U(6,6)$. As in the case of three-point functions in I, we calculate M_S by using free-field solutions of Bargmann-Wigner equations for Ψ and Duffin-Kemmer equations for Φ. Here α, β, $\tilde{\beta}$, γ are functions of s, t, and u subject to the restrictions due to crossing symmetry:

$$\begin{gathered} \alpha(s,t,u)=\alpha(u,t,s);\quad \beta(s,t,u)=\tilde{\beta}(u,t,s);\\ \gamma(s,t,u)=\gamma(u,t,s). \end{gathered} \tag{2.2}$$

In the present investigation, we make the simplest choice for these arbitrary functions, namely, we regard them as constants. The restrictions in (2.2) then imply that $\beta=\tilde{\beta}$, and hence we have effectively three free parameters.

To obtain M_S, it is convenient to define the "currents"

$$\begin{aligned} j_C{}^D&=\bar{\Phi}_E{}^D(-k')\Phi_C{}^E(k)+\Phi_E{}^D(k)\bar{\Phi}_C{}^E(-k')\,,\\ J_D{}^C&=\bar{\Psi}^{ABC}(p')\Psi_{ABD}(p)\,, \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} J_{ABC}&=\Phi_A{}^D(k)\Psi_{BCD}(p)\,,\\ \bar{J}^{ABC}&=\bar{\Psi}^{ABD}(p')\bar{\Phi}_D{}^C(-k')\,. \end{aligned} \tag{2.4}$$

M_S is then given by

$$M_S=\alpha J_C{}^C j_D{}^D+\beta J_D{}^C j_C{}^D+\gamma\bar{J}^{ABC}J_{ABC}\,. \tag{2.5}$$

As we are interested in pseudoscalar-meson scattering from the octet of baryons, we consider only the pseudoscalar-meson and baryon parts of Φ and Ψ in the currents. From I, we have[12]

$$\Phi_i{}^j{}_\alpha{}^\beta(k):\quad \left[\gamma_5\left(1-\frac{ik}{m_0}\right)\right]_i{}^j P_\alpha{}^\beta\,,$$

$$\Psi_{ijk,\alpha\beta\gamma}:\quad B_{ijk,\alpha\beta\gamma}=\tfrac{1}{3}[\chi_{ijk,\alpha}{}^\delta\epsilon_{\delta\beta\gamma}+\chi_{jki,\beta}{}^\delta\epsilon_{\delta\gamma\alpha}+\chi_{kij,\gamma}{}^\delta\epsilon_{\delta\alpha\beta}]\,,$$

where

$$\chi_{ijk,\alpha}{}^\delta=\tfrac{1}{2}[\gamma_5C(1+ip/M_\alpha{}^\delta)]_{jk}u_i(p)B_\alpha{}^\delta\,. \tag{2.6}$$

By using these free-field solutions, we compute the currents (2.3) and (2.4) and hence M_S. The results are summarized in the Appendix (A6) in terms of the conventional invariant amplitudes $A(s,t)$ and $B(s,t)$.[13]

Although M_S has been calculated in this fashion, it is instructive and convenient for the purposes of the next section to decompose the currents $j_C{}^D$ and J_{ABC} in a manner similar to that for $J_C{}^D$ in I [Eqs. (5.6, 5.7)].

$$\begin{aligned} j_C{}^D(\bar{P}P)=j_i{}^j{}_\gamma{}^\delta(\bar{P}P)=\tfrac{1}{4}[&(1)_i{}^j j^S(\bar{P}P)\\ &+(\gamma_\mu)_i{}^j j^V{}_\mu(\bar{P}P)+(\sigma_{\mu\nu})_i{}^j j^T{}_{\nu\mu}(\bar{P}P)\\ &+(\gamma_\mu\gamma_5)_i{}^j j^A{}_\mu(\bar{P}P)+(\gamma_5)_i{}^j j^P(\bar{P}P)]_\gamma{}^\delta\,, \end{aligned}$$

where

$$\begin{aligned} j^S(\bar{P}P)&=4\left(1-\frac{k'\cdot k}{m_0{}^2}\right)(\bar{P}P+P\bar{P})\,,\\ j^V{}_\mu(\bar{P}P)&=\frac{4i}{m_0}(k_\mu+k_\mu')(\bar{P}P-P\bar{P})\,,\\ j^T{}_{\mu\nu}(\bar{P}P)&=\frac{i}{4m_0}(q_\nu j^V{}_\mu(\bar{P}P)-q_\mu j^V{}_\nu(\bar{P}P))\,,\\ j^P(\bar{P}P)&=j^A(\bar{P}P)=0\,, \end{aligned} \tag{2.7}$$

where $q_\mu=k_\mu'-k_\mu$, and P and $\bar{P}$ are $SU(3)$ matrices for incoming and outgoing pseudoscalar mesons.

From $J_{ABC}(BP)$, we can project the parts $J^D{}_{ABC}$ and $J^B{}_{ABC}$ that have the $SU(3)$ transformation property of the decuplet and the octet and expand them as follows:

$$\begin{aligned} J^D{}_{ABC}(BP)&=J^D{}_{ijk,\alpha\beta\gamma}(BP)=\tfrac{1}{2}[(\gamma_\mu C)_{jk}J^D{}_{\mu,i}+\tfrac{1}{2}(\sigma_{\mu\nu}C)_{jk}J^D{}_{\mu\nu,i}]_{\alpha\beta\gamma}\,,\\ J^B{}_{ABC}(BP)&=J^B{}_{ijk,\alpha\beta\gamma}(BP)=\tfrac{1}{3}[J^B{}_{ijk,\alpha}{}^\delta\epsilon_{\delta\beta\gamma}+J^B{}_{jki,\beta}{}^\delta\epsilon_{\delta\gamma\alpha}+J^B{}_{kij,\gamma}{}^\delta\epsilon_{\delta\alpha\beta}]\,,\\ J^B{}_{ijk,\alpha}{}^\delta&=\tfrac{1}{2}[J^\xi{}_iC_{jk}+J^\eta{}_{\mu,i}(\gamma_\mu\gamma_5C)_{jk}+J^\psi{}_i(\gamma_5C)_{jk}]_\alpha{}^\delta\,. \end{aligned} \tag{2.8}$$

[11] M. A. B. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965).

[12] Because of the specific way in which mass splittings were introduced in I, $m_0=m_\rho$ and $M_\alpha{}^\delta$ corresponds to the physical mass of the baryon $B_\alpha{}^\delta$.

[13] In the absence of mass splittings these results contain the results of Cornwall *et al.* (Ref. 10) as special cases.

