Time Dependence of K_{e3}^0 Decays*

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An experiment was performed at the Brookhaven Cosmotron to detect the β decay of the products of the reaction $\pi^-+p\to\Lambda+K^0$. We have obtained spark-chamber photographs of 116 $K^0\to\pi^\mp+e^\pm+\nu$ events, and have determined the proper decay time of each K^0. The time distribution should be of the form $|1+x|^2e^{-\lambda_1 t}+|1-x|^2e^{-\lambda_2 t}-4\text{Im}(x)\sin\delta t\; e^{-(\lambda_1+\lambda_2)t/2}$, where λ_1 and λ_2 are the K_S and K_L decay rates, $\delta=m(K_L)-m(K_S)$, and x is the complex ratio of the $\Delta S=-\Delta Q$ amplitude to the $\Delta S=+\Delta Q$ amplitude. We find $\text{Re}(x)=0.17_{-0.35}^{+0.16}$, $\text{Im}(x)=0.0\pm0.25$, a result inconsistent with a "maximal" violation of CP invariance.

INTRODUCTION

THE Feynman–Gell-Mann current-current theory[1] of the weak interactions includes the hypothesis that the only strangeness-changing terms in the current are those in which the change in strangeness is of the same sign as the change in charge. This assumption is the $\Delta S=\Delta Q$ rule.

The absence of $\Delta S=-\Delta Q$ transitions is difficult to establish experimentally. One possibility of an experimental test is provided by K^0 leptonic decays, since $K^0\to\pi^-+e^++\nu$ is allowed by the $\Delta S=\Delta Q$ rule, while $K^0\to\pi^++e^-+\bar{\nu}$ is forbidden. Of course, an initially pure K^0 beam quickly becomes a $K^0-\bar{K}^0$ mixture, so that both electron and positron decays will occur. The total K_{e3}^0 decay rate (irrespective of the signs of the products) will be given by

$$P(t)=C[(1+x)^2e^{-\lambda_1 t}+(1-x)^2e^{-\lambda_2 t}]. \qquad (1)$$

C is a proportionality constant, λ_1 and λ_2 are the total K_L and K_S decay rates, and $x=g^*/f$, where g^* is the amplitude for $K^0\to\pi^++e^-+\bar{\nu}$ ($\Delta S=-\Delta Q$) and f is the amplitude for $K^0\to\pi^-+e^++\nu$ ($\Delta S=+\Delta Q$). CP invariance requires that x be a real number.

Early experimental evidence,[2] although later contradicted,[3] indicated that $x\neq0$, i.e., that the $\Delta S=\Delta Q$ rule was violated. Sachs and Treiman[4] then pointed out that the existence of a $\Delta S=-\Delta Q$ amplitude g^* would provide a test of CP invariance. Allowing for the possibility of a CP violation, $x=g^*/f=|x|e^{i\theta}$ is in general complex, and the K_{e3} decay rate is given by

$$P(t)=C[|1+x|^2e^{-\lambda_1 t}+|1-x|^2e^{-\lambda_2 t}-4|x|\sin\theta\,\sin\delta t\; e^{-(\lambda_1+\lambda_2)t/2}], \qquad (2)$$

where $\delta=m(K_L)-m(K_S)$. Recent experiments indicate that the mass difference δ, as defined here, is a positive number.[5] Equation (2) has been derived using the standard (but only conventional) plus sign in the Schrödinger equation $+i\hbar\partial\psi/\partial t=E\psi$, so that the oscillatory time dependence of an amplitude is $e^{-iEt/\hbar}$. Either adopting the opposite quantum-mechanical convention, or defining δ to be the negative of the above definition, would reverse the sign of the interference term in Eq. (2). An initial $\bar{K}^0$ beam would have the sign of the above interference term reversed.

With the discovery by Christenson *et al.*[6] of an apparent violation of CP invariance in the decay $K_L^0\to\pi^++\pi^-$, it is of great interest to find other examples of CP violation. Many suggestions have been put forward to explain the observed effect and to predict in which experiments other violations will be found.

It would, of course, be expected that the K^0 leptonic decay rate would show some CP violation, and the data should be re-examined on that basis, using Eq. (2) rather than Eq. (1). But, in general, one would expect the departure from Eq. (1) to be quite small, since the violation observed by Christenson *et al.* was only 0.2%. Sachs,[7] however, has proposed to explain the observed effect by a "maximal" violation of CP invariance in the leptonic decays of the K^0. The idea is that the small $K_L\to\pi^++\pi^-$ decay rate occurs not because of a small CP violation in the nonleptonic decay interaction, but rather as the result of off-diagonal terms in the $K^0-\bar{K}^0$ mass matrix caused by a violation of CP in leptonic decay. Since the K^0 leptonic decay rate is much smaller than the nonleptonic rate, the effect on the mass matrix is small, thus explaining the very small $K_L^0\to\pi^++\pi^-$

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[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] R. P. Ely, W. M. Powell, H. White, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, O. Fabbri, F. Farini, C. Filippi, H. Huzita, G. Miari, U. Camerini, W. F. Fry, and S. Natali, Phys. Rev. Letters **8**, 132 (1962); G. Alexander, S. P. Almeida, and F. S. Crawfor, Jr., *ibid.* **9**, 69 (1962).

