Generation of Resonances via Inelastic Effects*

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We study the N/D single-channel equations using a pole-dominated force and a simple model for the variation of the ratio of the total to the elastic partial-wave cross section. We find that inelasticity can lower and sharpen existing resonances, produce new resonances, and raise the maximum energy at which resonances are possible.

I. INTRODUCTION

MUCH attention has recently been devoted to the study of the effects of inelastic channels on the properties of resonances and bound states in two-body elastic-scattering channels. For some time it has been known[1,2] that introduction of inelasticity will narrow an already existing resonance and shift it to lower energies; in the case of bound states the binding energy increases and the effective coupling constant decreases. It remains of interest to study the situation where the forces are not sufficient by themselves to produce a resonance or bound state in an elastic calculation. What will be the effect of the introduction of inelasticity in this case? Can it produce resonances? Where? Etc.

In this short note we introduce the coupling to other (nonelastic) channels by using the ratio—assumed given—of the total to elastic cross sections in a two-body, equal-mass, single-channel N/D calculation.[3] We were motivated in our choice of form and numerical parameters by previous studies of the pion-pion interaction.[4,5] Using a pole-dominated force cut and a simple, reasonable model for the variation of the (total cross section)-(elastic cross section) ratio, we obtain results similar to those already known when the force is strong enough to produce a resonance without inelasticity. We also show, in the case of fixed forces too weak to produce a resonance or bound state by themselves, how a resonance can arise by increasing the coupling to inelastic channels. This should be expected because of the time-delay interpretation of resonances. Of some novelty, however, is our result that introduction of absorptive effects can, under appropriate circumstances, *increase* the energy at which a resonance is possible.

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[1] See, for example, P. Nath, Lecture delivered at the Conference on Strong Interactions and Elementary Particles, 1965 Summer School of Theoretical Physics, University of Colorado, Boulder, Colorado (unpublished). P. Nath and Y. N. Srivastava (unpublished).

[2] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).

[3] See, for example, G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1962), p. 48, or G. Frye and R. L. Warnock, Phys. Rev. **130**, 478 (1963).

[4] K. Smith and J. L. Uretsky, Phys. Rev. **131**, 761 (1963).

[5] A. Saperstein and J. L. Uretsky, Phys. Rev. **133**, B1340 (1964); **140**, B352 (1965).

In Sec. II of this paper, we summarize the appropriate N/D formalism. We work in dimensionless units so that the particular value chosen for the pole location is of little consequence; all that matters is the relative location of the pole and the inelastic threshold. In Sec. III, we introduce an appropriate range of values for the dynamical constants, and we obtain and discuss our results. Section IV contains a comparison of our model with some others which have recently been used in the discussion of the relation between inelasticity and resonances.

II. INELASTIC N/D FORMALISM

We first summarize briefly a form of the N/D[3] equations appropriate when inelastic channels are present and the ratio of the total-to-elastic cross sections is given. We consider, for simplicity, only P-wave scattering and write our scattering amplitude[4,5] as

$$A(\nu)=\left(\frac{\nu+1}{\nu}\right)^{1/2}e^{i\delta(\nu)}\sin\delta(\nu)=\left(\frac{\nu+1}{\nu}\right)^{1/2}\frac{S-1}{2i}. \qquad (1)$$

Here $S=\exp 2i\delta$, $\delta(\nu)$ is the phase shift (complex) in elastic scattering channel, and ν is the square of the barycentric momentum of one of the two particles expressed in units in which the mass of each of the two particles is 1.

Although the phase shift is no longer real when inelastic channels are open, we may—quite generally—write the amplitude (1) in terms of two real parameters $R(\nu)$ and $\theta(\nu)$:

$$A(\nu)=\left(\frac{\nu+1}{\nu}\right)^{1/2}\frac{1}{R(\nu)}e^{i\theta(\nu)}\sin\theta(\nu). \qquad (2)$$

From (2) it follows that

$$\mathrm{Im}A(\nu)=\left(\frac{\nu}{\nu+1}\right)^{1/2}R(\nu)|A(\nu)|^2, \qquad (3)$$

where Im denotes imaginary part. Using the optical theorem,[6] it follows from (3) that $R(\nu)$ can be interpreted as

$$R(\nu)=\sigma(\text{total})/\sigma(\text{elastic}), \qquad (4)$$

[6] See, for example, K. Nishijima, *Fundamental Particles* (W. A. Benjamin, Inc., New York, 1964), p. 121.

where σ refers to the appropriate partial-wave differential cross section integrated over all solid angles.