Then,

$$J^D{}_{\lambda,\alpha\beta\gamma}=\frac{-1}{54}\left[\left(1-2\frac{ik}{m_0}-\frac{p\cdot k}{Mm_0}\right)\gamma_\lambda+\frac{2ip_\lambda}{M}\left(2-\frac{ik}{M}\right)-\frac{2ik_\lambda}{m_0}\right]u(p)\sum_{P(\alpha\beta\gamma)}\epsilon_{\delta\sigma\alpha}B_\beta{}^\delta P_\gamma{}^\sigma,$$

$$J^D{}_{\mu\nu,\alpha\beta\gamma}=\frac{-1}{54}\left[\left(1+\frac{k\cdot p}{Mm_0}\right)\sigma_{\mu\nu}-2i\left(1+\frac{ik}{m_0}\right)\left(\frac{p_\mu\gamma_\nu}{M}-\frac{p_\nu\gamma_\mu}{M}\right)+\frac{2}{Mm_0}(k_\mu p_\nu-k_\nu p_\mu)\right]u(p)\sum_{P(\alpha\beta\gamma)}\epsilon_{\delta\sigma\alpha}B_\beta{}^\delta P_\gamma{}^\sigma,$$

$$\gamma_\mu J^D{}_\mu=0,\quad \gamma_\mu J^D{}_{\mu\nu}=J^D{}_\nu. \tag{2.9}$$

$$J^\psi{}_\alpha{}^\delta=\frac{1}{18}\left[\left(3-5\frac{ik}{m_0}-2\frac{p\cdot k}{Mm_0}\right)\gamma_5u(p)(P_\alpha{}^\sigma B_\sigma{}^\delta-\tfrac{1}{3}\delta_\alpha{}^\delta\,\mathrm{Tr}PB)-\left(\frac{ik}{m_0}+\frac{p\cdot k}{Mm_0}\right)\gamma_5u(p)(B_\alpha{}^\sigma P_\sigma{}^\delta-\tfrac{1}{3}\delta_\alpha{}^\delta\,\mathrm{Tr}BP)\right],$$

$$J^\eta{}_{\mu,\alpha}{}^\delta=\frac{1}{18}\left[\left\{2\left(1+\frac{p\cdot k}{Mm_0}\right)\gamma_\lambda-\frac{5ip_\lambda}{M}\left(1-\frac{ik}{m_0}\right)\right\}\gamma_5u(p)(P_\alpha{}^\sigma B_\sigma{}^\delta-\tfrac{1}{3}\delta_\alpha{}^\delta\,\mathrm{Tr}BP)\right.$$
$$\left.+\left\{\left(1+\frac{p\cdot k}{Mm_0}\right)\gamma_\lambda-\frac{ip_\lambda}{M}\left(1-\frac{ik}{m_0}\right)\right\}\gamma_5u(p)(B_\alpha{}^\sigma P_\sigma{}^\delta-\tfrac{1}{3}\delta_\alpha{}^\delta\,\mathrm{Tr}BP)\right],$$

and

$$J^\xi=\gamma_5J^\psi+\gamma_\mu\gamma_5J^\eta{}_\mu. \tag{2.10}$$

The decompositions (2.7), (2.9), and (2.10) show more clearly the structure of the $U(6,6)$ symmetric terms (J^Sj^S, $J^V{}_\mu j^V{}_\mu$, $J^\psi{}_BJ^\psi{}_B$, etc.). They also isolate the parts of the current that would couple to the vector-meson, baryon, and decuplet.

III. PROPAGATORS FOR Φ AND Ψ; ONE-PARTICLE EXCHANGES

There are two procedures that one can adopt in computing the one-particle exchange contributions. The first procedure is to regard $U(6,6)$ symmetry for the three-point functions (such as $\mathrm{Tr}\bar\Psi\Psi\Phi$, $\mathrm{Tr}\Phi\Phi\Phi$) as the symmetry that relates only the coupling constants and not the vertex functions. It is then a straightforward matter to use the conventional propagators for the physical particles that Ψ and Φ describe and compute the Born terms. Such a procedure [$SU(6)_W$ approach] has been the basis of several bootstrap models.[14] An alternative approach which is more consistent with the original formulation[4,6] of $U(6,6)$ symmetry is to compute the Born terms from the trilinear interaction terms and the propagators for Φ and Ψ derived from a Lagrangian.

Several authors[15] have discussed the Lagrangian formulation within the framework of $U(6,6)$ symmetry. In the present investigation, we follow Salam, Delbourgo, and Strathdee[4] with the appropriate modifications necessary to include the mass splittings. The essential difference between different approaches is in the contact terms (or nonpole terms) that invariably appear in the propagators. The contact terms are unique in the case of the field Φ because of the fact that Φ obeys the Duffin-Kemmer equations which can be derived from a Lagrangian. In the case of the Ψ field, the total symmetry of Ψ_{ABC} requires the presence of auxiliary variables in the Lagrangian. Different procedures lead to different contact terms, although the residues of the poles are the same in all cases. A prescription is therefore necessary to determine the contact terms. We obtain the required prescription by examining the meson-exchange contributions.

If we ignore the mass splittings, the propagator for Φ is given by

$$\langle\Phi_A{}^B,\Phi_C{}^D\rangle_+=\mathfrak{P}_{i\alpha,k\gamma}{}^{j\beta,l\delta}$$
$$=\frac{1}{2}\left[\delta_A{}^D\delta_C{}^B+\frac{(-iq+m)_A{}^D(iq+m)_C{}^B}{m^2+q^2}\right]. \tag{3.1}$$

It is easy to verify that

$$\tfrac{1}{2}[(iq+m)_l{}^n\delta_m{}^k+(-iq+m)_m{}^k\delta_l{}^n]\times\mathfrak{P}_{i,k}{}^{j,l}=\frac{1}{m}\delta_i{}^n\delta_m{}^j, \tag{3.2}$$

where Φ satisfies the Duffin-Kemmer equation

$$\tfrac{1}{2}[(iq+m)_l{}^n\delta_m{}^k+(-iq+m)_m{}^k\delta_l{}^n]\Phi_n{}^m=0, \tag{3.3}$$

and where, for convenience, we have suppressed the $SU(3)$ variables. Now to obtain the one-meson exchange contribution to meson-baryon scattering we take the time-ordered product of the trilinear $U(6,6)$-invariant terms $iG\bar\Psi^{EFA}\Psi_{EFB}\Phi_A{}^B$ and $(ig/4m_0)\Phi_C{}^D\times\Phi_D{}^G\Phi_G{}^C$, and contract once on the meson fields. The result in terms of (2.3), (2.4), and (3.1) is

$$-\frac{gG}{4m_0}\bar J_B{}^A\mathfrak{P}_{AC}{}^{BD}j_D{}^C.$$

[14] R. Gatto and G. Veneziano, Phys. Letters **19**, 512 (1965); R. H. Capps, Phys. Rev. Letters **16**, 1066 (1966).

[15] C. S. Guralnik and T. W. B. Kibble, Phys. Rev. **139**, B712 (1965); S. Kamefuchi and Y. Takahashi, Nuovo Cimento **44**, A1 (1966); A. Salam *et al.* (Ref. 4).

