and therefore the Σ channel can only contribute through the $T=1$ state of the four nuclei, which may be about 20 MeV above the ground state. This should result in a suppression of Λ-N and Λ-N-N forces. A similar effect arises when a Λ is in nuclear matter because of the Pauli principle between nucleons. In this case, consider the interaction between a nucleon and the Λ via two-pion exchange. All intermediate states of the nucleon which are below the Fermi level are forbidden. Hence the Λ-N interaction in nuclear matter would be different from the free case. However, if we calculate the binding energy of Λ in nuclear matter with these modified Λ-N and Λ-N-N interactions, we get the same result as with free Λ-N and Λ-N-N interactions up to fourth order in the pion coupling constant.[21] The above result is true only when three-body forces are included. It is also possible that by neglecting the Σ-channel suppression in $_{\Lambda}He^5$, but including the free three-body contribution to binding, Bodmer's effect is being taken into account.

[21] D. Kiang and Y. Nogami (to be published).

In conclusion, then, we think that the overbinding (by a central force) of the Λ in $_{\Lambda}He^5$ and in nuclear matter warrants the introduction of a short-range tensor force in the triplet Λ-N interaction. We plan to do a detailed calculation including the tensor force to verify this, and to investigate the other effects mentioned in the above paragraph.

Note added in proof. The repulsive contribution of 0.7 MeV was calculated taking only the central part of the two-pion-exchange Λ-N-N force. However, we have since then found that the tensor part of the Λ-N-N force is much more important and reduces the overbinding of B_{Λ} in $_{\Lambda}He^5$ substantially, without seriously affecting $_{\Lambda}H^3$. Details of this calculation are submitted for publication in Ann. Phys. (N. Y.).

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Determination of the S-Wave π-π Amplitude near the ρ Peak from the Reaction $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ *

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A fit to recent extensive data for the reaction $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at incident π^- momentum ~4 BeV/c and final two-pion center-of-mass energy $m_{\pi\pi} \sim m_\rho$ was made. The peripheral model with absorption was used in the fit. The asymmetry in the final two-pion distribution θ_π gives a quantitative determination of the π-π, S-wave, $I=0$ scattering amplitude. A constant phase shift of $\sim +60°$ gives as good a fit the to data as a resonance ϵ^0 (at 730 MeV with a width of 90 MeV), proposed by Durand and Chiu. A negative phase shift of $\sim -60°$ is ruled out by examining the distribution in θ_π as a function of $m_{\pi\pi}$.

I. INTRODUCTION

IT is known[1] that the angular distribution in θ_π for the final two pions in the reaction[2]

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n \tag{1}$$

near the final two-pion center-of-mass energy $m_{\pi\pi} \sim m_\rho$ requires a large S-wave phase shift[3] δ_0 interfering with the $l=1$ production.[4] Furthermore, the θ_π distribution of the final pions in the reaction[5]

$$\pi^\pm + p \rightarrow \pi^\pm + \pi^0 + p \tag{2}$$

near $m_{\pi\pi} \sim m_\rho$ yields a small negative value for the $I=2$, S-wave phase shift. Thus, reactions (1) and (2) indicate the presence of a large π-π phase shift $\delta_0{}^0$ near the ρ region.

The peripheral production model with absorptive corrections gives a good fit[6–8] to reaction (2), not only for the cross section as a function of the momentum

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[1] See, e.g., G. Shaw and D. Wong, Phys. Rev. **129**, 1379 (1963); M. Islam and R. Piñon, Phys. Rev. Letters **12**, 310 (1964).
[2] In this paper, we will be discussing data for incident π laboratory momentum ~4 BeV/c.
[3] A subscript will be used on the amplitudes and phase shifts to denote the l value, and a superscript to denote the isotopic spin.
[4] At these values for $m_{\pi\pi}$, d waves are neglected (but f^0 production probably becomes important at somewhat higher $m_{\pi\pi}$).