In the usual way[3] we now write

$$A(\nu)=\nu N(\nu)/D(\nu)\,, \tag{5}$$

where we assume D to be a real meromorphic function everywhere in the complex ν plane except for a cut along the positive, real axis ("physical" or "unitarity" cut). Similarly, $N(\nu)$ is a real analytic function, meromorphic on the ν plane except for the cuts, P, which generate the forces ("potential" cuts). From (3) it follows that

$$\mathrm{Im}[A(\nu)]^{-1}=-\left(\frac{\nu}{\nu+1}\right)^{1/2}R(\nu)\,, \tag{6}$$

so that

$$\mathrm{Im}D(\nu)=-\nu\left(\frac{\nu}{\nu+1}\right)^{1/2}R(\nu)N(\nu) \tag{7}$$

on the physical cut and

$$\mathrm{Im}N(\nu)=\nu^{-1}D(\nu)\,\mathrm{Im}A(\nu)\equiv\nu^{-1}\rho(\nu)D(\nu) \tag{8}$$

on the potential cut. We now assume the following dispersion relations [no Castillejo-Dalitz-Dyson (CDD) poles[7]]:

$$D(\nu)=1+\frac{\nu}{\pi}\int_0^\infty\frac{\mathrm{Im}D(\nu')d\nu'}{(\nu'-\nu)\nu'}\,, \tag{9a}$$

and

$$N(\nu)=\frac{1}{\pi}\int_P\frac{\mathrm{Im}N(\nu')d\nu'}{\nu'-\nu}\,, \tag{9b}$$

where we have taken one subtraction at the origin in D and none in N. Combining (7), (8), and (9) in the usual way and making the usual assumption about interchanging the order of integration, we obtain the desired integral equation

$$D(\nu)=1-\frac{\nu}{\pi^2}\int_P\frac{\rho(\nu')D(\nu')}{\nu'}M_R(\nu,\nu')d\nu'\,, \tag{10a}$$

and

$$M_R(\nu,\nu')=\int_0^\infty dx\left(\frac{x}{x+1}\right)^{1/2}\frac{R(x)}{(x-\nu)(x-\nu')}\,. \tag{10b}$$

To simplify matters, we take the real part of (10); the integral in (10b) becomes a Cauchy principle part integral and we then have an integral equation for $D_R(\nu)=\mathrm{Re}D(\nu)$ to be solved given $R(\nu)$ on the physical cut and $\rho(\nu)$ on the potential cut. Given the solution, $D_R(\nu)$, we obtain $N(\nu)$ by using (8) and (9b). Finally, going back to (9a), using $1/(\nu'-\nu)=P/(\nu'-\nu)+i\pi\delta(\nu'-\nu)$, and (7), we obtain, along the physical cut,

$$\mathrm{Im}D(\nu)=D_I(\nu)=-\left(\frac{\nu^3}{\nu+1}\right)^{1/2}R(\nu)N(\nu)\,. \tag{11}$$

Thus the physical scattering amplitude (5) is completely determined, on the physical cut, by the solution of Eq. (10).

Bound states or resonances in this partial wave are determined by the vanishing of $\mathrm{Re}[A(\nu)]^{-1}$, i.e.,

$$\frac{D_R(\nu)}{N(\nu)}=\left(\frac{\nu^3}{\nu+1}\right)^{1/2}R\cot\theta=0\,. \tag{12}$$

For the purely elastic case we obtain a resonance when $\cot\delta=0$, as expected. Since $N(\nu)$ has been assumed to have no poles along the physical cut or along the real ν axis between the threshold of the physical cut and the start of the potential cut—the region in which bound states are expected to occur—the condition for the existence of a resonance or bound state becomes $D_R(\nu)=0$. The width of a resonance is obtained by applying the Breit-Wigner single-level formula in the vicinity of a resonance, i.e.,

$$|A|^2=\frac{\nu^2N^2}{D_R^2+D_I^2}\approx\frac{\nu_R^2N^2(\nu_R)}{(\nu-\nu_R)^2[D_R'(\nu_R)]^2+D_I^2(\nu_R)}\sim\frac{1}{(\nu-\nu_R)^2+(\Gamma/2)^2}\,, \tag{13}$$

where the prime denotes the derivative with respect to ν and ν_R is the real root of

$$D_R(\nu)=0\,. \tag{14}$$

Using (11), we obtain, for the total width,

$$\Gamma=2\left(\frac{\nu_R^3}{\nu_R+1}\right)^{1/2}\frac{R(\nu_R)N(\nu_R)}{D_R'(\nu_R)}\,. \tag{15}$$