It is clear that the terms in $\mathfrak{P}_{AC}{}^{BD}$ proportional to $\delta_A{}^D\delta_C{}^B$ will produce a $U(6,6)$ invariant contribution to the one-meson exchange (obtained by letting $q_\mu=0$) which is proportional to the β term of Eq. (2.5). The parts of the propagator containing $i\mathbf{q}$'s will break the $U(6,6)$ symmetry. Therefore, it is natural to redefine the propagator so that the resulting "one-particle exchange" is a pure symmetry-breaking amplitude. This is accomplished by adding the contact term $-\delta_A{}^D\delta_C{}^D$ to the original propagator (3.1) to obtain the symmetry-breaking propagator

$$\langle\Phi_A{}^B,\bar\Phi_C{}^D\rangle_+ =\frac{1}{2}\left[-\delta_A{}^D\delta_C{}^B+\frac{(-i\mathbf{q}+m)_A{}^D(i\mathbf{q}+m)_C{}^B}{m^2+q^2}\right]. \quad (3.4)$$

Similar considerations lead us to define the propagator corresponding to the Ψ field as

$$\langle\Psi_{ABC},\bar\Psi^{DEF}\rangle_+ =\frac{1}{12}\sum_{P(ABC)}\left[\frac{(-i\mathbf{q}+M)_A{}^D(-i\mathbf{q}+M)_B{}^E(-i\mathbf{q}+M)_C{}^F}{M(q^2+M^2)} -\frac{(M-3i\mathbf{q})_A{}^D\delta_B{}^E\delta_C{}^F}{M}\right]. \quad (3.5)$$

Note that both propagators [(3.4) and (3.5)] vanish when the 4-momentum of the exchanged particle goes to zero. From (3.4) and (3.5), the propagators for different components of Φ and Ψ [compare Delbourgo *et al.*, Eqs. (4.29) and (4.30), Ref. 7] can be obtained by multiplying with appropriate combinations of γ matrices and the antisymmetric charge-conjugation matrix C. The inclusion of the mass splittings, based on the work in I, essentially amounts to introducing physical masses in the projected propagators. If we combine the symmetry-breaking propagators (3.4) and (3.5) with the vertex functions, we can write somewhat symbolically the effective meson-exchange and baryon-exchange matrix elements in momentum space as

$$-\tfrac{1}{4}Gg\bar\Psi^{ABC}(p')\Psi_{ABD}(p)\langle\Phi_C{}^D\bar\Phi_E{}^F\rangle_+ \times[\Phi_F{}^G(k)\bar\Phi_G{}^E(-k')+\bar\Phi_F{}^G(-k')\Phi_G{}^E(k)],$$

and

$$G^2\bar\Psi^{ABA'}(p')\Phi_{A'}{}^C(-k')\langle\Psi_{ABC}\bar\Psi^{DEF}\rangle_+\Phi_D{}^{D'}(k)\Psi_{D'EF}(p),$$

or in terms of the currents (2.3) and (2.4),

$$-\tfrac{1}{4}GgJ_D{}^C\langle\Phi_C{}^D\bar\Phi_E{}^F\rangle_+j_F{}^E,$$

and

$$G^2\bar J^{ABC}\langle\Psi_{ABC}\bar\Psi^{DEF}\rangle_+J_{DEF}. \quad (3.6)$$

Finally when we use the decompositions (2.7), (2.9), and (2.10) for the currents, we obtain

for *vector-meson exchange:*

$$-\frac{3Gg}{16}\frac{1}{m_V{}^2+q^2}\left[\frac{3}{2}(q_\lambda q_\mu-\delta_{\mu\lambda}q^2)J^V{}_\mu j^V{}_\lambda -\frac{2i}{m_0}J^T{}_{\mu\nu}(m_V{}^2-\tfrac{1}{2}q^2)q_\mu j_\nu\right], \quad (3.7)$$

for *baryon exchange:*

$$G^2\left[\left(\bar J^\psi+\frac{iq_\mu}{M_f}\bar J^\eta{}_\mu\right)\frac{M_E-i\mathbf{q}}{M_E{}^2+q^2}\left(J^\psi-\frac{iq_\lambda}{M_i}J^\eta{}_\lambda\right) -\bar J^\psi\frac{M_E-i\mathbf{q}}{M_E}J^\psi+\frac{i}{M_E}(\bar J^\psi q_\mu J^\eta{}_\mu-\bar J^\eta{}_\mu q_\mu J^\psi)\right]. \quad (3.8)$$

for *decuplet exchange:*

$$G^2\left[\left(\bar J^D{}_\mu+\frac{iq_\nu}{M_f}\bar J_{\nu\mu}\right) \times\left(\delta_{\mu\lambda}+\frac{q_\mu q_\lambda}{M_E{}^2}\right)\frac{M_E(M_E-i\mathbf{q})}{M_E{}^2+q^2}\left(J_\lambda+\frac{iq_\rho}{M_i}J_{\lambda\rho}\right) -\bar J_\mu\frac{M_E-2i\mathbf{q}}{M_E}J_\mu+\tfrac{1}{2}\bar J_{\mu\nu}\frac{i\mathbf{q}}{M_E}J_{\nu\mu}\right]. \quad (3.9)$$

In (3.7), (3.8), and (3.9), a summation over $SU(3)$ variables is implied. The initial, final, and exchange masses M_i, M_f, and M_E, $m_v{}^2$ can be identified easily by the $SU(3)$ variables. The invariant amplitudes A and B obtained from these expressions are summarized in the Appendix [(A7), (A8), and (A9)]. It is to be noted that because of the contact terms in the propagators, the terms that we characterize as "one-particle exchanges" contain, in addition to the conventional exchange terms, some extra nonpole contributions. A characteristic feature of these symmetry-breaking "one-particle exchange" amplitudes is that they vanish at threshold in elastic scattering.

IV. NUMERICAL RESULTS

Before proceeding, certain limitations of the model should be considered. The "one-particle exchanges" that have been included represent only the nearest singularities to the elastic threshold of meson-baryon scattering in the sense that they contain the lowest-lying $SU(6)$ multiplets as intermediate states. Furthermore, these "exchange" terms are real functions of s and t that contain divergent contact parts. The $U(6,6)$-symmetric terms are polynomials in s and t that also diverge for high energies, and if α, β, and γ are chosen to be real constants, the corresponding amplitudes will be real. Hence, without modifications to account for elastic unitarity and higher resonance states, we expect our model to apply only in the low-energy region of the physical cut, where the amplitudes are necessarily small and real. This rules out the $\bar KN$ system,

TABLE I. Experimental and theoretical values (using $\alpha=-\beta=-0.0957$) of the S-wave (a_{2T}) and P-wave ($a_{2T,2J}$) scattering lengths for πN and KN elastic scattering.

Scattering length	Experimental values	Ref.	Calculated values
a_3	-0.114 ± 0.003	20	-0.080
	-0.088 ± 0.004	16	
	-0.096	17	
a_1	0.205	20	0.160
	0.171 ± 0.005	16	
	0.157	17	
a_{31}	-0.042 ± 0.004	20	-0.071
	-0.038 ± 0.005	16	
	-0.034	17	
a_{33}	0.215 ± 0.005	16	0.194
a_{11}	-0.101 ± 0.007	16	-0.089
	-0.039	17	
a_{13}	-0.029 ± 0.005	16	-0.047
	-0.030	17	
a_2	-0.205 ± 0.005	18	-0.194
a_0	$0.03\ \pm0.03$	19	0.709
a_{21}			-0.034
a_{23}			-0.002
a_{01}			0.020
a_{03}			-0.026

which is known to be absorptive near threshold, and leaves πN and KN elastic scattering as the most likely candidates for consideration. Since experimental data for these reactions are parameterized as partial-wave amplitudes and scattering lengths, we will compute these from our model. We take the low-energy regions to be $P_{\text{lab}}=0-300$ MeV/c and $P_{\text{lab}}=0-400$ MeV/c for πN and KN, respectively, slightly above the inelastic regions for both (P_{lab} is the laboratory momentum of the incident meson).