[3] L. Kirsch, R. J. Plano, J. Steinberger, and P. Franzini, Phys. Rev. Letters **13**, 35 (1964).

[4] R. G. Sachs and S. B. Treiman, Phys. Rev. Letters **8**, 137 (1962).

[5] J. Canter *et al.*, Phys. Rev. Letters **17**, 942 (1966); R. H. Good *et al.*, Bull. Am. Phys. Soc. **11**, 767 (1966); G. W. Meisner, B. B. Crawford, and F. S. Crawford, Jr., Phys. Rev. Letters **17**, 492 (1966); J. V. Jovanovich, T. Fujii, F. Turkot, G. T. Zorn, and M. Deutsch, *ibid.* **17**, 1075 (1966).

[6] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[7] R. G. Sachs, Phys. Rev. Letters **13**, 286 (1964).

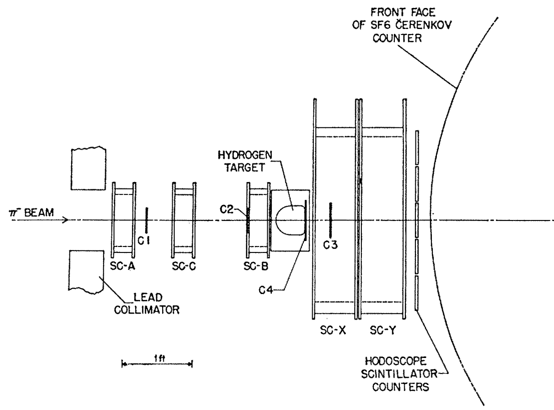

FIG. 1. Plan view of experimental apparatus.

rate. In fact, in order to explain the effect being as large as it is, it is necessary that $|g^*|\approx|f|$, and that the relative phase be close to 90°. This then leads naturally to the aesthetic "maximal" violation hypothesis $g^*=\pm if$.

We report here a test of the above suggestion, based on the analysis of the time of decay of 116 $K^0\to\pi^\pm+e^\mp+\nu$ events obtained in a spark chamber experiment.[8] We find no evidence to support the hypothesis of a maximal violation of CP invariance in leptonic K^0 decay.

EXPERIMENT

An experiment to study the β-decays of the products of the reaction $\pi^-+p\to\Lambda^0+K^0$ was performed at the Cosmotron at Brookhaven National Laboratory. A schematic drawing of the apparatus appears in Fig. 1.

A beam of 1030 MeV/c pions was focused onto a liquid-hydrogen target. The momentum spread of the beam was 5% (full width at half-maximum), and its size at the target was 1.4 by 1.2 in. Scintillation counters C1 and C2 detected the incident pions. The absence of a count in C4, a 7 by 7 by $\frac{3}{16}$-in. anticoincidence counter, indicated that the pion had interacted to form neutral secondaries. By using dc coupling in the anticoincidence counter and its associated circuitry, a rejection efficiency of better than 0.99995 was achieved. Located 4.1 in. farther downstream inside spark chamber X was counter C3, 6 by 6 by $\frac{1}{16}$ in.; it detected the subsequent decay of a neutral secondary into charged particles in the spark-chamber region. Since C3 was completely shielded from the beam by the anticoincidence counter, the requirement that a particle hit it eliminated the large number of scattering events which missed C4 but hit the hodoscope.

The hodoscope of ten 6- by 15- by $\frac{1}{4}$-in. counters allowed us to require that a specific number of charged particles pass through it. In this experiment one would normally have required three particles (designated 3/10), but, in an attempt to gather single $\Lambda\to p+e^-+\bar{\nu}$ events without requiring a simultaneous $K_S\to\pi^++\pi^-$, most of the experiment was run with 2/10 triggering.

Finally, a large pressurized gas Čerenkov counter (Č) detected electrons, while rejecting pions and other charged particles with an efficiency better than one part in 10^3. Details of the construction and performance of this counter have been published.[9] In order to maximize the small expected counting rates, the hodoscope and Čerenkov counter were designed to subtend a half angle of 45° at the target. The resulting 8-ft diameter of Č, together with a constraint that it occupy only 2 ft along the beam, required that the front face of the counter be $\frac{3}{8}$-in. steel in order to contain 150 psi of SF_6 safely. This front thickness caused the counter to be rather sensitive to γ rays, and its electron efficiency to be energy-dependent, particularly below 100 MeV/c. Monte Carlo calculations show that the variable efficiency had no appreciable effect on the space or time distribution of events. The pion beam passed through a thin window ($\frac{1}{16}$ in. stainless steel), as well as through a gap in the hodoscope.