[5] Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **25**, 365 (1962).
[6] K. Gottfried and J. Jackson, Nuovo Cimento **34**, 735 (1964).
[7] L. Durand and Y. Chiu, Phys. Rev. **137**, B1530 (1965).
[8] M. Bander and G. Shaw, Phys. Rev. **139**, B956 (1965).

transfer (to the nucleon) t, but to the measured[9–11] angular distributions in θ_π (the polar scattering angle in the two-pion center-of-mass system) and ϕ_π (the Treiman-Yang azimuthal angle). Thus, a similar, detailed calculation of (1) should give quantitative values for δ_0^0. Such a calculation was performed by Durand and Chiu,[12] who found that the data averaged in $m_{\pi\pi}$ over the ρ peak required an $I=0$, S-wave resonance (ϵ^0) located at 730 MeV with a width of 90 MeV.

More extensive data[13] are now available, so that we have repeated the calculation of Durand and Chiu in greater detail. We find that not only their resonant solution ϵ^0, but also a constant δ_0^0 of $\sim\pm 60°$, give equally good fits to the data.[9–11,13] Furthermore, when the distributions[13] in θ_π and ϕ_π are fitted as a function of $m_{\pi\pi}$, instead of averaging over the peak, the negative value (or equivalently $\delta_0^0 \sim 120°$) for δ_0^0 is ruled out.[14] (A large negative value for δ_0^0, as is pointed out by Chew,[15] would have been quite significant with regard to the vacuum Regge trajectory.)

In conclusion, we find that reaction (1) does not require the S-wave resonance ϵ^0 proposed by Durand and Chiu. In addition, the slowly varying value of $\sim +60°$ in the region 650–850 MeV appears to fit smoothly with most of the lower-energy determinations of δ_0^0.

II. CALCULATIONS AND CONCLUSIONS

We consider the reaction (1) to proceed via the one-pion-exchange diagram shown in Fig. 1 for the production of an S-wave π^+-π^- pair and the neutral ρ. Let the π-π scattering amplitudes which are functions of the invariant $s=m_{\pi\pi}^2$ be[3] $A_0(s)$ and $A_1(s)$. Then the amplitudes for diagrams in Fig. 1 in the peripheral model with absorptive corrections have the form $A_0\langle\lambda|\lambda'\rangle$ and $A_1\langle\lambda|\lambda'\mu\rangle Y_1^\mu(\theta_\pi,\phi_\pi)$, where the matrix elements $\langle\lambda|\lambda'\rangle$ and $\langle\lambda|\lambda'\mu\rangle$ are a function of s, the momentum transfer t, and the incident-pion momentum k. Thus, the cross section for (1) is

$$\sigma=\sum_{\lambda,\lambda'}\{|A_0(s)|^2|\langle\lambda|\lambda'\rangle|^2 +2\,\mathrm{Re}[A_0(s)A_1^*(s)\sum_\mu\langle\lambda|\lambda'\rangle\langle\lambda|\lambda'\mu\rangle^* Y_1^\mu(\theta_\pi,\phi_\pi)] +\sum_{\mu,\nu}\langle\lambda|\lambda'\mu\rangle\langle\lambda|\lambda'\nu\rangle^* Y_1^\mu(\theta_\pi,\phi_\pi)^* Y_1^\nu(\theta_\pi,\phi_\pi)|A_1(s)|^2\} \times dtdsd\cos\theta_\pi d\phi_\pi. \quad (3)$$

[9] Z. Guiragossian, Phys. Rev. Letters **11**, 85 (1963).

[10] V. Hagopian, W. Selove, J. Alitti, J. Baton, and M. Neven-Rene, Phys. Rev. **145**, 1129 (1966).

[11] Aachen-Birminghan-Bonn-Hamburg-London (I. C.)-München Collaboration, Phys. Rev. **138**, B897 (1965).

[12] L. Durand and Y. Chiu, Phys. Rev. Letters **14**, 329, 680(E) (1965).

[13] R. Birge, R. Ely, T. Schumann, Z. Guiragossian, and M. Whitehead, in *Proceedings of the 12th Annual International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965), p. 153; Z. Guiragossian (private communication).

[14] The distribution in θ_π is more sensitive to the S-wave π-π parameters than is the distribution in ϕ_π.

[15] G. Chew, Phys. Rev. **140**, B1427 (1965).