The "$U(6,6)$-symmetric" effective matrix elements, as can be seen from (A4) and (A6), contribute to $l=0$, 1, 2 partial waves. In the case of elastic scattering ($M_i=M_f=M$, $m_i=m_f=m$),

$$C(s,t)+D(s,t)+D(u,t)=0 \tag{4.1}$$

and the contribution from the γ term can be written as

$$A^{(\gamma)}(s,t)=(\gamma/36)[(3\mathfrak{D}-\mathfrak{C})D(s,t)+(3\mathcal{E}-\mathfrak{C})D(u,t)],$$
$$B^{\gamma}(s,t)=\tfrac{3}{8}\gamma(\mathcal{E}-\mathfrak{D})b_V(t).$$

It can be verified that at threshold, viz. $t=0$ and $s=(M+m)^2$, $u=(M-m)^2$,

$$A^{(\gamma)}+mB^{(\gamma)}=0. \tag{4.2}$$

It was previously noted that the "one-particle exchanges" also vanish at threshold. Consequently, in elastic scattering at threshold and hence for s-wave scattering lengths, it is only the α and β terms that contribute. It is apparent from (A6) that the α term has the $SU(3)$ and charge-conjugation structure of a singlet scalar meson exchange in the t channel (without a pole). The β term is a combination of two terms, one of which has the structure of an exchange in the t channel of an octet of scalar mesons and the other, an octet of vector mesons. These properties of the α and β terms will be exploited later.

The "one-particle exchanges," in general, contribute to all partial waves. When the nonpole parts are ignored, the "one-particle exchanges" are just the standard vector-meson exchange, baryon exchange, and decuplet exchange. The parameters G and g and hence all the required coupling constants, as discussed in I, are determined from the π-N and ρ-$\pi\pi$ coupling constants:

$$g_{PP\pi^0}{}^2=\frac{50}{81}\left(1-\frac{m_\pi{}^2}{4M_N{}^2}\right)\left(1+\frac{2M_N}{m_\rho}\right)^2G^2,$$
$$g^2=\frac{8}{81}g_{\rho\pi\pi}{}^2. \tag{4.3}$$

From $g_{PP,\pi^0}{}^2/4\pi=15$ and $g_{\rho\pi\pi}{}^2/4\pi=\frac{1}{2}$, we have $G^2/4\pi=2.05$ and $g^2/4\pi=0.0494$. The vector-meson couplings needed in πN and KN scattering are given by the following relations:

$$g_{\rho\pi\pi}=2g_{\rho KK}=(9/2\sqrt{2})g,$$
$$g_{\rho PP}=\frac{G}{\sqrt{2}}\frac{2}{3}\left(1-\frac{1}{3}\frac{m_\rho}{M_N}-\frac{5}{12}\frac{m_\rho{}^2}{M_N{}^2}\right),$$
$$g_{\omega PP}=\frac{G}{\sqrt{2}}\frac{2}{3}\left(3+\frac{m_\rho}{M_N}-\frac{1}{4}\frac{m_\rho{}^2}{M_N{}^2}\right),$$
$$g'_{\rho PP}=\frac{G}{\sqrt{2}}\frac{4}{9}\left(1+\frac{5M_N}{m_\rho}-\frac{3}{4}\frac{m_\rho}{M_N}\right), \tag{4.4}$$
$$g'_{\omega PP}=\frac{G}{\sqrt{2}}\frac{4}{9}\left(-3+\frac{3M_N}{m_\rho}-\frac{9}{4}\frac{m_\rho}{M_N}\right),$$
$$g_{\phi PP}=g'_{\phi PP}=0,\quad m_\rho=m_\omega.$$

The Yukawa-type meson-baryon coupling constants and the trilinear decuplet-baryon-meson coupling constant are tabulated in I.

Now we calculate the S-wave scattering lengths for all isospin states of πN and $KN(a_{2T})$. Since a_{2T} only depends on the threshold values of the α and β amplitudes, we have

$$a_3=\frac{-8M_N}{M_N+m_\pi}\left\{\left(1+\frac{m_\pi{}^2}{m_0{}^2}\right)(\alpha+\beta)+\frac{2}{3}\frac{m_\pi}{m_0}\beta\right\},$$
$$a_1=\frac{-8M_N}{M_N+m_\pi}\left\{\left(1+\frac{m_\pi{}^2}{m_0{}^2}\right)(\alpha+\beta)-\frac{4}{3}\frac{m_\pi}{m_0}\beta\right\},$$
$$a_2=\frac{-8M_N}{M_N+m_K}\left\{\left(1+\frac{m_K{}^2}{m_0{}^2}\right)(\alpha+\tfrac{2}{3}\beta)+\frac{4}{3}\frac{m_K}{m_0}\beta\right\}, \tag{4.5}$$
$$a_0=\frac{-8M_N}{M_N+m_K}\left(1+\frac{m_K{}^2}{m_0{}^2}\right)\alpha.$$

(Note that α and β have dimensions of inverse mass.) If we eliminate the parameters and put in the numerical

