Two-Channel Model of P_{11} π-N Partial-Wave Amplitude

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A simple two-channel model of the P_{11} π-N channel and a phenomenological second channel, the σ-N, is constructed. It is shown that this model reproduces the detailed structure of the π-N phase shift and produces the nucleon pole as a dynamical bound state with the proper residue. On the basis of this model it is argued that the forces required in the second channel are comparable to those in the π-N channel, and a significant portion of the binding of the nucleon is produced by the σ-N channel. Finally, it is observed that the Roper resonance (600 MeV) corresponds to a pole in the S matrix, and should be fitted into an SU_3 multiplet.

RECENT analyses of the pion-nucleon elastic-scattering data have revealed a number of interesting features for the phase shifts up to the neighborhood of incident-pion kinetic energies of 1 BeV.[1–4] Of particular interest is the P_{11} partial wave which contains a resonance in addition to the nucleon pole: The real part of the P_{11} ($I=\frac{1}{2}$, $J=\frac{1}{2}$, $l=1$) phase shift δ is negative at low energy, reaches a minimum of approximately $-2°$ at $\sim$100 MeV, changes sign at $\sim$180 MeV, and rises up through 90° at $\sim$600 MeV. In addition, the absorptive coefficient η rapidly becomes small above 500 MeV. Since these detailed features are not reproduced in the Chew-Low–type bootstrap calculations,[5–11] one might question whether it is possible to construct a simple theoretical model which fits the phase-shift data and at the same time contains the nucleon pole in the form of a bound state.

First, we recall that the P_{11} phase shift is consistent with a model[12] in which the nucleon pole is taken to be a part of the potential and the absorptive coefficient is included phenomenologically by using the Frye-Warnock equation.[13] There, it was also shown that the solution of the Frye-Warnock equation does not reproduce the phase shift if the nucleon pole was not included in the potential. This result indicates that the inelastic contribution is sufficiently large so that the solution of the Frye-Warnock equation requires the addition of an elementary-particle pole either in the form of a nucleon pole in the potential or in the form of a usual Castillejo-Dalitz-Dyson (CDD) pole in the D function. On the other hand, such a pole may not be necessary if one employs the multichannel ND^{-1} formalism instead of the one-channel Frye-Warnock equation.[14]

In this paper, we show that a simple two-channel model can accommodate the nucleon as a bound-state pole while giving a reasonably good fit to the scattering data.[15] The results of our model indicate that the inelastic channel is as important as the elastic channel in producing the nucleon bound state and the Roper resonance (90° phase shift at 600 MeV). We shall later discuss the implication of this result.

In view of the rapid decrease of the absorptive coefficient above $\sim$300 MeV, we approximate the inelastic channels by a single two-body channel in a

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[12] P. Coulter and G. Shaw, Phys. Rev. **141**, 1419 (1966).
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[14] See, e.g., M. Bander, P. Coulter, and G. Shaw, Phys. Rev. Letters **14**, 207 (1965).
[15] For a qualitative consideration of this problem, see D. Atkinson and M. B. Halpern, Phys. Rev. **150**, 1377 (1966). (They favor πN^* rather than σN as the second channel coupled to the $P_{11}\pi N$ channel.) After the present work was completed, it was brought to our attention that a similar model was constructed by I. Bender, D. Heiss, and E. Tränkle [and by J. H. Schwarz, Phys. Rev. **152**, 1325 (1966)]. In each case the zero of the amplitude is fitted to the data but the phase shifts differ strongly from the phase-shift analyses of Refs. 1–4. In particular, the phase shift passes through $\pi/2$ immediately after going through zero. Their inability to fit the phase shift is presumably due to some constraints on the input parameters.

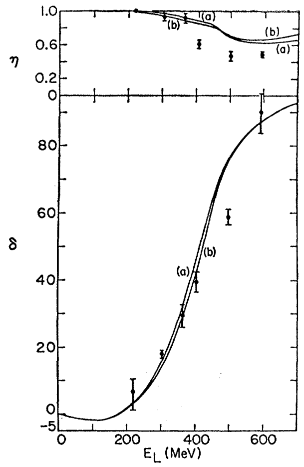

FIG. 1. Plots of calculated values (solid curves) for η and δ versus E_L. The experimental points shown are from Ref. 4. The solid curves for δ fall on top of low-energy, energy-dependent phase-shift analysis of Ref. 1 for $E_L<200$ MeV. Curve (a) corresponds to the πN phase-space factor (a) with $W_0=-285\ m_\pi$, $\gamma_{11}=2360$, $\gamma_{12}=191$, $\gamma_{22}=800$, whereas (b) corresponds to phase-space factor (b) with $W_0=5.0\ m_\pi$, $\gamma_{11}=108$, $\gamma_{12}=121$, $\gamma_{22}=261$.

