(3) the sets $\Gamma_{\mu\nu}-qg_{\mu\nu}+\Gamma_{\mu\nu}{}^{(1)}$ and $J_{\mu\nu}$ or the sets $\Gamma_{\mu\nu}-qg_{\mu\nu}-\Gamma_{\mu\nu}{}^{(1)}$ and $J_{\mu\nu}$ each generate groups with the structure

$$\begin{pmatrix} Z_1 & Z_2 \\ 0 & Z_1{}^* \end{pmatrix},$$

with $\tilde{Z}_1=-Z_1$, $\tilde{Z}_2=-Z_2$ or its transpose. Such a group is a nonsimple group with the Abelian normal subgroup

$$\begin{pmatrix} 0 & Z_2 \\ 0 & 0 \end{pmatrix}.$$

The operator $\mathfrak{M}$ defined in Eq. (4.1) has a continuous spectrum. This can be seen by calculating the commutator

$$[\mathfrak{M},A_k{}^l]=i[C_{ii}-D^{ii},A_k{}^l]=2iA_k{}^l. \tag{B20}$$

If $\mathfrak{M}$ possessed a discrete eigenvalue, say m, then (B20) indicates that $m+2i$ is also an eigenvalue, which contradicts the assumption that $\mathfrak{M}$ is Hermitian. Incidentally, it may be noted that the three operators $\mathfrak{M}$, $A_i{}^i+B^i{}_i$, $A_i{}^i-B^i{}_i$ form a subgroup which is isomorphic to the two-dimensional Lorentz group $O(2,1)$, and it is well known that in the unitary representations of that group, only one generator has a discrete spectrum. In this case, that generator is $A_i{}^i+B^i{}_i$.

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Analysis of the Λ-N Interaction*

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We examine whether the recent low-energy Λ-N scattering data of Alexander *et al.* are compatible with the binding energies of ${}_\Lambda H^3$ and ${}_\Lambda He^5$, if only central Λ-N forces are assumed. We set up Λ-N and N-N potentials, in the spirit of the Moszkowski-Scott separation method, to fit the low-energy scattering parameters. It is found that although these forces are compatible with the ${}_\Lambda H^3$ binding, ${}_\Lambda He^5$ is grossly overbound. This is interpreted as an indication for the existence of an appreciable tensor component in the triplet Λ-N potential.

I. INTRODUCTION

RECENTLY, the scattering length and effective range of the singlet and the triplet lambda-proton (Λ-p) interaction have been determined directly for the first time from the Λ-p scattering data.[1] Assuming charge symmetry, we would refer to these numbers as the low-energy parameters of the lambda-nucleon (Λ-N) interaction. An important feature of these results is that the low-energy parameters in the singlet and triplet Λ-N s-wave scattering are not too different, in contradiction to previous analyses.

Previously, these parameters had been estimated from the data on spins and binding energies of light hypernuclei.[2–5] If one assumes an effective central Λ-N potential, the binding energy B_Λ of the Λ in a light hypernucleus is determined primarily by the s-wave interaction. The most reliable analyses of the binding energies of hypernuclei are those from ${}_\Lambda H^3$ and ${}_\Lambda He^5$. For ${}_\Lambda H^3$, the spin-averaged interaction for a Λ-N bond is $(3V_s+V_t)/4$ or V_t, in terms of the singlet and triplet potentials, V_s and V_t, depending on whether the total spin is $\frac{1}{2}$ or $\frac{3}{2}$. The fact that the observed spin of ${}_\Lambda H^3$ is $\frac{1}{2}$ implies that the singlet potential V_s is more attractive than the triplet one V_t. For ${}_\Lambda He^5$, the spin-averaged interaction for a Λ-N bond is $(V_s+3V_t)/4$. Assuming the shape and the range of the potential, V_s and V_t can be determined from the binding energies of ${}_\Lambda H^3$ and ${}_\Lambda He^5$. The Λ-N low-energy scattering parameters can then be estimated by using these potentials. So far, several sets of V_s and V_t have been proposed.[2–5] They all show strong spin dependence, the singlet scattering length a_s being much larger (in absolute value) than the triplet one a_t, and the triplet effective range r_t being much larger than the singlet one r_s. In contrast to this, the newly determined parameters indicate only weak spin dependence (see Table II of Ref. 1).