Calculating the elastic partial width Γ_e from the complete form of (13) in a similar manner, we find $\Gamma/\Gamma_e=R$, which gives us another check on the original interpretation, (4).

To obtain useful results easily, we assume a simple model for the dependence of R, viz.,

$$R(\nu)=1+r\left(\frac{\nu+1}{\nu}\right)^{1/2}\frac{\nu-\nu_T}{\nu}\theta(\nu-\nu_T)\,, \tag{16}$$

where ν_T is the threshold for inelastic processes ($\nu_T>0$), and $\theta(\nu)$ is the unit step function. Then, (10b) may be written as

$$M_R(\nu,\nu')=M(\nu,\nu')+rm(\nu,\nu')\,, \tag{17}$$

[7] L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1956). See also Ref. 3.

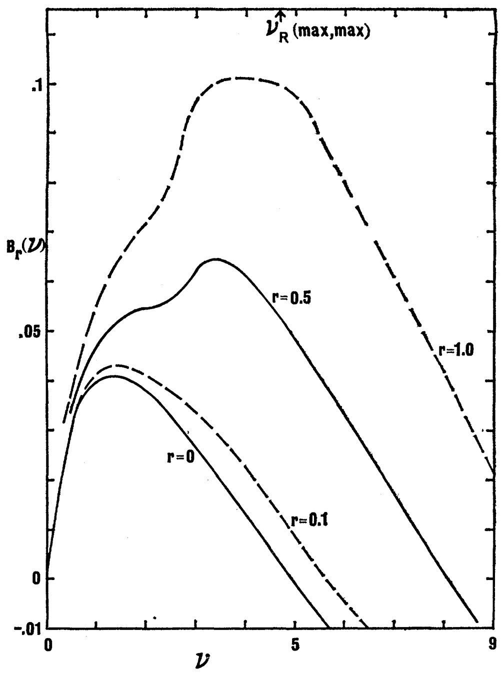

FIG. 1. Plot of $B_r(\nu)$ versus ν for various values of r. Inelastic threshold $\nu_T=3$; pole at $\nu_P=-3$.

where

$$M(\nu,\nu')=P\int_0^\infty dx\left(\frac{x}{x+1}\right)^{1/2}\frac{1}{(x-\nu)(x-\nu')}$$
$$=\frac{h(\nu')-h(\nu)}{\nu-\nu'}, \tag{18a}$$

$$h(\nu)=2\left(\frac{\nu}{\nu+1}\right)^{1/2}\ln[\nu^{1/2}+(\nu+1)^{1/2}],$$

and

$$m(\nu,\nu')=P\int_{\nu_T}^\infty dx\frac{x-\nu_T}{x(x-\nu)(x-\nu')}$$
$$=\frac{\nu_T}{\nu\nu'}\ln\nu_T-\frac{\nu-\nu_T}{\nu(\nu-\nu')} \tag{18b}$$
$$\times\ln|\nu_T-\nu|-\frac{\nu'-\nu_T}{\nu'(\nu'-\nu)}\ln|\nu_T-\nu'|.$$

Here r is the asymptotic value of the ratio of inelastic to elastic cross sections. To simplify matters still more, we replace the potential cut by a single pole at the negative value ν_P, i.e., we assume that in (10a)

$$\rho(\nu)=a\delta(\nu-\nu_P), \tag{19}$$

where the real parameter a represents the strength of the interaction producing the scattering. Equation (10a) then reduces to an algebraic equation which can be simply solved to give

$$D_R(\nu)=C^{-1}[C+B_r(\nu)], \tag{20a}$$

where

$$C=\nu_P[h'(\nu_P)-rm(\nu_P,\nu_P)+\pi^2/a]/\pi^2$$
$$=1/D_R(\nu_P), \tag{20b}$$

$$B_r(\nu)=\frac{\nu}{\pi^2}\left[\frac{h(\nu_P)-h(\nu)}{\nu-\nu_P}+rm(\nu,\nu_P)\right]. \tag{20c}$$