relative S state. We call this the "σ-N" channel and choose $m_\sigma\simeq 3.5m_\pi$. For simplicity, we approximate the potential for the $\pi N\to\pi N$, $\pi N\to\sigma N$ and the $\sigma N\to\sigma N$ amplitudes by a single pole at a fixed total energy W_0. Together with the three residues of the pole (the T matrix is symmetric) we have a four-parameter representation in the ND^{-1} form

$$T=ND^{-1}, \tag{1}$$

$$N_{ij}=\gamma_{ij}/(W-W_0);\quad i,\ j=1,\ 2,\quad \gamma_{ij}=\gamma_{ji} \tag{2}$$

$$D_{ij}=\delta_{ij}-\gamma_{ij}d_i(W);\quad i,\ j=1,\ 2 \tag{3}$$

$$d_i(W)=\frac{W-W_0}{\pi}\int_{a_i}^{\infty}dW'\frac{\rho_i(W')}{(W'-W_0)^2(W'-W-i\epsilon)}, \tag{4}$$

where $\rho_i(W')$ is the phase-space factor for the ith channel and a_i is the corresponding threshold.

For the σN channel, we use an S-wave phase-space factor with a spread of the σ mass to avoid a cusp behavior at the σN threshold. Specifically, we take

$$\begin{aligned}\rho_2(W)&=[K(W)-K(M+2m_\pi)]\\&\quad\times\{(W+M+m_\sigma)[W^2-(M-m_\sigma)^2]\}^{1/2}/4W^2 \qquad W>M+2m_\pi\\&=0, \qquad\qquad W<M+2m_\pi\end{aligned} \tag{5}$$

and

$$K(W)=\{W-M-m_\sigma+[(W-M-m_\sigma)^2+\Gamma^2]^{1/2}\}^{1/2},$$

where Γ is the width of the Briet-Wigner resonance describing the unstable particle σ. This expression is similar to that obtained by Nauenberg and Pais,[16] differing in that we have forced $\rho_2=0$ at the physical inelastic threshold $M+2m_\pi$. The width used, $\Gamma=0.1m_\pi$, was used only to obtain a smooth behavior of the phase shift around $W=M+m_\sigma$ and is not intended to be a good representation of a physical scalar meson.

For the πN channel, we consider two choices for $\rho_1(W)$:

$$\text{(a)}\qquad \rho_1(W)=(k/W)^3,$$

and

$$\begin{aligned}\text{(b)}\qquad \rho_1(W)&=(k/M)^3, && W<18m_\pi\\ &=0, && W>18m_\pi.\end{aligned}$$

A cutoff is necessary in choice (b) but the solution is not very sensitive to the cutoff since there are still four adjustable parameters in the model. Thus we discuss the (representative) case with the cutoff at $W=18m_\pi$. Note that the factor W^{-3} in choice (a) can also be considered as a form of a cutoff. As we shall see below, solutions with (a) and (b) are not very different.

For a given value of W_0, we adjust the three residues γ_{11}, γ_{22}, and γ_{12} to produce (i) a zero of the determinant of the D matrix at $W=M$, (ii) a zero in the T_{11} amplitude at $W=8.75m_\pi$ ($E_L=180$ MeV), and (iii) a zero in the real part of the T_{11} amplitude at $W=10.8m_\pi$ ($E_L=600$ MeV). One can easily verify that these three conditions give rise to three linear equations for γ_{11}, γ_{22}, and $(\gamma_{11}\gamma_{22}-\gamma_{12}^2)$. Therefore, the solution is unique for a given W_0. (The sign of γ_{12} is undetermined since only the square enters into the equations.)

With both choices of the πN phase space (a) and (b), we adjust W_0 to obtain a good over-all fit to δ and η. The results are shown in Fig. 1. As one can see, the major discrepancy between our results and the experimental analyses lies in the fact that we do not have a sufficiently small η. The inability to maintain a small η over a wide range of energies is presumably due to the neglect of higher inelastic channels.

After fitting the scattering data, we also calculate the residue of the nucleon pole and deduce a pion-nucleon coupling constant. We find our calculated f^2 to be 0.072 in case (a) and 0.073 in case (b), as compared to the empirical value of 0.08.[17–19]

The significance of a resonance and a bound-state pole in the two-channel model considered here is most easily understood by examining the two channels when uncoupled. In this case each channel has a resonance or bound state. When the coupling between channels is turned on, the lower of the two poles moves to lower energy, while the higher-energy pole moves up. A gen-

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[17] The calculated f^2 is not as insensitive to the phase-space factor as one might infer from the quoted results. We have found a variation of $\sim$30% among a number of reasonably good fits to the scattering data.