A small gas Čerenkov counter at the first focus of the beam rejected events initiated by electrons in the beam.

The spark chambers were triggered by the coincidence scheme $12\bar{4}3(2/10)\check{C}$. Pictures taken without the Č requirement provided a sample of nonleptonic ΛK decays for testing and calibration.

Spark chambers A, B, and C, of ten $\frac{1}{4}$-in. gaps each, served to locate the trajectory of the incident pion; chambers X and Y, of fourteen $\frac{3}{8}$-in. gaps each, displayed the decay products of the Λ and K. The plates of these chambers were stainless steel mesh having an average thickness of 0.019 g/cm². The chambers were photographed through two 20-ft focal-length field lenses, one for each of two 90° stereo views. A better than average spark-chamber photograph is shown in Fig. 2.

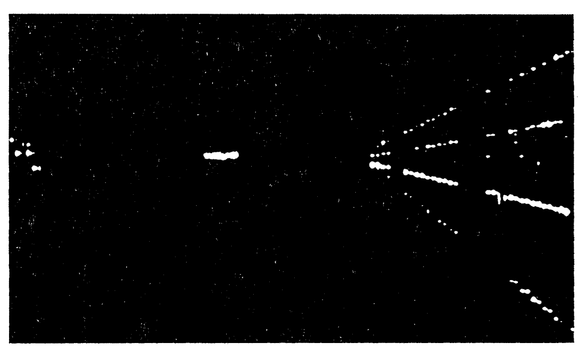

FIG. 2. Spark chamber photograph of a double-V event. Only one view is shown. Note that the heavily ionizing proton is easily identifiable.

[8] A detailed discussion of this experiment is to be found in D. M. Wolfe, Ph.D. thesis, University of Pennsylvania (1966). A short report of the results was published in Bull. Am. Phys. Soc. **11**, 767 (1966), and reported in Proceedings of the 1966 High-Energy Physics Conference at Berkeley, 1966 (to be published).

[9] S. Frankel, V. Highland, T. Sloan, O. Van Dyck, W. Wales, and D. Wolfe, Rev. Sci. Instr. **37**, 15 (1966).

The liquid hydrogen was contained in a 0.010-in. Mylar cylinder, 5 in. in diameter and 5 in. long, mounted inside an insulating vacuum chamber (see Fig. 1). In order to have the anticoincidence counter C4 as close to the interaction region as possible, it was placed inside the vacuum chamber and in contact with the hydrogen container. The counter then served additionally as a mechanical support for the downstream end of the target, which it held flat over a circle 3 in. in diameter. The proximity of the scintillator to the hydrogen (0.010 in. of Mylar and 0.015 in. of insulation and counter wrapping) maximized the number of Λ's and K's that decayed beyond the veto counter.

FILM ANALYSIS

During the experiment, 113 000 useful pictures were taken in the normal triggering mode. In addition, 10 000 pictures were taken without requiring a count in Č, providing a sample of nonleptonic ΛK decays. (These two batches of film will be referred to hereafter as "leptonic" and "nonleptonic." It was necessary to winnow from these two sets of film the following two double-V events:

$$\pi^- + p \rightarrow \Lambda + K^0,$$

with

$$\text{(a)}\quad \begin{array}{l} \Lambda \rightarrow \pi^- + p\,, \\ K^0 \rightarrow \pi^{\pm} + e^{\mp} + \nu\,. \end{array} \qquad (\Lambda K_{e3})$$

or

$$\text{(b)}\quad \begin{array}{l} \Lambda \rightarrow \pi^- + p\,, \\ K^0 \rightarrow \pi^+ + \pi^-\,. \end{array} \qquad (\Lambda K)$$

Since the particles were not in a magnetic field, it was not possible to determine the signs of their charges.

A triple scan of the film for events in which two V's appeared yielded 12 000 "leptonic" pictures and 750 "nonleptonic" pictures. The only selection criteria were that the vertex of a V have the same z coordinate (along the beam) in both stereo views, and that both vertices occur downstream from the anticoincidence counter.

Physicists reviewed the selected events. In addition to rechecking the scanning criteria, they identified the interacting pion trajectory and required that it pass between the two vertices, a necessary condition for a two-body production reaction. Because of the high-incident pion flux [10^5 π's/(5-msec burst)], it was often impossible to identify the interacting pion unambiguously; such events were rejected.

The surviving events (3950 leptonic and 389 nonleptonic) were measured on an MPS-θ digitized measuring machine.[10] A computer program reconstructed the measured events in space and set the following geometric criteria: (a) The apparent vertices must be true intersections in space, within the errors. (b) The two vertices and the incident pion must be coplanar, within the errors. (c) Vertices must lie more than 0.25 in. from the pion trajectory (in order to eliminate possible "stars"). 1388 leptonic events and 159 nonleptonic events passed these criteria.

[10] Manufactured by Nuclear Research Instruments, Berkeley, California.