From (8) and (9b) we obtain

$$1/N(\nu)=\pi C(\nu_P-\nu), \tag{21}$$

so that (15) becomes

$$\frac{\Gamma}{2}=\left(\frac{\nu_R^3}{\nu_R+1}\right)^{1/2}\frac{R(\nu_R)}{\pi|(\nu_P-\nu_R)B_r'(\nu_R)|}, \tag{22}$$

and, finally, from (21) and (20a),

$$\left(\frac{\nu^3}{\nu+1}\right)^{1/2}R(\nu)\cot\theta(\nu)=\frac{D_R(\nu)}{N(\nu)}=\pi(\nu_P-\nu)$$
$$\times[C+B_r(\nu)]. \tag{23}$$

III. RESULTS

We are interested in what the formalism, developed in Sec II, will tell us about the development of resonances and bound states as we vary the interaction strength a and the inelasticity parameter r. We choose simple representative values:

$$\nu_P=-3,\quad\text{and}\quad\nu_T=3, \tag{24}$$

and note that $D_R(\nu)$ vanishes when $C=-B_r(\nu)$. In Fig. 1, we plot $B_r(\nu)$ with r as a parameter with the choice of constants (24). The curves are all concave

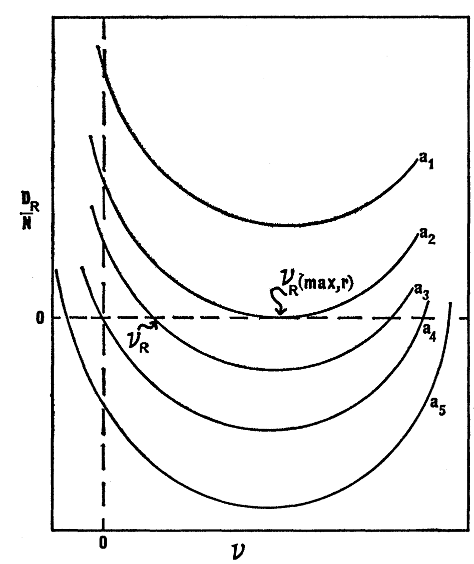

FIG. 2. Sketch of $[\nu^3/(\nu+1)]^{1/2}R\cot\theta=D_R/N$ as a function of ν with a as a parameter. Here, $a_1<a_2<a_3<a_4=a_B<a_5$.

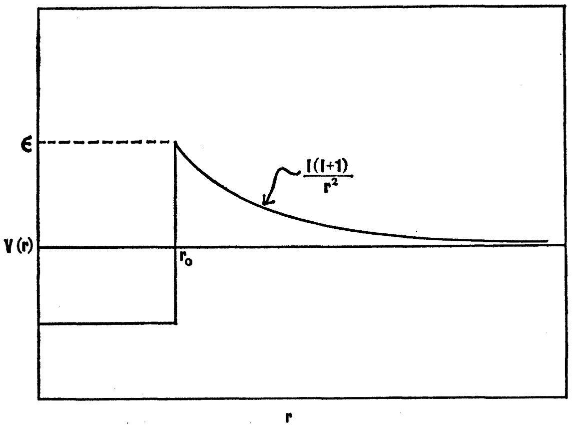

FIG. 3. Sketch of simple attractive-potential well with repulsive angular-momentum barrier.

downwards, pass through the zero axis only at $\nu=0$ and some positive value of ν, and have a positive maximum at $\nu_R(\max,r)$; they go to $-\infty$ as ν goes to $\pm\infty$. With the choice (24), $\pi^2C/3=-0.266-0.102r+\pi^2/a$ so that $C\lesseqgtr 0$ as $\pi^2/a\lesseqgtr 0.266+0.102r$. For

$$0<a<a_B=\pi^2(0.266+0.102r)\,,\quad C<0\,;$$

in this range, $|C|$ decreases as a or r increases, and the horizontal line $|C|=\text{constant}$ moves down from $+\infty$ so as to eventually intersect the curve $B_r(\nu)$. We thus have either zero or two positive values of ν (depending upon a and r) at which $D_r(\nu)=0$. One of these two roots is to be interpreted as a resonance.