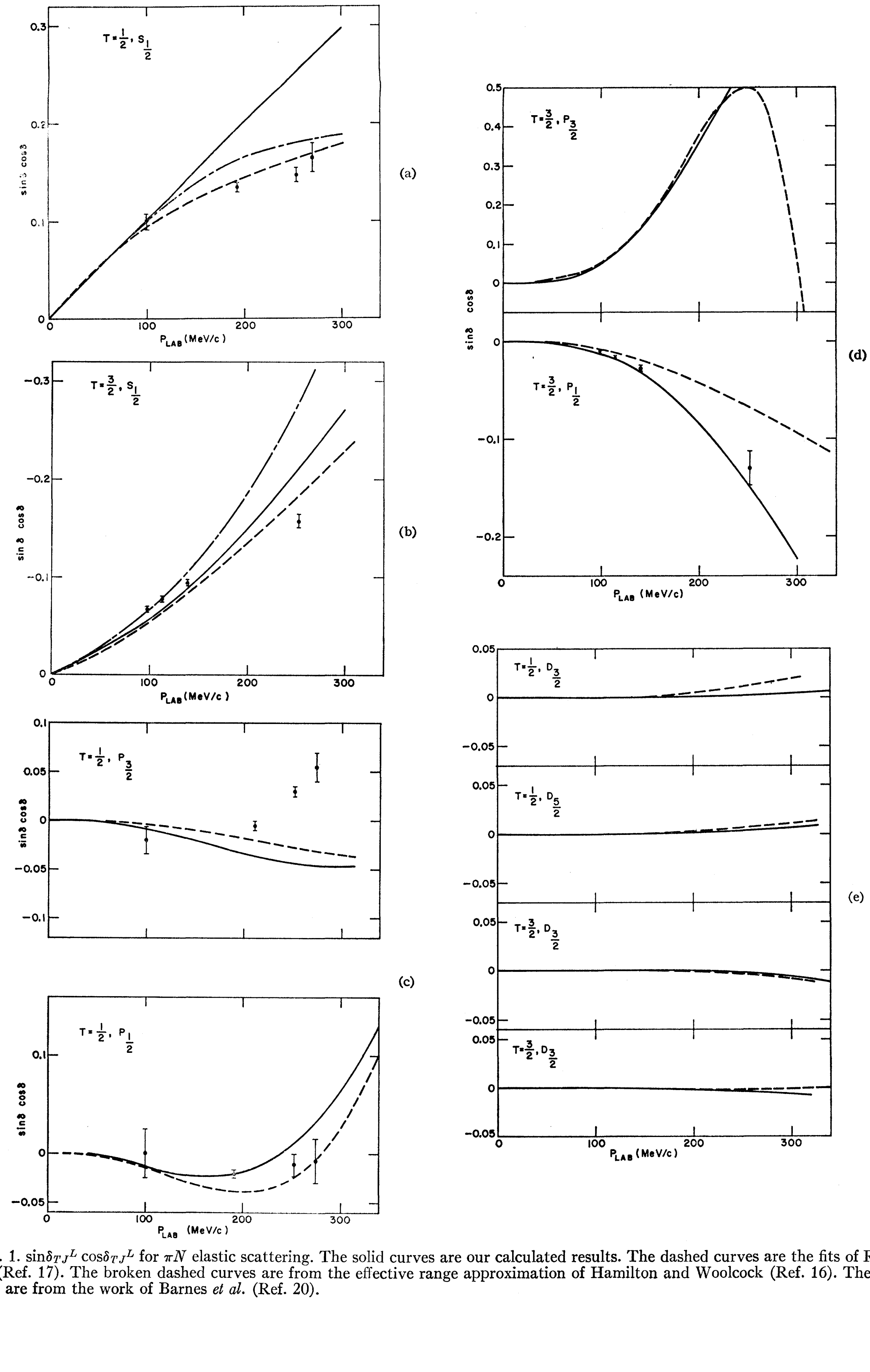

FIG. 1. $\sin\delta_{TJ}{}^{L}\cos\delta_{TJ}{}^{L}$ for πN elastic scattering. The solid curves are our calculated results. The dashed curves are the fits of Roper *et al.* (Ref. 17). The broken dashed curves are from the effective range approximation of Hamilton and Woolcock (Ref. 16). The data points are from the work of Barnes *et al.* (Ref. 20).

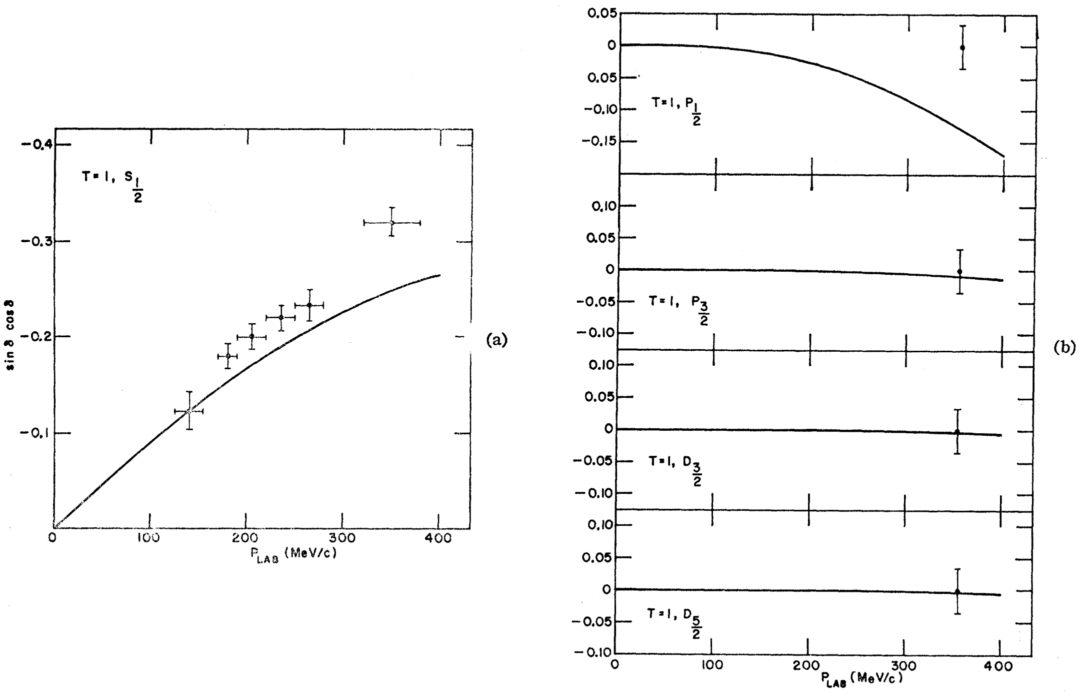

FIG. 2. $\sin\delta_{TJ}{}^{L}\cos\delta_{TJ}{}^{L}$ for $T=1$, KN elastic scattering. The solid curves are our calculated results. The data points are from the work of Goldhaber *et al.* (Ref. 18).

values of the masses, we obtain the relations

$$a_2=-0.468a_1+1.497a_3\,,\qquad a_0=3.303a_1-2.273a_3\,. \tag{4.6}$$

The scattering lengths determined from experimental data by several authors have been listed in Table I. If we use the Hamilton and Woolcock[16] values for πN S waves, we obtain $a_2=-0.212\pm0.008$ and $a_0=+0.765\pm0.025$. Using the Roper, Wright, and Feld[17] values gives $a_2=-0.217$, $a_0=+0.737$. In both cases, the a_2 values are close to the result of Goldhaber *et al.*[18] The a_0, however, is very large and far from the extrapolation of Stenger *et al.*,[19] whose a_0 is consistent with zero. This serious disagreement for the $T=0$ s wave at zero energy will persist away from the threshold and is the major fault of the model. We will defer further discussion of the $T=0$, KN system to the concluding section, and discuss only $T=\frac{1}{2}$, $\frac{3}{2}$ for πN and $T=1$ for KN.

Because the scattering lengths in Table I are extrapolated from the lowest-energy experimental data, there is a rather wide variation of values from one set to another. Therefore, it is reasonable to determine our parameters α, β, γ not from the scattering lengths, but by requiring a fit to the S-wave data at the lowest measured energies. One salient feature of the πN S-wave scattering lengths should first be noted: For all three values listed, $a_1\simeq-2a_3$. This is often taken to be evidence for the dominance of vector-meson exchange near threshold. We can simulate this behavior and eliminate one parameter by requiring $\beta=-\alpha$. For πN this has the effect of canceling that part of the β term which has the $SU(3)$ structure of a scalar octet exchange, leaving the part that has the structure of vector exchange. For KN this cancellation does not occur, and in fact for $T=0$ it is only the α term that contributes. To fix the remaining parameters, α and γ, unambiguously we require the $T=\frac{1}{2}$ and $T=1$ S waves to pass through the lowest experimentally determined points. With the definition $L_{2T,2J}=\sin\delta_{TJ}{}^{L}\cos\delta_{TJ}{}^{L}=f_{L\pm}{}^{T}$ (where $L=S$, P, D, etc.), we require

$$S_{11}(P_{\rm lab}=100\text{ MeV}/c)=+0.100\,,$$ [20]

$$S_{21}(P_{\rm lab}=140\text{ MeV}/c)=-0.123\,.$$ [18]

This fixes the parameters (in units of inverse pion masses),

$$\alpha=-\beta=-0.0957\,,$$

$$\gamma=3.00\,,$$

[16] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[17] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965).