The purpose of this paper is to investigate whether the new scattering data are compatible with the binding energies of ${}_\Lambda H^3$ and ${}_\Lambda He^5$, if only central forces are assumed. In Sec. II, we set up Λ-N and N-N central s-wave potentials in the spirit of the Moszkowski-Scott[6] separation method, to fit the new low-energy scattering

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data. In Sec. III, the binding energy of $_{\Lambda}H^3$ is estimated by using the Feshbach-Rubinow (FR) method.[7,8] The accuracy of the method is examined. In Sec. IV, the separation energy B_Λ of the Λ in $_{\Lambda}He^5$ is obtained, assuming the α core to be rigid. It is found that the Λ in $_{\Lambda}He^5$ is grossly overbound. In conclusion, this is interpreted as an indication for the existence of an appreciable short-range tensor component in the triplet Λ-N potential.

II. THE Λ-N AND N-N INTERACTIONS

A. The Λ-N Interaction

Alexander *et al.*[1] quote the following low-energy parameters by analyzing $\Lambda p \to \Lambda p$ cross sections:

$$a_s = -2.46 \text{ F}, \quad a_t = -2.07 \text{ F}, \quad r_s = 3.87 \text{ F}, \quad r_t = 4.50 \text{ F}.$$

These are their most likely values, but no errors are quoted. By comparing with the results of the previous analyses, we find that the singlet Λ-N interaction has become slightly weaker than its earlier value, since r_s has increased while a_s remains about the same.[9] The triplet force has increased in strength, since a_t has a larger value now, while r_t has not changed much. This has resulted in an approximate equality of the two interactions, although the singlet interaction is still the stronger of the two. The large values for effective range mean big "holes" in the zero-energy wave functions, implying that there is strong, short-range repulsion in these forces. Reproducing these data in a potential model would then require introduction of hard cores,[10] or possibly strong soft cores. Rather than use such strong potentials, one can alternatively use weak, long-range potentials in the Moszkowski-Scott[6] spirit. At any given energy, the weak, long-range potential can be adjusted to give the same phase shift as the potential with hard core, provided the over-all potential is attractive. This is so, since the strong, short-range repulsion is "cancelled," so far as phase shift is concerned, by part of the strong, subsequent attraction, leaving only a weak, attractive tail. It is this long-range tail that is responsible for the observed phase shift in scattering. If the Λ is loosely bound in a hypernucleus, then it is again this long-range tail that is responsible for this binding.

We therefore choose the Λ-N potential of the following form:

$$\begin{aligned} V_{s,t}(r) &= 0 \quad \text{for} \quad r < d_{s,t} \\ &= -A_{s,t} e^{-\nu r}/\nu r \quad \text{for} \quad r > d_{s,t}, \end{aligned} \tag{1}$$

where the subscripts s, t stand for singlet and triplet, respectively. Since the "separation distances" d_s and d_t turn out typically ~ 1 F, it is reasonable to choose for ν the two-pion range ($\nu = 1.3992$ F^{-1}). The two adjustable parameters A and d are determined by requiring (1) to fit the low-energy parameters a and r, for singlet and triplet potentials separately. Since we are only fitting the zero-energy data, d is a constant and not energy-dependent. It is hoped, however, that since the Λ is loosely bound to the nucleus, zero-energy parameters will suffice to determine the binding energy B_Λ accurately. This assumption has also been made in all the previous work. By this procedure, we have bypassed the construction of a complete hard-core potential, and yet have reproduced the experimental low-energy data. This results in a great simplification in the calculation of binding energies of $_{\Lambda}H^3$ and $_{\Lambda}He^5$.

TABLE I. The parameters of the three sets of Λ-N potentials.