In Fig. 2, we sketch Eq. (23) with C as a parameter; as $|C|$ increases, the curves move down. The intersections of the D_R/N curve with the horizontal zero axis correspond to the two intersections of $|C|$ and $B_r(\nu)$. The delay time in scattering Q can be written as[8]

$$Q=\frac{d\arg A}{dE}\sim\frac{d\theta(\nu)}{d\nu}\,, \tag{25}$$

where we have made use of (2). Associating a resonance with a positive delay time, we can say that resonances are possible in those parts of Fig. 2 in which $\theta(\nu)$ is increasing. We thus have a resonance when $\cot\theta$ goes *down* through the zero line. Therefore, the first intersection in Fig. 1 corresponds to a resonance; the second marks the return of θ back down through $\pi/2$ suggested by the Levinson theorem.[9] From (25) it follows that the lifetime of the resonance state increases as the $\cot\theta$ curve crosses zero more steeply, or from (23) as $B_r(\nu)$ increases more steeply at the intersection point. Thus, for a resonance at a fixed point ν_R, the time delay increases as $B_r'(\nu_R)$ increases. From (22) we see that this implies a decreasing total width Γ, as expected.

At $a=a_B$, $C=0$ and we have a zero-energy resonance; from (22) we obtain zero width as we should. For $a>a_B$, $C>0$, the first intersection in Fig. 1 occurs for negative ν and we have a bound state. The second intersection gives a rising $\cot\theta$—hence is not a resonance—and is again suggested by the Levinson theorem. This model can have, thus, at most one resonance or bound state.

Studying Fig. 1, we may come to the following conclusions:

1. With r fixed there may or may not be a resonance i.e., depending upon the potential strength, the horizontal line $|C|=\text{constant}$ may or may not intersect $B_r(\nu)$, but if there is a resonance it must occur for $\nu_R<\nu_R(\max,r)$. There is, thus, a maximum energy at which a resonance is possible, defined by $B_r'[\nu_R(\max,r)]=0$. As the strength a is increased, ν_R decreases until, at $a=a_B$, a zero-energy bound state is formed.

The existence of an upper limit on the energy of possible resonances may be interpreted in terms of a simple potential picture. In Fig. 3 we sketch a simple attractive potential with range r_0, together with an angular-momentum barrier. There will be no time delay (no barrier penetration necessary) and hence no resonance for energies $E\gtrsim\epsilon$, where $\epsilon\sim l(l+1)/r_0^2$. Letting the range be determined by the mass of an exchanged particle, we have $r_0\sim|\nu_P|^{-1/2}$ so that $\epsilon\sim l(l+1)|\nu_P|$. For our numbers we obtain $\epsilon\sim 6$ which compares favorably as an order-of-magnitude result with $\nu_R(\max, r=0)\sim 1.5$.

2. Fixing the potential strength a, $|C|$ decreases as r increases, whereas $B_r(\nu)$ increases monotonically. Hence, given *any* a in the range $0<a<a_B$, we can find an $\bar{r}$ such that a resonance will result for $r>\bar{r}$. No matter how weak the attractive potential is, a resonance is possible if there is sufficient coupling to other channels. Physically, this can be interpreted in terms of the time spent in the other coupled channels before the return to the elastic channel; the argument is similar to that usually given for compound-nucleus formation.[10] As in result 1, once an r has been found which produces a resonance, the resonance can be shifted to smaller ν by increasing the strength.

3. As r increases, $\nu_r(\max,r)$ increases. Thus, the range of ν in which a resonance may occur for varying potential strength may be increased by increasing the inelasticity of the channel. In this sense, absorption from the elastic channel may be said to lead to a shift *upward* in resonance energy. This upward shift does not proceed indefinitely with increasing r, reaching an absorption-dominated limit when r is great enough for the first term of $B_r(\nu)$, (20c), to be negligible compared to the second term. The limiting value is the single

[8] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), p. 494.

[9] N. Levinson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **25**, No. 9 (1949); or see Ref. 8, p. 284.

[10] See, for example, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 340.