[18] S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1962).

[19] V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, Phys. Rev. **134**, B1111 (1964).

[20] S. W. Barnes, B. Rose, G. Giacomelli, J. Ring, K. Miyake, and K. Kinsey, Phys. Rev. **117**, 226 (1960); S. W. Barnes, H. Winick, K. Miyake, and K. Kinsey, *ibid.* **117**, 238 (1960).

and allows the calculation of all partial waves that contribute to low-energy πN and KN elastic scattering. The results for the scattering lengths are listed in Table I.

In Figs. 1 and 2, the resulting S, P, and D waves are given as functions of laboratory momentum for $\pi N(T=\frac{1}{2},\frac{3}{2})$ and $KN(T=1)$. For comparison, some of the experimentally determined amplitudes are also given. Note that the πN data of Barnes *et al.*[20] come from fitting differential and total cross sections below $E_\pi=170$ MeV (laboratory kinetic energy), whereas the curves of Roper *et al.*[17] were determined by including polarization data up to $E_\pi=350$ MeV. Because the latter fit includes higher-energy data than Barnes *et al.*, the details of the two fits differ in the region of the N^*, where data are less certain, particularly in the P_{13} and P_{31}. The Hamilton and Woolcock[16] curves for πN S waves are taken from their effective-range fit to data below $E_\pi=45$ MeV. The KN data of Goldhaber *et al.*[18] come from their experiments on K^+p elastic scattering, from $P_L=140-624$ MeV/c.

In general, our results are in reasonably good agreement with the data. All of the πN partial waves have the same sign and qualitative behavior as the data of Roper *et al.* The agreement is particularly good below $P_{\text{lab}}=200$ MeV/c, where N^* no longer dominates the differential and total cross sections. Except for the small negative P_{21}, the KN, $T=1$ partial waves are consistent with the dominance of S-wave scattering reported by Goldhaber *et al.* These results emphasize the importance of the background terms, particularly in the S waves, where the α and β amplitudes dominate, and in the P_{11} partial wave where the large γ term counterbalances the large negative nucleon pole contribution (Table II).

TABLE II. Calculated values of πN partial-wave amplitudes ($\sin\delta_{TJ}{}^L \cos\delta_{TJ}{}^L$) with one-particle exchanges and background terms separately tabulated to illustrate their relative importance.

P_{lab} (MeV/c)	Sum of all one-particle exchanges	Sum of all background terms	Total partial-wave amplitude
S_{11}			
60	−0.0010	0.0616	0.0606
100	−0.0044	0.1051	0.1007
140	−0.0114	0.1512	0.1398
200	−0.0227	0.2246	0.2019
240	−0.0369	0.2762	0.2393
300	−0.0618	0.3601	0.2983
S_{31}			
60	0.0022	−0.0340	−0.0318
100	0.0103	−0.0679	−0.0576
140	0.0266	−0.1136	−0.0870
200	0.0544	−0.2049	−0.1505
240	0.0838	−0.2793	−0.1955
300	0.1359	−0.4070	−0.2711
P_{11}			
60	−0.0125	0.0086	−0.0039
100	−0.0508	0.0380	−0.0128
140	−0.1206	0.0991	−0.0215
200	−0.2840	0.2650	−0.0190
240	−0.4310	0.4316	+0.0006
300	−0.7094	0.7718	+0.0624
P_{31}			
60	−0.0022	−0.0013	−0.0035
100	−0.0083	−0.0064	−0.0147
140	−0.0185	−0.0117	−0.0302
200	−0.0386	−0.0477	−0.0863
240	−0.0536	−0.0795	−0.1331
300	−0.0769	−0.1462	−0.2231
P_{13}			
60	−0.0020	−0.0002	−0.0022
100	−0.0077	−0.0009	−0.0086
140	−0.0164	−0.0016	−0.0180
200	−0.0323	−0.0015	−0.0338
240	−0.0427	+0.0007	−0.0420
300	−0.0558	+0.0088	−0.0470
P_{33}			
60	0.0070	0.0030	0.0100
100	0.0305	0.0131	0.0436
140	0.0813	0.0330	0.1143
200	0.2679	0.0850	0.3529
240	0.6190	0.1347	0.7537

V. DISCUSSION AND SUMMARY

The physical motivation for our model is provided by the fact that tensors of $U(6,6)$ subjected to Bargmann-Wigner equations lead to a supermultiplet structure which corresponds exactly to that of $SU(6)$ symmetry. As emphasized by Salam *et al.*,[7] this is entirely equivalent to assuming that the known particles at rest correspond to the representations of a compact $U(6)\otimes U(6)$ group. Given the possibility of such an assumption, one can write down tensor combinations of three-, four-, and higher-point functions which are formally invariant under $U(6,6)$. In the lowest-order perturbation theory, they can be evaluated by substituting the free-field solution for the tensors. Such a procedure has considerable support from experimental observations in the case of three-point functions constructed from Ψ_{ABC} and $\Phi_A{}^B$ which describe the presently known low-lying baryonic and mesonic states. When applied to four-point functions, however, it leads to contradictions and conflict with the basic principle of unitarity. Even in the case of three-point functions, as pointed out in I, there is no theoretical justification for the procedure except as a working hypothesis.

Since, in general, perturbation theory in strong interactions is questionable, we adopt the alternative approach based on dispersion relations. In several calculations[3,16] based on the latter approach, it has been found that it is necessary to introduce background terms which approximate distant singularities in addition to one-particle exchange contributions. It is interesting to investigate whether such background terms possess higher symmetries. Within the framework of $U(6,6)$ symmetry, the contact terms (four-point functions evaluated in the lowest-order perturbation theory) provide a representation of such background terms.[7] If these are supplemented by symmetry-breaking one-particle exchange contributions, one has a starting point to relate a large number of scattering processes. With this point of view, we have investigated only a limited version of the model as applied to pseudoscalar meson-baryon scattering, confining our attention only to the states that form the 35-dimensional and 56-dimensional representation of $SU(6)$ (we could also consider associated production, vector-meson production, and baryonic resonance production in the same context). Further, the model is incomplete for the following reasons. First of all, we have made no attempt to calculate unitarity corrections. Secondly, for simplicity, we have chosen α, β, $\tilde{\beta}$, γ to be real constants. It is more natural to choose functional forms with appropriate cutoff properties in them so that they define a range over which the $U(6,6)$-symmetric terms dominate. This would enable one to use the N/D procedures to obtain unitary scattering amplitudes, thus greatly extending the predictive power of the model. The model then could be tested for higher-energy behavior and polarization predictions.