Set	d_s (F)	A_s (MeV)	a_s (F)	r_s (F)	d_t (F)	A_t (MeV)	a_t (F)	r_t (F)
I	1.017	204.1	−2.46	3.87	1.180	223.3	−2.07	4.50
II	1.222	295.8	−3.46	3.87	0.940	147.2	−1.52	4.50
III	0.645	108.8	−1.46	3.87	1.278	266.2	−2.37	4.50

Table I gives the parameters of three sets of Λ-N potentials used in subsequent calculations, together with the corresponding scattering length and effective range. The symbols used in the table have all been explained earlier in the text. Set I corresponds to the interaction that fits the low-energy parameters quoted by Alexander *et al.*[1] In sets II and III, a_s and a_t are varied arbitrarily within certain limits about the most likely experimental values, so that the zero-energy Λ-p cross section $[\propto (a_s^2 + 3a_t^2)]$ remains constant. This is done with the realization that there may be considerable uncertainty in the experimental values given in set I.

B. The N-N Interaction

For the calculation of $_{\Lambda}H^3$ binding, we also need to know the n-p interaction in the triplet s state which fits the deuteron data. Again, in the Moszkowski-Scott spirit, a superposition of one-pion-exchange potential (OPEP) and a Yukawa potential of two-pion range is used which acts only beyond a separation distance d. The strength of the OPEP is taken as fixed from theory, and again the separation distance and the strength of the two-pion part are varied to fit the scattering length a_t and the effective range r_t. The n-p potential is thus

$$\begin{aligned} V_{np}^{\text{triplet}}(r) &= 0 \quad \text{for} \quad r < 0.722\text{F}, \\ &= -10.742 \frac{e^{-0.6996r}}{0.6996r} - 336.2 \frac{e^{-1.3992r}}{1.3992r} \text{MeV} \\ &\qquad \text{for} \quad r > 0.722\text{F}. \end{aligned} \tag{2}$$

This gives $a_t = 5.38$ F and $r_t = 1.72$ F.

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[7] H. Feshbach and S. I. Rubinow, Phys. Rev. 98, 188 (1955).
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III. BINDING ENERGY OF $_\Lambda H^3$

$_\Lambda H^3$ is the lightest hypernucleus with Λ-deuteron separation energy $B_\Lambda = 0.21 \pm 0.2$ MeV. The total binding energy is given by $B_\Lambda + B_d = 2.226$ MeV. Since the spin of $_\Lambda H^3$ is $\frac{1}{2}$, the interaction for the Λ-N bond is $(3V_s + V_t)/4$, dominated by the singlet interaction.

We examine $_\Lambda H^3$ using the FR[7,8] method. The method was first developed for the triton assuming three nucleons of equal mass interacting via a central, s-state two-body interaction. The wave function Ψ of the triton is taken to be a function of a single variable R:

$$\Psi = \phi(R), \quad R = \tfrac{1}{2}(r_1 + r_2 + r_3),$$

where r_1 is the distance between particles 2 and 3, and similarly for r_2 and r_3. Here it is assumed that the interaction between each pair of nucleons is indentical. The three-body Schrödinger equation can then be reduced to an equation for $\phi(R)$, which resembles a two-body Schrödinger equation. This gives an improvement over the Irving trial function $\Psi = \exp(-\lambda R)$, with a variational parameter λ, and yields excellent results for the binding energy.

To see the accuracy of the FR method in the triton, let us compare it with the variational calculation of Tang *et al.*[11] Assuming the s-wave potentials $V_s = V_t = -V_0 e^{-\kappa r}$ with $V_0 = 96.995$ MeV and $\kappa = 1.156$ F^{-1}, they found, by an extensive numerical calculation, the binding energy of the trition to be between 7.65 and 7.84 MeV. They believe that the true value is between 7.65 and 7.70 MeV. The FR method yields 7.57 MeV; thus the accuracy in this case is quite satisfactory.

The FR method has been applied to $_\Lambda H^3$ by Abou-Hadid and Higgins,[12] who have chosen R to be

$$R = \tfrac{1}{2}(r_1 + r_2 + \eta r_3),$$

where r_3 now stands for the n-p distance. The parameter η is introduced to take account of the asymmetry in the interactions and the masses. η is varied until the maximum binding energy is obtained. Abou-Hadid and Higgins have calculated the volume integral of the Λ-N interaction of a given range which gives the required binding energy B_Λ in $_\Lambda H^3$. It is found that the FR method is considerably better than a simple exponential-type two-parameter wave function of the earlier Dalitz-Downs[13] variational calculation, but much less accurate than their later six-parameter variational result.[2] The actual values of the volume integral given by the FR method lie almost midway between the above two variational results.