We used the method of Ref. 8 to calculate the amplitudes $\langle\lambda|\lambda'\rangle$ and $\langle\lambda|\lambda'\mu\rangle$. The assumption of total absorption in the relative $l=0$ state of the final two pions and the nucleon was seen to give the best fit to the charged ρ production (2).[16] It is expected that the process in Fig. 1(a) will not be as sensitive to the details of the absorption because of its simpler helicity structure. Thus, we used the same absorption parameters for the diagrams in Fig. 1 as for reaction (2).[8]

The $l=1$, π-π amplitude A_1 was taken as

$$A_1(s)=\frac{1}{\sqrt{2}}\left[\frac{m_\rho^2-s}{(s-4m_\pi^2)\gamma_\rho}-i\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\right]^{-1}, \quad (4)$$

with $m_\rho=760$ MeV and γ_ρ corresponding to the width $\Gamma_\rho=100$ MeV. We included a (small) $I=2$ amplitude as well as the (large) $I=0$ amplitude[3] in A_0:

$$(4\pi)^{1/2}A_0=\frac{1}{\sqrt{3}}\left[\alpha_0^0(s)-i\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\right]^{-1} +\frac{1}{\sqrt{6}}\left[\alpha_0^2(s)-i\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\right]^{-1}, \quad (5)$$

with

$$\alpha_0^I(s)=\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\cot\delta_0^I. \quad (6)$$

$\alpha_0^2(s)$ was taken to be a large negative constant α_0^2 corresponding to $|\delta_0^2|\lesssim 15°$.[5,17] We considered two forms for δ_0^0:

$$\alpha_0^0(s)=\alpha_0^0, \quad (7)$$

and

$$\alpha_0^0(s)=(m_0^2-s)/\gamma. \quad (8)$$

The effects in the present problem of including S-wave π-π production can be observed only in the θ_π and ϕ_π distributions. If these distributions are averaged in $m_{\pi\pi}$ over the ρ peak, we find three types of solutions which give equally good fits to the data: (i) the resonant solution ϵ^0 found by Durand and Chiu; (ii) a large

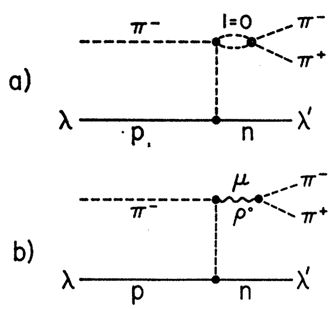

FIG. 1. One-pion exchange diagrams for the reaction (1). λ and λ' are the helicity states of the proton and neutron, respectively. Diagram (a) corresponds to the production of an S-wave π-π pair. Diagram (b) corresponds to the production of the ρ resonance in a helicity state μ.

[16] We note that total absorption in the final ρN state is not necassary if the ρN elastic scattering amplitude is strongly helicity dependent. See M. Bander and G. Shaw, Bull. Am. Phys. Soc. **11**, 23 (1966).

[17] The factor $[(s-4m_\pi^2)/s]^{1/2}$ is essentially a constant in the energy region we are concerned with, so that $\alpha=$constant is approximately the same as $\delta=$constant.

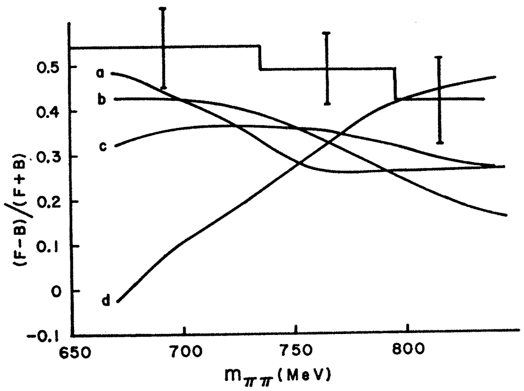

FIG. 2. Plots of the forward-backward asymmetry $(F-B)/(F+B)$ as a function of $m_{\pi\pi}$. The experimental data ($|t|<10m_\pi^2$) are those of Birge *et al.* (Ref. 13). The theoretical curves calculated using the peripheral model with absorption correspond to the S-wave parameters: (a) an $I=0$ resonance at 730 MeV with a width of 100 MeV, no $I=2$ amplitude included; (b) $\alpha_0^0=0.4$ (i.e., $\delta_0^0\approx66°$) and no $I=2$; (c) $\alpha_0^0=0.4$ and $\alpha_0^2=-3.0$ (i.e., $\delta_0^2\approx-17°$); (d) $\alpha_0^0=-0.4$ and no $I=2$.