In spite of these shortcomings, it is encouraging from the results in the previous section that we obtain reasonably good agreement with the presently available experimental information in πN and KN ($T=1$ state only) scattering. There is a serious disagreement in KN ($T=0$ state) in that the model predicts large S-wave scattering in contradiction with the results of Stenger *et al.*[19] Their results, however, are extrapolated from K^+d scattering at comparatively higher energies ($P_{\text{lab}}=350$–812 MeV/c). If further experiments (preferably $K_2{}^0P$) confirm the results of Stenger *et al.*, then our model in its present form is untenable. In the work of Warnock and Frye[3] on the KN system, in which the Stenger results are used, there is an indication of significant contributions from higher resonances. One may then be led to include higher $SU(6)$ multiplets in the scheme considered here. This and other possible modifications discussed above need further investigation.

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APPENDIX

The purpose of this Appendix is to collect together the results for the invariant amplitudes $A(s,t,u)$ and $B(s,t,u)$ from various terms in the model. To this end, we note that A and B are defined by[21]

$$T=-A(s,t,u)+i\frac{q_i+q_f}{2}B(s,t,u)\,, \tag{A1}$$

where T is the T matrix for pseudoscalar meson-baryon scattering, and q_i and q_f are, respectively, the 4-momenta of the initial and final meson. The corresponding 4-momenta of the baryons will be denoted by p_i and p_f. Let m_i, m_f, and M_i, M_f denote the masses of the initial and final meson and baryon masses, respectively. Then, s, t, and u are the Mandelstam variables, $s=-(p_i+q_i)^2$, $t=-(q_i-q_f)^2$, $u=-(p_i-q_f)^2$, and $s+t+u=M_i{}^2+M_f{}^2+m_i{}^2+m_f{}^2$. When necessary, we denote the mass of the exchanged meson (baryon) by $m_e(M_E)$. The partial-wave projections of A and B are given by

$$A_l(s)=\frac{1}{2}\int_{-1}^{1}AP_l(x)dx\,,\quad B_l(s)=\frac{1}{2}\int_{-1}^{1}BP_l(x)dx\,, \tag{A2}$$

where $x=\cos\theta$, θ being the scattering angle in the c.m. system. The partial-wave scattering amplitude, $f_{l\pm}$, for a state with total angular momentum $J=l\pm\frac{1}{2}$ and parity $(-1)^{l+1}$ are given in terms of A_l and B_l by the following equation:

$$\begin{aligned}f_{l\pm}=&\frac{1}{8\pi W}(E_i+M_i)^{1/2}(E_f+M_f)^{1/2}\\&\times\left[A_l+\left(W-\frac{M_i+M_f}{2}\right)B_l\right]\\&+\frac{1}{8\pi W}(E_i-M_i)^{1/2}(E_f-M_f)^{1/2}\\&\times\left[-A_{l\pm1}+\left(W+\frac{M_i+M_f}{2}\right)B_{l\pm1}\right]\,,\end{aligned} \tag{A3}$$

where W is the c.m. energy and

$$E_{i,f}+M_{i,f}=((W+M_{i,f})^2-m_{i,f}{}^2)/2W\,.$$

$U(6,6)$-Symmetric Contributions

We would like to emphasize that the term "$U(6,6)$ symmetric" is used for convenience in the sense that the relevant contributions can be derived from an effective Lagrangian which is $U(6,6)$ invariant when expressed in terms of the fields Ψ and Φ. The use of the Bargmann-Wigner equations violates this invariance. In addition, it should be noted that we include $SU(3)$ mass splittings.

To list the invariant amplitudes from different terms

[21] A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964).

in (2.4), we define the following functions of s, t, and u:

$$F(t)=\left(1-\frac{t-M_i^2-M_f^2}{2M_iM_f}\right)\left(1-\frac{t-m_i^2-m_f^2}{2m_\rho^2}\right),$$

$$a_V(s,t)=\frac{2}{3}\frac{(u-s)}{m_\rho^2}\frac{((M_i+M_f)^2-t)}{2M_iM_f},$$

$$\bar{a}_V(s,t)=\frac{1}{9}\frac{(M_i+M_f+2m_\rho)}{m_\rho}\times\left[\frac{(M_i+M_f)(u-s)-(m_i^2-m_f^2)(M_i-M_f)}{M_iM_fm_\rho}\right],$$

$$b_V(t)=\frac{2}{9}\frac{(2m_\rho+M_i+M_f)((M_i+M_f)^2-t)}{M_iM_fm_\rho^2}, \tag{A4}$$

$$C(s,t)=\frac{-2}{M_iM_fm_\rho^2}[m_\rho(M_i-M_f)^2(m_\rho+M_i+M_f)+(M_i+M_f)(m_i^2M_f+m_f^2M_i)-t(M_i+m_\rho)(M_f+m_\rho)-st-(M_f^2+m_f^2-s)(M_i^2+m_i^2-s)],$$

$$D(x,t)=2\left[\left(1-\frac{t-M_i^2-M_f^2}{2M_iM_f}\right)\left(1+\frac{2x-M_i^2-M_f^2}{2m_\rho^2}\right)-2\left(1-\frac{x-M_f^2-m_i^2}{2M_fm_\rho}\right)\left(1-\frac{x-M_i^2-m_f^2}{2M_im_\rho}\right)\right],$$

where $x=u$, s.

To denote the $SU(3)$ dependences, we need the following definitions: $(\bar{B}B)_W$ (where $W=F$, D, or S),

$$(\bar{B}B)_F=B\bar{B}-\bar{B}B;\quad (\bar{B}B)_D=\bar{B}B+B\bar{B}-\tfrac{2}{3}\,\mathrm{Tr}B\bar{B};\quad (\bar{B}B)_S=\mathrm{Tr}B\bar{B}$$

W	F	D	S
a_W	1	0	1
c_W	-1	3	-2

$$\begin{aligned}\mathfrak{C}&=\mathrm{Tr}(-\bar{B}BP\bar{P}-\bar{B}B\bar{P}P+\bar{B}\bar{P}BP+\bar{B}PB\bar{P})+\mathrm{Tr}(\bar{B}B)\,\mathrm{Tr}(\bar{P}P),\\ \mathfrak{D}&=\mathrm{Tr}(-\bar{B}\bar{P}PB)+\mathrm{Tr}(\bar{B}\bar{P})\,\mathrm{Tr}(BP),\\ \mathcal{E}&=\mathrm{Tr}(-\bar{B}P\bar{P}B)+\mathrm{Tr}(\bar{B}P)\,\mathrm{Tr}(B\bar{P}).\end{aligned} \tag{A5}$$

Then,

$$A^{(\alpha)}(s,t)=-4\alpha F(t)\,\mathrm{Tr}(\bar{B}B)\,\mathrm{Tr}(\bar{P}P),$$

$$B^{(\alpha)}(s,t)=0,$$

$$A^{(\beta)}(s,t)=\beta\sum_W(\bar{B}B)_W[-\tfrac{4}{3}a_WF(t)(P\bar{P}+\bar{P}P)-(a_Wa_V(s,t)+c_W\bar{a}_V(s,t))(P\bar{P}-\bar{P}P)],$$

$$B^{(\beta)}(s,t)=-\beta\sum_W(\bar{B}B)_W(3a_W+c_W)b_v(t)(P\bar{P}-P\bar{P}), \tag{A6}$$

$$A^{(\gamma)}(s,t)=\frac{\gamma}{36}[\mathfrak{C}C(s,t)+3\mathfrak{D}D(s,t)+3\mathcal{E}D(u,t)],$$

$$B^{(\gamma)}(s,t)=\tfrac{3}{8}\gamma(\mathcal{E}-\mathfrak{D})b_V(t).$$

"One-Particle-Exchange" Contributions

In pseudoscalar-meson–baryon scattering, we have contributions due to vector meson, baryon, and decuplet exchange. We denote the corresponding invariant amplitudes by $A^{(V)}$, $A^{(B)}$, $A^{(D)}$, etc.