We have further examined the FR method in comparison with Dalitz and Downs's[2] six-parameter calculation. The Λ-N interaction is taken to be a Yukawa potential with two-pion range. In one case, Dalitz and Downs's calculation yields 2.476 MeV for the binding energy of $_\Lambda H^3$ and in the other it yields 3.226 MeV. Using the same potentials, the FR method gives $B = 1.908$ and 2.760 MeV, respectively. These are smaller than Dalitz and Downs's values by 0.568 and 0.466 MeV, respectively. Thus we have to admit at least an inaccuracy of 0.5 to 0.6 MeV in this range of binding energy of $_\Lambda H^3$. This inaccuracy tends to increase as the binding energy of the system is reduced. It is our feeling that the accuracy of the FR method is marred by the asymmetry in the interactions between the three particles.

We have used the FR method in $_\Lambda H^3$ despite the fact that it is less accurate than extensive variational calculations. This is done primarily because the FR method is very simple, and yet sufficient to estimate whether the calculated binding energy is compatible with the experimental value. We have estimated the total binding energy of $_\Lambda H^3$, using the forces set up in Sec. II. The results are summarized in Table II. In view of the inaccuracy of the method, sets I and II can be compatible with the observed binding energy of $_\Lambda H^3$, whereas set III probably cannot bind $_\Lambda H^3$.

TABLE II. Results of the binding-energy calculations with the Λ-N potentials given in Table I. In column 2, the first number is the one obtained by the FR method, and the bracketed figure is a rough estimation of the error involved compared with the six-parameter variational calculation of Downs and Dalitz (Ref. 2). The latter calculation, for example, would yield with set I the $_\Lambda H^3$ binding energy ~2.25 MeV. The equilibrium η (defined in Sec. III) is 2.82, 2.70, and 3.33 for sets I, II, and III, respectively.

Λ-N potential	Binding energy of $_\Lambda H^3$ (MeV)	Separation energy of Λ in $_\Lambda He^5$ (MeV)
I	1.60(0.65)	6.45
II	1.78(0.60)	6.02
III	1.28(0.70)	6.06

IV. BINDING ENERGY OF $_\Lambda He^5$

$_\Lambda He^5$ is the commonest of all hypernuclei and its Λ-α separation energy (3.1 MeV) is accurately known. The interaction for a Λ-N bond in $_\Lambda He^5$ is given by $(V_s + 3V_t)/4$. Thus the triplet potential is dominant in the interaction, and it is this part of the force that has changed most from its earlier estimates. Following Dalitz and Downs[14] and Bodmer and Sampanthar,[15] we calculate the average field that the Λ feels in this nucleus, assuming the α core to be undistorted. Since α is a rather tightly bound system, this approximation is not a bad one. Dalitz and Downs[14] have estimated the error due to radial compression of the α core, and found t to be small. Measuring all the vectors from the center of the nucleus, the one-body potential for the Λ, generated by all the nucleons (denoted by subscript i), is

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given by

$$U(r_\Lambda)=\sum_j\int\varphi_j^*(\mathbf{r}_i)V(|\mathbf{r}_i-\mathbf{r}_\Lambda|)\varphi_j(\mathbf{r}_i)d^3r_i. \tag{3}$$

Here the Λ-N potential is $V=V_s+3V_t$, since there are four Λ-N bonds; the φ_j's are the single-particle nucleon orbitals and the sum is taken over all occupied states. The expression (3) can be rewritten as

$$U(r_\Lambda)=\int\rho(r_i)V(|\mathbf{r}_i-\mathbf{r}_\Lambda|)d^3r_i. \tag{4}$$