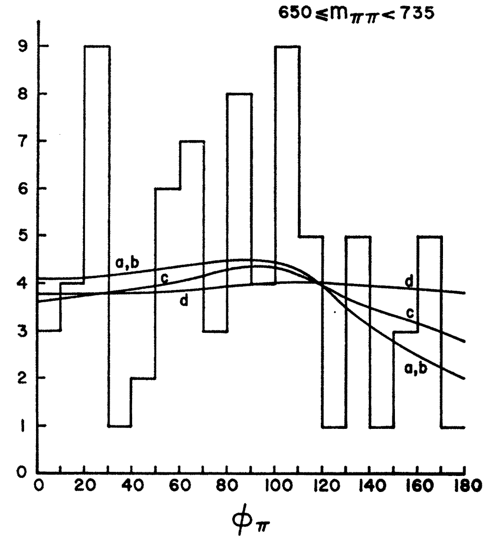

FIG. 4. Plots of ϕ_π in the bin $650<m_{\pi\pi}<735$ for the solutions (a)–(d) described in Fig. 2.

positive constant δ_0^0; (iii) a large negative constant δ_0^0. Our fits to the data of Birge *et al.*[13] as a function of $m_{\pi\pi}$ are shown in Figs. 2–4.[18] Although the S-wave contribution is important in fitting the ϕ_π distribution, we note in Fig. 4 that it cannot be used to distinguish between the types of solutions. On the other hand, the forward-backward asymmetry

$$\frac{\sigma(\theta_\pi<\pi/2)-\sigma(\theta_\pi>\pi/2)}{\sigma(\theta_\pi<\pi/2)+\sigma(\theta_\pi>\pi/2)}=\frac{F-B}{F+B}$$

as a function of $m_{\pi\pi}$, shown in Fig. 2, seems to rule out the negative phase-shift solution (d).[19] [Note that as the (negative) $I=2$ phase shift δ_0^2 is made larger in magnitude, the fit with a negative δ_0^0 gets worse.]

Thus, a detailed fit to the data for reaction (1) yields two solutions for δ_0^0 in the energy range $650\lesssim m_{\pi\pi}\lesssim850$: the narrow resonance ϵ^0 found by Durand and Chiu, and a constant positive value of $\sim60°$. We feel that the latter, "simpler," solution is more likely to be correct. This solution seems to fit smoothly with most of determinations of δ_0^0 at smaller $m_{\pi\pi}$.[20]

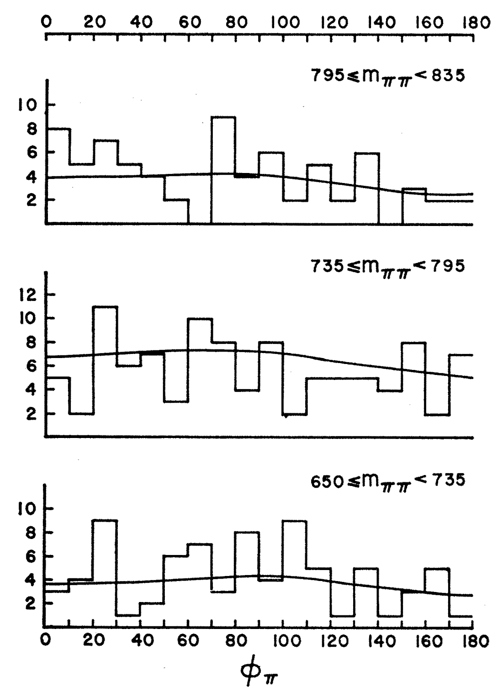

FIG. 3. Plots of the ϕ_π distribution for three bins in $m_{\pi\pi}$. The data are those of Ref. 13. The calculated (smooth) curve corresponds to the S-wave parameters (b) described in Fig. 2.

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[18] In addition to the results presented, we calculate the full θ_π, ϕ_π distributions.

[19] Note that *all* the theoretical determinations of $(F-B)/(F+B)$ are lower than the data. However, because of the "background," the slope of this quantity is probably better determined experimentally than is the absolute normalization.

[20] See, e.g., L. Brown and P. Singer, Phys. Rev. **133**, B812 (1964); C. Lovelace, R. Heinz, and A. Donnachie, Phys. Letters **22**, 332 (1966); Y. Fujii, University of Tokyo (unpublished). For a full list of references, see P. Singer, Finnish Summer School, 1966 (to be published).