Vector-Meson Exchange

$$A^{(V)}(s,t)=-\frac{3Gg}{4}\sum_W\mathrm{Tr}[(\bar{B}B)_W(P\bar{P}-\bar{P}P)]\left[a_W\left(\frac{3m_V^2}{m_V^2-t}a_V-a_V+\frac{[(M_i+M_f)^2-t](M_f-M_i)(m_f^2-m_i^2)}{M_iM_fm_\rho(m_V^2-t)}\right)+c_W\left(\frac{3m_V^2}{m_V^2-t}\bar{a}_V-\bar{a}_V+\frac{(t-4m_V^2)}{9m_\rho M_iM_f}\frac{[(M_i+M_f)(u-s)-(M_i-M_f)(m_i^2-m_f^2)]}{m_V^2-t}\right)\right], \tag{A7}$$

$$B^{(V)}(s,t)=-\frac{3Gg}{4}\sum_W(\bar{B}B)_W(P\bar{P}-\bar{P}P)(3a_W+c_W)\left[\frac{3m_V^2}{m_V^2-t}b_V(t)-b_V(t)+\frac{2}{3}\frac{(t-4m_V^2)}{m_\rho M_iM_f}\frac{[(M_i+M_f)^2-t]}{m_V^2-t}\right].$$

Conventional vector exchange is obtained by taking only the pole terms.

In the case of baryon and decuplet exchanges, it is sufficient to compute the invariant amplitudes $A^{(\ )}$ and $B^{(\ )}$ due to the direct exchanges in the s channel. The corresponding u-channel pole contributions can be obtained by the substitutions $s\leftrightarrow u$, $m_i\leftrightarrow m_f$, $P\leftrightarrow\bar{P}$, and $B(s,t)=-B(u,t)$.

Baryon Exchange

$$A^{(B)}(s,t)=\frac{G^2}{81}\left[M_Ef_i(s)f_f(s)\frac{A_1(s)}{M_E^2-s}-[f_i(s)+f_f(s)-1]A_2(s)-\frac{A_3(s)}{M_E}\right]\times(25\,\mathrm{Tr}\bar{B}\bar{P}PB+5\,\mathrm{Tr}\bar{B}\bar{P}BP+5\,\mathrm{Tr}\bar{B}PB\bar{P}+\mathrm{Tr}\bar{B}BP\bar{P}),$$

$$B^{(B)}(s,t)=\frac{G^2}{81}\left[M_E f_i(s) f_f(s)\frac{B_1(s)}{M_E^2-s}-[f_i(s)+f_f(s)-1]B_2(s)-\frac{B_3(s)}{M_E}\right]$$
$$\times(25\,\mathrm{Tr}\bar{B}\bar{P}PB+5\,\mathrm{Tr}\bar{B}\bar{P}BP+5\,\mathrm{Tr}\,\bar{B}PB\bar{P}+\mathrm{Tr}\bar{B}BP\bar{P}),$$

where

$$f_{i,f}(s)=\frac{2M_{i,f}M_E+M_{i,f}^2-m_{i,f}^2+s}{2M_{i,f}M_E},$$

$$A_1(s)=-\left(1+\frac{M_i+M_E}{m_\rho}\right)\left(1+\frac{M_f+M_E}{m_\rho}\right)\left(M_E-\frac{M_i+M_f}{2}\right)-(4m_\rho+M_i+M_f+2M_E)(s-M_E^2)/2m_\rho^2,$$

$$A_2(s)=-\left(1+\frac{2M_E^2-M_i^2-M_f^2}{2m_\rho^2}+\frac{s-M_E^2}{m_\rho^2}\right),$$

$$B_1(s)=\left(1+\frac{M_i+M_E}{m_\rho}\right)\left(1+\frac{M_f+M_E}{m_\rho}\right)+\frac{s-M_E^2}{m_\rho^2},$$

$$B_2(s)=\left(\frac{2}{m_\rho}+\frac{M_i+M_f}{m_\rho^2}\right),$$

and

$$A_3(s)=A_1(s)-M_EA_2(s),\quad B_3(s)=B_1(s)-M_EB_2(s). \tag{A8}$$

Decuplet Exchange

The results in Eqs. (A7) and (A8) follow from Eqs. (3.7) and (3.8). As explained in Sec. III, these contributions include (i) parts due to the conventional vector or baryon exchanges calculated using the coupling-constant relations implied by $U(6,6)$ or $SU(6)_W$ symmetry [I, Appendix Eqs. (A2), (A3), (A11)]; (ii) parts that arise because of contact terms in the propagators. In the decuplet case, for the sake of conciseness, we present below only the contributions of the type (i), although the complete results from (3.9) have been included in the numerical calculations.

$$A^{(D)}(s,t)=\frac{2}{81}\frac{1}{M_E^2-s}\frac{G^2}{M_iM_f}\left(1+\frac{M_i+M_E}{m_\rho}\right)\left(1+\frac{M_f+M_E}{m_\rho}\right)$$
$$\times\left[\left(M_E-\frac{M_i+M_f}{2}\right)(M_i+C_i)(M_f+C_f)+\frac{3}{2}\left(M_E+\frac{M_i+M_f}{2}\right)(2C_iC_f-M_i^2-M_f^2+t)\right]$$
$$\times\{3(\mathrm{Tr}\bar{B}B\,\mathrm{Tr}\bar{P}P-\mathrm{Tr}\bar{B}P\,\mathrm{Tr}\bar{P}B)+\mathrm{Tr}(3B\bar{B}P\bar{P}-3\bar{B}B\bar{P}P-B\bar{B}\bar{P}P-\bar{B}BP\bar{P}+BP\bar{B}\bar{P}+B\bar{P}\bar{B}P)\},$$

$$B^{(D)}(s,t)=-\frac{2}{81}\frac{1}{M_E^2-s}\frac{G^2}{M_iM_f}\left(1+\frac{M_i+M_E}{m_\rho}\right)\left(1+\frac{M_f+M_E}{m_\rho}\right)$$
$$\times[(M_i+C_i)(M_f+C_f)-\tfrac{3}{2}(2C_iC_f-M_i^2-M_f^2+t)]\{\text{same } SU(3) \text{ dependence as in } A^{(D)}\},$$

where C_i and C_f are constants given by

$$C_i=\frac{M_E^2+M_i^2-m_i^2}{2M_E},\quad C_f=\frac{M_E^2+M_f^2-m_f^2}{2M_E}. \tag{A9}$$

Note that when the partial waves are projected out, all of the $U(6,6)$-symmetric and one-particle exchange terms have the correct threshold dependences, i.e., $f_{l\pm}(q)$ goes as q^{2l} as q (the c.m. momentum) goes to zero.