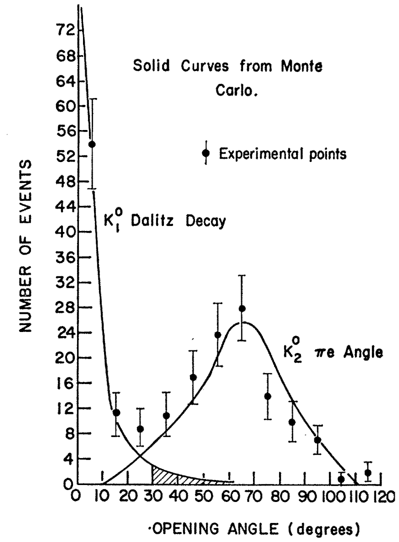

FIG. 3. Experimental opening angle distribution of supposed K_{e3}^0 decays. Solid curves show the Monte Carlo prediction for true K_{e3}^0 decays, and for $K_1^0 \rightarrow \pi^0+\pi^0$ followed by Dalitz decay of one π^0.

The point of production of an event was located by finding the intersection of the incident pion track and the plane formed by the products of a two-body decay of the Λ or K^0. Since we were searching for leptonic three-body decays, we expected that one of the V's would be useless for determining the production point. The computer program tried to fit the events under the four hypotheses provided by assuming that one or the other V was a two-body decay, and that one or the other was the Λ or the K^0.

Fits were accepted if (a) The reconstructed production point lay inside the hydrogen target. (b) The reconstructed pion momentum was between 0.9 and 1.25 BeV/c. (c) The momentum of a particle determined from its (assumed) two-body decay agreed with the momentum as calculated from the production kinematics. (d) The calculated mass of the Λ was in the range 1115±15 MeV. Good fits were found for 452 leptonic events and 110 nonleptonic events.

Fiducial volume requirements were also imposed on the events. Λ's were required to decay between C4 and C3 (4.1 in.). K's were accepted up to 2 in. farther downstream, since tests showed an apparent lack of both triggering and scanning bias throughout chamber X.

Events were classified as possible ΛK_{e3} events if either the K momentum determined from decay kinematics disagreed with the momentum determined from production kinematics or the decay was noncoplanar

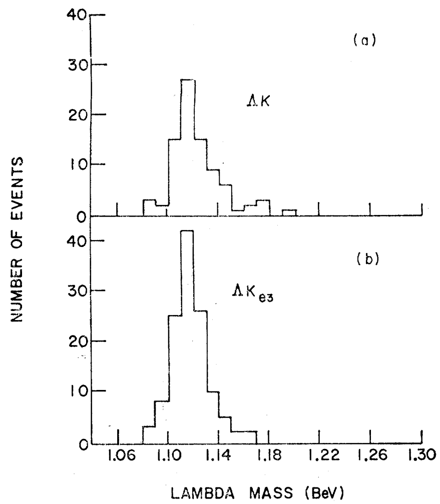

FIG. 4. Reconstructed Λ mass in reaction $\pi^- + p \rightarrow \Lambda + K^0$ for (a) nonleptonic K^0 decays and (b) $K_{e3}{}^0$ decays.

well outside the errors. 336 ΛK_{e3} candidates were found. In the nonleptonic ΛK events five apparent three-body K^0 decays were found, where three were expected.

Physicists carefully studied the remaining events, and rejected suspicious pictures, such as those with large angle scatters, obvious gamma-ray pair production, or unexplained extra tracks associated with an event. This left 190 ΛK_{e3} events and 89 ΛK events.

A serious background in this experiment was the Dalitz decay[11–13] ($\pi^0 \rightarrow \gamma + e^+ + e^-$) of the π^0's from the decay $K_S{}^0 \rightarrow \pi^0 + \pi^0$. The opening angle distribution of the Dalitz pairs is peaked at small angles, but not as sharply as pair production by real γ rays. In order to reduce to an acceptable level the probable contamination of our ΛK_{e3} sample from this effect, it was necessary to reject K^0 decays which had an opening angle of less than 30°, leaving 116 ΛK_{e3} events (see Fig. 3).

DATA ANALYSIS

The identification of the events was checked in various ways. We have found in a subsequent $\Lambda \rightarrow p + \pi^-$ experiment that the decay proton is almost always identifiable by its heavier track density. Visually checking the Λ's found in this experiment verified that the proton (and hence the Λ) had always been correctly identified by the analysis. The up-down asymmetry of the protons with respect to the production plane was found to be $\alpha P = 0.57 \pm 0.19$ in the ΛK_{e3} events and $\alpha P = 0.55 \pm 0.21$ in the ΛK events. The accepted value is $\alpha P = 0.62 \pm 0.07$.[14] The mean lifetime of the Λ's was found to be $(2.6 \pm 0.24) \times 10^{-10}$ sec in the ΛK_{e3} events and $(2.7 \pm 0.30) \times 10^{-10}$ sec in the ΛK events, to be compared with the accepted value of $(2.61 \pm 0.02) \times 10^{-10}$ sec.[15] A comparison of the values of the reconstructed Λ mass in the two cases is shown in Fig. 4.