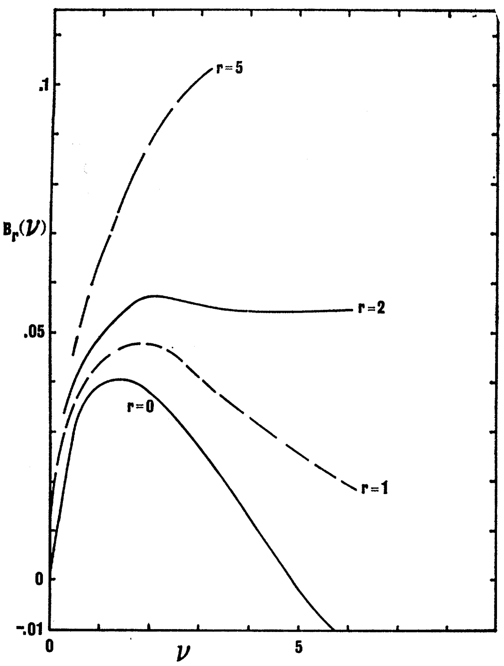

FIG. 4. Plot of $B_r(\nu)$ versus ν for various values of r. Inelastic threshold $\nu_T=10$; pole at $\nu_P=-3$.

root of

$$[\nu m(\nu,\nu_P)]'=0; \tag{26}$$

for the choice of parameters (24) we obtain ν_R(max,max) $\sim$4.7. The "absorption-dominated" resonance is completely independent of the potential parameters, i.e., we could have no force ($a\to 0$, $ar>0$) and still get the same result, a resonance maximum somewhat above the inelastic threshold.

4. For a fixed strength a, and an r for which a resonance exists, the resonance can be sharpened and moved to lower ν by increasing r; or it may be sharpened and kept at the same ν_R by increasing r while decreasing a. These results have been obtained in various other ways before.[1,2] In our formalism they follow both from the decrease in $|C|$ and from the increase in ν_R(max,r). For large enough r, only the first factor is operable and the rate of change with r diminishes.

The general validity of our four conclusions can be tested by comparison with numerical solutions obtained with potentials more complicated than a simple pole. For example, the general behavior illustrated in Fig. 2 is demonstrated for increasing a with fixed $r=0$ in Fig. 2 of Smith and Uretsky.[4] Similar results, keeping the potential fixed and varying r, were obtained in unpublished calculations of Saperstein using the third-order pion-pion interaction.[5] Here it was seen that an increase in r allows resonances at larger ν_R and gives smaller widths for a given ν_R with smaller values of the coupling constant.

Finally, we must investigate the effect of moving the inelastic threshold. In Fig. 4, we give the same curves as in Fig. 1, but plotted now with the new threshold, $\nu_T=10$. Qualitatively, our previous results are unchanged. For small values of r, we see that moving the inelastic threshold so far out effectively decouples it insofar as determining resonances is concerned; changes in r produce very small changes in ν_R and Γ. Increasing r indefinitely, we again come to the region of absorption-dominated resonances; as before, ν_R (max, max) is somewhat above the threshold, in this case $\sim$13.6.

Lowering the inelastic threshold to $\nu_T=1$, we find ν_R(max,max)$\sim$2.1, and so the discussion and results will be very similar to those obtained with $\nu_T=3$.

To lend a sense of completeness to our discussion, we consider the case where the inelastic and elastic thresholds coincide, $\nu_T=0$. Our model, Eq. (16), cannot be used in this case because of the singularity at $\nu=0$. To get a qualitative insight into what occurs in this case, we simply assume $R=$constant. Going through the formalism of II again, we find that it is equivalent to putting $r=0$ and letting a become Ra. Thus, in this extreme—and perhaps unphysical—model, the opening of inelastic channels is exactly equivalent to increasing the strength of the potential and nothing more.

IV. COMPARISON WITH MULTICHANNEL N/D AND ANALYTICALLY CONTINUED ANGULAR MOMENTUM MODELS

In order to judge the trustworthiness of results from our simple model, it is useful to discuss the assumptions we have made in the light of some more complex models which have been recently proposed.