Here $\rho(r_i)$ is the density distribution of the nucleons in α; it is spherically symmetric. We take the normalized density distribution as

$$\rho(r_i)=(\beta^3/\pi^{3/2})\exp(-\beta^2r_i^2), \tag{5}$$

with $\beta=0.85056$ F^{-1}, the same value as that of Bodmer and Sampanthar.[15] V_s and V_t are given in Eq. (1), with the values of the parameters shown in Table I. The integration in (4) can be done analytically and the average potential can be written as

$$U=(U_s+3U_t), \tag{6}$$

with

$$U_s(r_\Lambda)=-\frac{A_s}{2\nu r_\Lambda}e^{\nu^2/4\beta^2}\left\{e^{-\nu r_\Lambda}\left[1-\mathrm{erf}\left(\frac{\nu}{2\beta}+\beta(d_s-r_\Lambda)\right)\right]\right.$$
$$\left.-e^{\nu r_\Lambda}\left[1-\mathrm{erf}\left(\frac{\nu}{2\beta}+\beta(d_s+r_\Lambda)\right)\right]\right\}, \tag{7}$$

with an exactly similar expression for U_t. After $U(r_\Lambda)$ is obtained, it is a simple matter to calculate the eigenenergy of the bound-state Λ-α system in this potential. It will be seen from Table II that this binding energy for all three sets of Λ-N potentials turns out to be too large when compared with the experimental value of about 3.1 MeV. It is clear, then, that if the low-energy scattering data are fitted by purely central forces of the form we have assumed, then the binding energy of the Λ-α system turns out to be too large, although ${}_\Lambda H^3$ results for two of the three sets considered are still compatible with experiments.

V. DISCUSSION

Purely central Λ-N potentials were constructed to fit the new Λ-p scattering data of Alexander *et al.*[1] By studying Table II, it is clear that ${}_\Lambda He^5$ is grossly overbound by such potentials. Even varying the scattering lengths a_s and a_t within reasonable limits about the quoted values does not remove this discrepancy. Another discrepancy is found when the depth of the Λ-particle potential is estimated in nuclear matter with these potentials. Actually, in nuclear matter, states other than the s state contribute significantly, as is not true in light hypernuclei. However, the pure s-state contribution should give a fair indication of whether the total result is going to be compatible with the estimated depth. This depth is believed to be about 30 MeV[16,17] from extrapolation of binding-energy data of hypernuclei. If we denote this depth by D, then, in first order,

$$D=\rho\Omega, \tag{8}$$

where $\rho=0.17$ F^{-3}, the density of nuclear matter, and Ω is the volume integral of $(V_s+3V_t)/4$ for the Λ-N potential. Higher-order corrections are small for the potentials that we have taken. Our estimates of depth D according to Eq. (8), using potentials given in Table I, range, roughly, from 80 to 90 MeV. It will again be noticed that D is grossly overestimated.

We are of the opinion that these discrepancies arise because we have taken a purely central Λ-N force in the triplet s-state. A purely central force contributes to the binding energy of a spherical system in the first order, whereas a tensor force starts contributing only in the second order. This results in a suppression of the contribution of the tensor force to the binding energy of spherical systems.[18] It has been shown[19] that the tensor force in the Λ-N interaction should be predominantly of very short range, since the two-pion exchange part of the tensor force is very weak, and hence it should originate mainly from the exchange of heavier mesons. Further, since it starts contributing only in the second order, its range is essentially halved in a binding-energy calculation. This leads us to believe that if the triplet Λ-N potential is taken as partly central and partly tensor, one can fit the low-energy scattering data and the binding energies of ${}_\Lambda H^3$ and ${}_\Lambda He^5$ simultaneously. The introduction of a tensor force would not alter the ${}_\Lambda H^3$ binding energy appreciably, since it comes mainly from the singlet Λ-N force. At the same time, there is a good chance that the binding energy of the Λ-α system and the depth of the Λ potential in nuclear matter can then be fitted.

There are, however, some other effects which can contribute to the above discrepancy. The effect of the three-body force on the binding of the ${}_\Lambda He^5$ may be appreciable. The two-pion-exchange part of the three-body force is repulsive in ${}_\Lambda He^5$, and a crude estimate suggests that it may reduce the binding of the system by about 0.7 MeV (see note added in proof).