An extensive series of Monte Carlo calculations was performed as a further check on the data. Events were generated subject to all the conditions and known biases of the actual experiment. In general, the distributions of quantities so generated were in excellent agreement with the data. For example, the Λ, K, and incident pion momentum spectra for both ΛK and ΛK_{e3} events, and the distribution of coplanarity angles for K_{e3} decays were all in good agreement with the data.

The agreement was not as good for certain other parameters. These were quantities expected to be very sensitive to film measurement errors. The Monte Carlo calculations were extended to include these measuring errors, which had been subject to careful independent study. The Monte Carlo tracks were mathematically projected onto two perpendicular planes (the 90° stereo views) and changed slightly both in an angle and position. This "wobbling" of the projected tracks was done in accordance with a Gaussian distribution corresponding to the measuring errors. These wobbled events were then treated in the same way as real data by the reconstruction and analysis programs. After this extension, the Monte Carlo calculations were in good agreement with the data for such quantities as the distribution of production points in the hydrogen for both ΛK and ΛK_{e3}, and the distribution of lifetimes and flight-path lengths for the Λ^0's and K^0's in the ΛK events (see Figs. 5 and 6). This agreement gave us

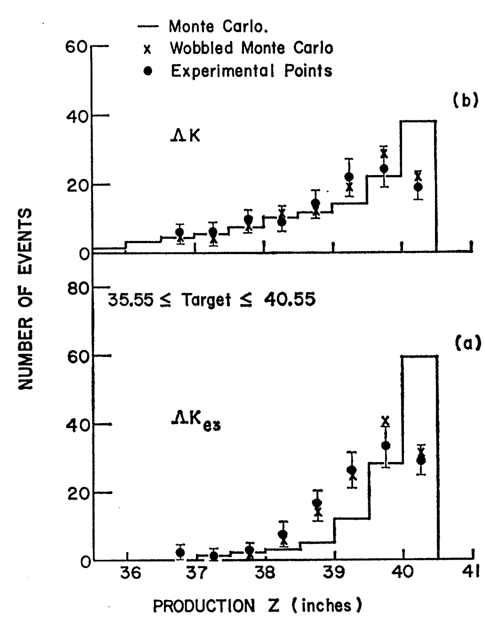

FIG. 5. Measured production point of ΛK^0 events for (a) nonleptonic K decays and (b) $K_{e3}{}^0$ decays. Also shown are the predictions of the ordinary Monte Carlo calculation, and of the "wobbled" Monte Carlo calculation, which takes the measuring errors into account.

[11] R. H. Dalitz, Proc. Phys. Soc. (London) **A64**, 667 (1951).
[12] N. M. Kroll and W. Wada, Phys. Rev. **98**, 1355 (1955).
[13] N. P. Samios, Phys. Rev. **121**, 275 (1961).
[14] J. W. Cronin and O. E. Overseth, Phys. Rev. **129**, 1795 (1963).
[15] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965).

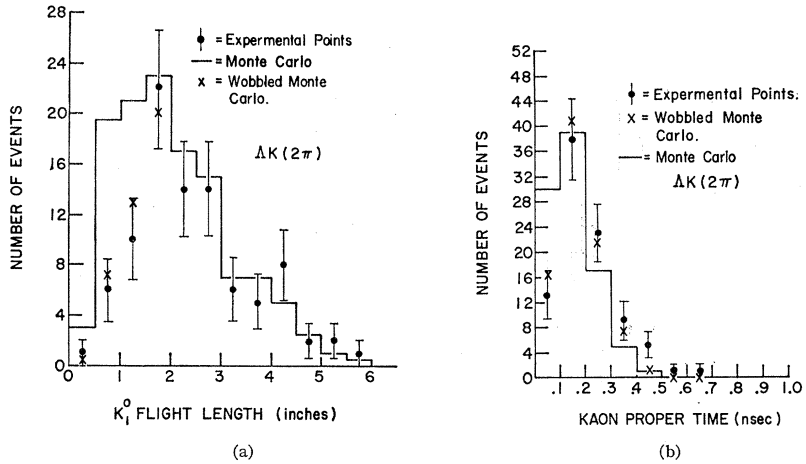

FIG. 6. Experimental distribution for nonleptonic ΛK^0 events of (a) length of K^0 flight path from production to decay and (b) corresponding elapsed time in K^0 frame of reference. Also shown are the predictions of the ordinary Monte Carlo calculation, and of the "wobbled" Monte Carlo calculation, which takes the measuring errors into account.

confidence that we understood the biases and measuring errors in the experiment, and knew how to take them into account in the Monte Carlo technique. This was important, since the comparison of the K_{e3} decay-time distribution with theory made use of further Monte Carlo calculations.