Bander, Coulter, and Shaw[11] have used the Frye and Warnock[3] inelastic N/D equations [in which the imaginary part of the phase shift ($-\frac{1}{2}\ln\eta$), rather than R, must be specified] in order to compare with results obtained from a simple two-channel N/D calculation. Both models were specified to have no CDD singularities and simple pole forces. The inelastic, single-channel scattering amplitude agrees with the corresponding results from the two-channel calculation so long as the force in the second channel is not strong enough to produce a bound state in the second channel when severed from the initial channel. After the two models disagree, the authors assume that they can be brought back into agreement by the addition of appropriate CDD singularities to the single-channel calculation. The resultant hypothesis is that single-channel inelastic N/D calculations without CDD poles are meaningless in the presence of coupling to a strongly bound channel.

Bander, Coulter, and Shaw[11] have shown that the breakdown in agreement between the two models occurs when zeros in the elastic first-channel S matrix

[11] M. Bander, P. W. Coulter, and G. L. Shaw, Phys. Rev. Letters 14, 270 (1965); see also D. Atkinson, K. Dietz, and D. Morgan, Ann. Phys. (N.Y.) 37, 77 (1966).

element S_{11} move from the second sheet, through the inelastic cut, onto the physical sheet. At this real zero of the scattering matrix element, as determined by the two-channel calculation, η also vanishes; the result is a singularity of the Frye-Warnock equations—a breakdown of the Fredholm nature of the equations. Thus, the need for CDD poles may be associated with the fact that the corresponding single-channel integral equation is no longer Fredholm.

In contrast, the integral equation of our model, Eq. (10), shows no change in character at a zero of S_{11}; if the left-hand cut is such that the equation was Fredholm before the development of a zero in S_{11}, the equation remains Fredholm during and after the development of such a zero. In other words, the change in R resulting from a zero in S_{11} cannot change the Fredholm nature of the integral equation so long as R remains smooth and bounded, which it does. Hence, there can be no discontinuous need for CDD singularities. In view of this contrast, it is reasonable to hope that our model—without CDD poles—gives a fair indication of what may happen when *many* inelastic channels are present, combining to give a smooth function R similar to that assumed in this paper.

As another different approach, we consider the work of Hartle and Jones.[12] They consider first the Frye-Warnock η equations for very large angular momentum where no bound states, resonances, or CDD singularities are allowed. *Assuming* the possibility of analytic continuation in angular momentum L, they continue the Frye-Warnock η equations to small L where resonances are possible. They distinguish between two types of resonance poles on the unphysical sheet reached by passing through the elastic cut. "Elastic resonances" are defined as those which move to the left-hand cut as L becomes large; if the pole retreats through the right-hand inelastic cut in the limit of large L, it is an "inelastic resonance." They point out that an "inelastic resonance" can only be produced by a fixed-L single-channel N/D calculation if CDD singularities are included. "Elastic resonances" can occur without CDD poles and, if they do so, they require left-hand cuts unlike the "inelastic resonances" which—because of the CDD poles—can exist with no potential cuts. In a manner very similar to the results of Bander, Coulter, and Shaw, the "inelastic resonance" becomes evident—and the CDD poles become necessary—when η passes through a real zero, i.e., when the Fredholm nature of the Frye-Warnock equation breaks down.

It is evident that the "inelastic resonances" of Hartle and Jones are quite different in nature from the resonances considered in Secs. II and III of this paper. *Both give resonance behavior which would not occur in a purely elastic calculation.* The "inelastic resonance" of Hartle and Jones is truly inelastic in that it requires no potential; however, it is necessary to specify the CDD parameters, as well as the inelasticity parameter η, and it might be necessary to specify a potential as well. Our results require only the specification of a potential and the inelasticity parameter R. There seems to be no *a priori* reason to rule out one or the other of these two distinct types of resonance. It may even be that our resonances will fit into the category designated "elastic resonance" by Hartle and Jones.

From the point of view of practical calculations, e.g., bootstrapping, it seems most reasonable to start the calculation using methods analogous to those developed in this paper since no arbitrary CDD parameters will be necessary. Later on, it may prove necessary to include CDD poles in some manner. In any case, it seems fairly evident from the results of this paper and others (e.g., Refs. 1, 2, 11, 12) that calculations made without reference to inelasticity are likely to be faulty and misleading.

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[12] J. B. Hartle and C. E. Jones, Phys. Rev. **140**, B90 (1965).