[18] It is interesting to note that the input pole position for case (b) was 5.0 m_π (see caption of Fig. 1), which is close to the value of 4.6 m_π that one would get from N_{33}^* exchange in the static model. The input strength γ_{11} for this case is also close to the value given by N_{33}^* exchange. The feature that case (a) requires an input pole far out on the left is due to the convergence properties of the phase-space factor (a).

[19] Since our simple model gives a good description of all the features of the P_{11} partial wave, it is worthwhile to use the D function from this model in the Dashen-Frautschi [Phys. Rev. **137**, B1331 (1965)] calculation of the n-p electromagnetic mass splitting. Defining D by the relation $T_{11}=[\gamma_{11}/(W-W_0)]/D$, we obtain D's which enhance the N_{33}^* exchange even more than models considered by G. Shaw and D. Y. Wong [Phys. Rev. **147**, 1028 (1966)].

eral result of this is that if the 600-MeV resonance and the nucleon-bound state appear in a coupled two-channel scattering amplitude as in our model, the strengths of the forces in the uncoupled channels must be such as to produce in each channel a bound state or a resonance in the range $6.7m_\pi < W < 10.8m_\pi$. This indicates that the forces in the higher-mass channel (σ-N) must be comparable to those in the π-N channel, particularly in view of the higher threshold for this channel. We believe that these results are more general than the particular model considered here and depend primarily on two channels being a good approximation to the π-N P_{11} scattering amplitude.

The strength of the σ-N channel would seem to be sufficient to satisfy the above requirement if one considers the strength of the σ-N coupling which measures the strength of the nucleon exchange as determined in fits to N-N scattering data.[20]

[20] A. Scotti and D. Y. Wong [Phys. Rev. **138**, B145 (1965)] find $g_\sigma^2=3.0$; J. S. Ball, A. Scotti, and D. Y. Wong [Phys. Rev. **142**, 1000 (1966)] find $g_\sigma^2=5.1$–4.1. These values correspond to an $f_\sigma^2=0.7$–1.5.

In our model, the strengths of the potentials were such that the uncoupled πN amplitude has a resonance at $9m_\pi$ and $7.8m_\pi$ for cases (a) and (b), respectively, and the uncoupled σN channel has a bound-state pole at $8.45m_\pi$ (a) and $7.2m_\pi$ (b). In both cases (a) and (b) the coupling constant associated with the uncoupled πN amplitude is within a factor of 2 of the coupled case. This indicates that although the two-channel model is essential to account for the phase-shift data, the original Chew-Low–type calculations of coupling-constant relations still have semiquantitative significance.

As stated above, the uncoupled problem contained two poles in the S matrix. Thus, regardless of the coupling mechanism, the Roper resonance must be associated with an SU_3 multiplet of the 8 or higher representation. It will probably be necessary to perform detailed phase-shift analyses to find the other members of the multiplet.

Finally, we remark that it will also be useful to apply the simple multichannel ND^{-1} model to the analyses of other π-N partial-wave amplitudes.

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n-p Mass Difference in an SU_2 Bootstrap Model*

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We demonstrate the possibility that $M_n > M_p$ in an off-shell, SU_2-symmetric, reciprocal bootstrap model of the N and the Δ [$N^*(1238)$]. Only the static electrical potential between charged pions and the proton is used as a driving term. However, the change in the Δ-exchange contribution to the strong forces (due to the electromagnetic mass shifts in the Δ isoquartet) may be more than enough to reverse the "intuitive" result, $M_p > M_n$.

THE ubiquitous problem of the n-p mass difference has recently been re-examined by several authors within the framework of various versions of the bootstrap model of the nucleon[1–5] or through "propagator" methods.[6–9] In a critique of the Dashen-Frautschi approach to mass shifts, Sawyer[10] has proposed an off-shell Lippman-Schwinger approximation to the Bethe-Salpeter equation as a favorable alternative to N/D, for use in exploring questions pertaining to bootstrap theory. In this paper, we adopt this formalism to study the n-p mass difference, and have obtained the following qualitative results: (a) The difference $M_n - M_p \equiv \delta_{np}$ cannot be calculated numerically without a rather complete theory of the strong interactions. This is *in addition* to a knowledge of the electromagnetic driving terms. (b) However, it turns out to be entirely plausible, as a result of the bootstrap, and contrary to "intuition," that $\delta_{np} > 0$ with *only* the electrical Coulomb force operating without retardation between the $\pi^\pm$ and p as a "driving term."

By "bootstrap," we mean only (a) the compositeness of the N and Δ in terms of π's and N's, in an SU_2 framework, and (b) the explicit recognition of the importance of Δ exchange to bind the N, and vice versa, as was first pointed out by Chew.[11]

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