In Fig. 3 the observed opening angle between the charged particles in the supposed K_{e3} decays is compared with a Monte Carlo calculation for K_{e3} decays together with the Dalitz decay background mentioned above. The K_{e3} decay was assumed to be a pure vector interaction;[16] the details of the Dalitz decay interaction were taken from the work of Kroll and Wada.[12] The agreement between data and calculation is excellent.

BACKGROUNDS

The spark chamber pictures taken in the "leptonic" triggering mode may include any type of double-V event in which one of the decay particles is an electron. Also, in any double-V event that has a γ ray associated with it, there is an approximately 30% probability that the γ ray will convert in the front face of the Čerenkov counter, and thus have a chance of satisfying the triggering requirements. The probability of various such events simulating a ΛK_{e3} event is discussed below. The most serious of such backgrounds would be those involving a $K_S{}^0$ decay, since they would sharply enhance the number of events with a short K^0 lifetime.

The π^0 Dalitz decay background, $K_S{}^0 \to \pi^0+\pi^0$, $\pi^0 \to \gamma+e^++e^-$, is very large, as seen in Fig. 3. Nevertheless, the 30° cutoff on the opening angle eliminates most of it, while sacrificing relatively few real events. We estimate that 3.5 Dalitz decays were included in the final data, and that five real events were lost by the cutoff. The Dalitz decays of π^0's from the reactions $K_L{}^0 \to 3\pi^0$ and $\Lambda \to n+\pi^0$ were calculated to contribute only 0.3 events.

Other reactions which produce electrons are $\Lambda \to p + e^- + \bar{\nu}$ and the $\pi \to \mu \to e$ decay chain of any of the charged π's produced in ΛK decays. The former effect has a very small branching ratio,[17] and the latter has very little probability of the decays taking place colinearly and thus not being rejected at some stage of the analysis. The probability of either of these two effects simulating a ΛK_{e3} event is calculated to be negligible.

Among the events having an associated γ ray is $K_L{}^0 \to \pi^+ + \pi^- + \pi^0$, which is calculated to contribute 1 event to our data. All of the types of events mentioned in the discussion of Dalitz decay have the possibility of appearing as a double V via pair production in the chambers, but they are completely eliminated by the 30° opening angle criterion.

The Monte Carlo calculations show that the decay $\Sigma^0 \to \Lambda + \gamma$ has only a 5% chance of reconstructing as a Λ. The number of Σ's produced was not large, since the incident pion momentum spectrum was centered just at ΣK threshold. The contamination from this source is estimated as 0.5 events.

Finally, there is the decay $K^0 \to \pi^+ + \pi^- + \gamma$, about which rather little is known. As discussed by Franzini *et al.*,[18] there is experimental evidence indicating that the direct radiative decay rate is less than 1% of the rate $K_L \to \pi^+ + \pi^- + \pi^0$, so that it would be negligible in this experiment. There is also, however, the possibility of "inner bremsstrahlung" in the decay $K_S{}^0 \to \pi^+ + \pi^-$. The photon spectrum for this process, presented by Franzini *et al.*,[18] is peaked like $1/k$ at small c.m. photon energy k. Since the efficiency of our Čerenkov counter dropped off rapidly below 50-MeV lab energy,[9] this decay mode was sharply discriminated against. In addition, Monte Carlo calculations show that since the γ-ray energy was generally small, only 3% of such events would have been considered three-body decays. Using a cutoff of 5 MeV on the photon spectrum, we

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[16] S. B. Treiman, in *Weak Interactions and Topics in Dispersion Physics*, edited by C. Fronsdal (W. A. Benjamin, Inc., New York, 1963), p. 69.

[17] R. P. Ely *et al.*, Phys. Rev. **131**, 868 (1963).

[18] P. Franzini, L. Kirsch, P. Schmidt, J. Steinberger, and R. J. Plano, Phys. Rev. **140**, B127 (1965).

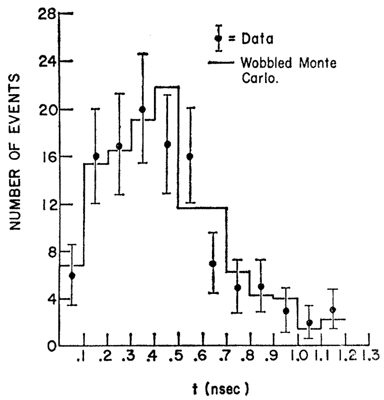

FIG. 7. Elapsed K^0 proper time from production to decay for K_{e3} events. The histogram is the "wobbled" Monte Carlo calculation, which takes into account the measuring errors, using the values $\text{Re}(x)=0.17$, $\text{Im}(x)=0.0$ determined by the maximum likelihood analysis.

calculate a background of 1.4 events from $K_S{}^0 \to \pi^+ + \pi^- + \gamma$. The result is insensitive to the value of the cutoff.