Bodmer[20] has pointed out that the Λ-N interaction in ${}_\Lambda He^5$ may not be the same as the free Λ-N interaction. This is so since in ${}_\Lambda He^5$, the virtual process $He^4+\Lambda \to He^4+\Sigma^0$ is forbidden because of isospin conservation,

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[20] A. R. Bodmer, Phys. Rev. **141**, 1387 (1966).

and therefore the Σ channel can only contribute through the $T=1$ state of the four nuclei, which may be about 20 MeV above the ground state. This should result in a suppression of Λ-N and Λ-N-N forces. A similar effect arises when a Λ is in nuclear matter because of the Pauli principle between nucleons. In this case, consider the interaction between a nucleon and the Λ via two-pion exchange. All intermediate states of the nucleon which are below the Fermi level are forbidden. Hence the Λ-N interaction in nuclear matter would be different from the free case. However, if we calculate the binding energy of Λ in nuclear matter with these modified Λ-N and Λ-N-N interactions, we get the same result as with free Λ-N and Λ-N-N interactions up to fourth order in the pion coupling constant.[21] The above result is true only when three-body forces are included. It is also possible that by neglecting the Σ-channel suppression in ${}_\Lambda He^5$, but including the free three-body contribution to binding, Bodmer's effect is being taken into account.

[21] D. Kiang and Y. Nogami (to be published).

In conclusion, then, we think that the overbinding (by a central force) of the Λ in ${}_\Lambda He^5$ and in nuclear matter warrants the introduction of a short-range tensor force in the triplet Λ-N interaction. We plan to do a detailed calculation including the tensor force to verify this, and to investigate the other effects mentioned in the above paragraph.

Note added in proof. The repulsive contribution of 0.7 MeV was calculated taking only the central part of the two-pion-exchange Λ-N-N force. However, we have since then found that the tensor part of the Λ-N-N force is much more important and reduces the overbinding of B_Λ in ${}_\Lambda He^5$ substantially, without seriously affecting ${}_\Lambda H^3$. Details of this calculation are submitted for publication in Ann. Phys. (N. Y.).

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Determination of the S-Wave π-π Amplitude near the ρ Peak from the Reaction $\pi^-+p\rightarrow\pi^++\pi^-+n$ *

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A fit to recent extensive data for the reaction $\pi^-+p\rightarrow\pi^++\pi^-+n$ at incident π^- momentum $\sim$4 BeV/c and final two-pion center-of-mass energy $m_{\pi\pi}\sim m_\rho$ was made. The peripheral model with absorption was used in the fit. The asymmetry in the final two-pion distribution θ_π gives a quantitative determination of the π-π, S-wave, $I=0$ scattering amplitude. A constant phase shift of $\sim+60°$ gives as good a fit the to data as a resonance ϵ^0 (at 730 MeV with a width of 90 MeV), proposed by Durand and Chiu. A negative phase shift of $\sim-60°$ is ruled out by examining the distribution in θ_π as a function of $m_{\pi\pi}$.

I. INTRODUCTION

It is known[1] that the angular distribution in θ_π for the final two pions in the reaction[2]

$$\pi^-+p\rightarrow\pi^++\pi^-+n \tag{1}$$

near the final two-pion center-of-mass energy $m_{\pi\pi}\sim m_\rho$ requires a large S-wave phase shift[3] δ_0 interfering with the $l=1$ production.[4] Furthermore, the θ_π distribution of the final pions in the reaction[5]

$$\pi^\pm+p\rightarrow\pi^\pm+\pi^0+p \tag{2}$$

near $m_{\pi\pi}\sim m_\rho$ yields a small negative value for the $I=2$, S-wave phase shift. Thus, reactions (1) and (2) indicate the presence of a large π-π phase shift δ_0^0 near the ρ region.

The peripheral production model with absorptive corrections gives a good fit[6–8] to reaction (2), not only for the cross section as a function of the momentum

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[1] See, e.g., G. Shaw and D. Wong, Phys. Rev. **129**, 1379 (1963); M. Islam and R. Piñon, Phys. Rev. Letters **12**, 310 (1964).

[2] In this paper, we will be discussing data for incident π laboratory momentum $\sim$4 BeV/c.

[3] A subscript will be used on the amplitudes and phase shifts to denote the l value, and a superscript to denote the isotopic spin.

[4] At these values for $m_{\pi\pi}$, d waves are neglected (but f^0 production probably becomes important at somewhat higher $m_{\pi\pi}$).

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