In summary, we expect our final data to include a background contamination of 5.7 events with a K_S lifetime, and less than one event with a K_L lifetime.

RESULTS

A histogram of the lifetimes of the 116 K_{e3} events is shown in Fig. 7. The probability of decay of a K^0 into $\pi^\pm e^\mp \nu$ is expected to be given by Eq. (2). A contamination of nonleptonic $K_S{}^0$ decays can be easily included in the formula (ignoring the small probability of $K_L{}^0$ background) by adding a term proportional to $e^{-\lambda_1 t}$:

$$P'(t)=C\{[|1+x|^2+Y]e^{-\lambda_1 t}+|1-x|^2 e^{-\lambda_2 t} -4|x|\sin\theta \sin\delta t\, e^{-(\lambda_1+\lambda_2)t/2}\}, \quad (3)$$

where Y is a measure of the $K_S{}^0$ background.

A maximum likelihood calculation could now be made to determine the values of $|x|$, θ, and δ that best fit the data, taking into account the errors in Y. But the data were known to be biased at short K^0 times, since the K^0 lifetime curve as determined in the ΛK events had events missing at short times. As discussed above, this could be accounted for entirely by the measuring errors. The effect of such errors was greatest for K^0's decaying after a very short flight path, since the resultant large angular errors caused the events to fail the kinematic tests. The discrepancy between observed and Monte Carlo K^0 flight length is shown in Fig. 6(a). Also shown is the agreement obtained with the wobbled Monte Carlo technique.

Using these Monte Carlo results, the probability $p(L)$ of a K^0 surviving the measuring errors as a function of flight length L was fit well by a straight line, $p(L)=0.2L$, over the range 0 to 5 cm, with $p(L)$ remaining unity after 5 cm. One can then translate $p(L)$ into a probability $\alpha(t)$ of a K^0 surviving the measuring errors as a function of proper time:

$$\alpha(t)=0.2\beta\gamma ct, \quad t<t_0,$$

and

$$\alpha(t)=1, \quad t>t_0,$$

where $t_0=5/\beta\gamma c$, the time for a K^0 with momentum $p=\beta\gamma m$ to travel 5 cm. The probability $P''(t)$ of detecting a K_{e3} decay at proper time t is then

$$P''(t)=\alpha(t)P'(t), \quad (4)$$

where $P'(t)$ is given by Eq. (3). This is the function to which the maximum likelihood analysis was applied.

The likelihood function was maximized simultaneously as a function of $\text{Re}(x)$, $\text{Im}(x)$, and δ. A maximum was obtained for

$$\text{Re}(x)=0.17_{-0.35}^{+0.16}, \quad \text{Im}(x)=0.0\pm0.25.$$

The errors are statistical, including the statistical error in the $K_S{}^0$ background Y; the values quoted are those at which the likelihood has fallen by $1/e$. A plot of the $1/e$ contour in the complex x plane is shown in Fig. 8. The values of the K^0 mean lifetimes used were $\lambda_1^{-1}=0.90\times10^{-10}$ sec, and $\lambda_2^{-1}=56\times10^{-9}$ sec.[19]

Since the maximum of the likelihood function occurred at $\text{Im}(x)=0$, no information was obtained on the value of δ. In other words, the fit to our data is not sensitive to the actual value of δ. From other experiments, the value of δ now seems to be approximately 0.55.[20] If we move slightly away from the best value of $\text{Im}(x)$ to $\text{Im}(x)=0.16$ ($\theta=10°$), our best value of δ would be $0.4_{-0.3}^{+0.25}$.

In Fig. 7 a (wobbled) Monte Carlo calculation for the K_{e3} time distribution expected in this experiment using $\text{Re}(x)=0.17$, $\text{Im}(x)=0$ is compared to the data. A χ^2 comparison of the Monte Carlo calculation to the data gives 7.14 with 11 degrees of freedom, for a χ^2 probability of 75%. By varying the values of $\text{Re}(x)$ and $\text{Im}(x)$ individually, it was found that $\text{Re}(x)=0.17$ and $\text{Im}(x)=0$ indeed give the minimum χ^2.

These calculations were repeated with different values of the K_{e3} opening angle cutoff (and corresponding background estimate). They were also repeated for more

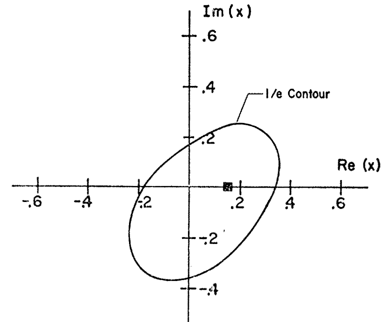

FIG. 8. Plot in the complex x plane of point giving maximum likelihood fit to the $K_{e3}{}^0$ decay time distribution, and of the contour of points at which the likelihood has fallen to $1/e$ of its maximum value.

[19] See Ref. 15. Recently, the measured value of λ_1 seems to be changing, but the resultant change in our result is small compared to our errors. The result is still less sensitive to the value of λ_2.

[20] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. **140**, B74 (1965). A summary of the mass difference data is given by M. L. Good, in Proceedings of the International Conference on Weak Interactions, 1965, Argonne National Laboratory Report No. ANL-7130 (unpublished).

restricted K^0 fiducial volumes. In all cases, the same results were obtained, well within the errors.

Other systematic effects were considered. Reasonable upper limits on errors in the short-flight-length calculation varied Im(x) between $+0.1$ and -0.2 and Re(x) between 0.1 and 0.2. Varying the expected K_S background by ± 2 standard deviations varied the result over the same range. If, instead, these extra events had the Λ lifetime, the result for x varied over roughly half the above range. Similar amounts of a K_L type of background had virtually no effect.

In general, we estimate that any plausible systematic error would move the measured value of x around well within the contour shown in Fig. 6 for the statistical error.

These calculations have shown that a general feature of these experiments is for a spurious excess of K_S events to give Im$(x)>0$ (destructive interference), and a shortage of K_S events to give Im$(x)<0$ (constructive interference).

DISCUSSION

Our results—Re$(x)=0.17_{-0.35}^{+0.16}$, Im$(x)=0.0\pm 0.25$—can be compared to the requirement of the $\Delta S=\Delta Q$ rule—Re$(x)=0$, Im$(x)=0$— to the requirement of CP invariance—Im$(x)=0$— and to Sachs' maximal CP violation hypothesis—Re$(x)=0$, Im$(x)=\pm 1$. Although the errors are substantial, the results are inconsistent with the maximal CP violation hypothesis, and indicate that any violations of the $\Delta S=\Delta Q$ rule or of CP invariance in K_{e3} decays are not very large.

In Fig. 9, our result is plotted in the complex x plane, along with the results of other experiments.[18,21–23] Note that the result of Ref. 18 has been plotted with the reverse of the published sign of Im(x). In the past, there have been some differences in phase convention; we believe that the plotted results are all consistent with Eq. (2).[22]

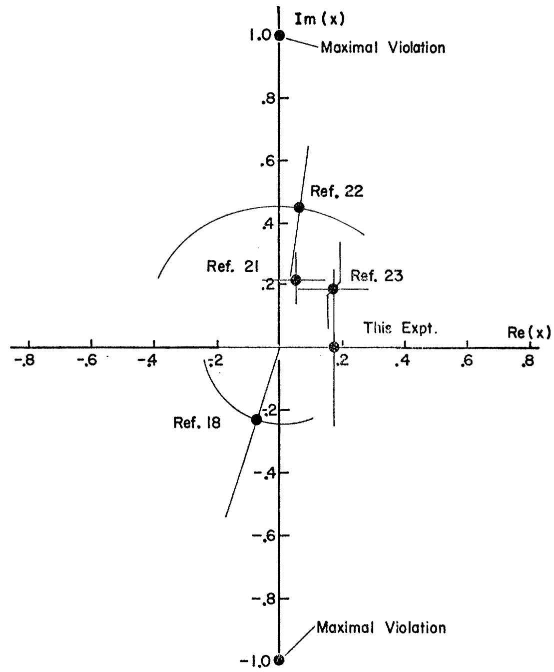

FIG. 9. Plot in the complex x plane of the results of this experiment along with those of other experiments. The points $x=\pm i$ predicted by the maximal CP violation hypothesis are also illustrated.

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[21] B. Aubert, L. Behr, F. L. Canavan, J. P. Lowys, P. Mittner, and C. Pascaud, Phys. Letters **17**, 59 (1965); in Proceedings of the International Conference on Weak Interactions, 1965, Argonne National Laboratory Report No. ANL-7130 (unpublished).

[22] M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, C. Filippi-Filosofo, H. Hutzita, F. Mattioli, and G. Miari, Nuovo Cimento **38**, 684 (1965). Their equivalent of Eq. (2) has the opposite sign for the interference term, but this is said to be an error, and the published sign is correct.

[23] Y. Cho *et al.*, in Proceedings of the International Conference on High-Energy Physics, Berkeley, 1966 (to be published).

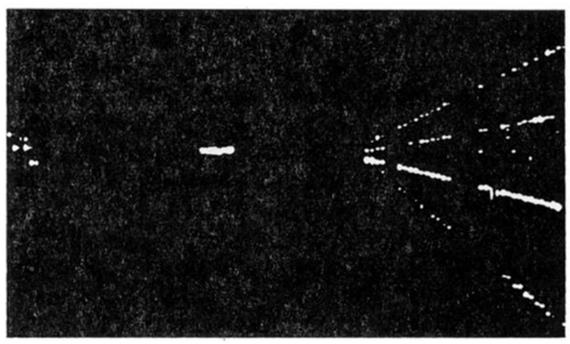

FIG. 2. Spark chamber photograph of a double-V event. Only one view is shown. Note that the heavily ionizing proton is easily identifiable.