similar to (4) and gives the coupling as[10]

$$h_{rs}\left(\begin{array}{cc|c} 8 & s & r \\ i & j & k \end{array}\right)+\gamma_{rs}\left(\begin{array}{cc|c} 8 & 8 & 8_s \\ 0 & i & i \end{array}\right)\left(\begin{array}{cc|c} 8 & s & r \\ i & j & k \end{array}\right). \quad (11)$$

If both the tensor meson and the vector meson belong to octets, then C conservation requires that only the antisymmetric coupling is present. We then see that keeping i fixed but varying j and k, the coupling constant (11) transforms like an $SU(3)$ symmetric quantity. Thus there is no renormalization for the ratio[11] of coupling constants

$$\frac{g(A_2\rho\pi)}{g(K^{**}K^*\pi)},\quad \frac{g(K^{**}\rho K)}{g(K^{**}\omega_8 K)}. \quad (12)$$

Therefore the ratios of decay widths,

$$\frac{\Gamma(A_2\to\rho\pi)}{\Gamma(K^{**}\to K^*\pi)},\quad \frac{\Gamma(K^{**}\to\rho K)}{\Gamma(K^{**}\to\omega K)} \quad (13)$$

are the same as those calculated by Glashow and Socolow,[1] even after including first-order breaking in the coupling constants. The experimental information about the second ratio is scanty but the first ratio is in good agreement[1] with the experimental number.

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[10] Note that there is only one pion to disperse.
[11] This result is similar to the one obtained in Ref. 5 for meson couplings with two baryons.

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Baryon Mass Splitting in a Boson-Fermion Model*

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A model is proposed in which a baryon consists of a boson and fermion deeply bound in a potential. The fermion can be regarded as a quark and the boson as a bound state of two quarks. In the model, the mass splitting of the different isospin multiplets among the baryons arises partly from mass splittings within the boson and fermion multiplets and partly from a symmetry-breaking interaction. The isospin-conserving mass splittings within the boson and fermion multiplets, as well as the symmetry-breaking interaction, are assumed to be proportional to the hypercharge. Under the assumption that these mass splittings and symmetry-breaking interactions are small, it is found that the seven baryon mass splittings which conserve isospin are given in terms of four parameters, and that the Gell-Mann–Okubo mass formula holds. Some effects of representation mixing are considered.

1. INTRODUCTION

A NUMBER of authors have considered models in which a baryon is assumed to be a bound state of three quarks. In particular, Morpurgo[1] discussed the possibility that the quark-quark interaction might be described by a nonrelativistic potential, even though the binding energies are comparable to the quark masses. At first glance, it is attractive to add to this hypothesis the assumption that the mass differences among the baryon isospin multiplets arise solely from an intrinsic mass splitting of the quark masses themselves, and that the quark-quark interaction is invariant under SU_3. However, this point of view leads to two predictions in contradiction to experiment, as remarked by Dalitz[2] and others. The first of these predictions is that the Σ-Λ mass splitting is zero, and the second is that the Ω-N^* mass difference is $\frac{3}{2}$ times the Ξ-N mass difference. Dalitz,[2] Federman, Rubinstein, and Talmi,[3] and others therefore assumed that the quark-quark interaction must break the symmetry.

In this paper, we propose an alternative model which gives the seven mass splittings (neglecting electromagnetic effects) of the baryon octet and decuplet in terms of four parameters. In this model a baryon of the octet or decuplet is a bound state of a boson of spin 1

* Work supported in part by the U. S. National Science Foundation.
[1] G. Morpurgo, Physics 1, 95 (1965).
[2] R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Berkshire, England, 1966), p. 157.
[3] P. Federman, H. R. Rubinstein, and I. Talmi, Phys. Letters 22, 208 (1966).

and SU_3 multiplicity 6, and a fermion of spin $\frac{1}{2}$ and SU_3 multiplicity 3. The boson and fermion each are assumed to have strong-interaction mass splittings proportional to the hypercharge. We obtain the Gell-Mann–Okubo octet mass formula and the decuplet equal-spacing rule by assuming that the interaction breaks the symmetry in a simple way: namely, the symmetry-breaking term is proportional to the hypercharge. Thus, all SU_3 symmetry-breaking effects which conserve isospin are taken to be proportional to the hypercharge.

One reason for our taking a boson-fermion model rather than a quark model is a desire to obtain a baryon as a bound state of two particles rather than three. Although a boson-fermion model is conceptually more complicated than the three-quark model, it is simpler from a computational standpoint, and we can explore in more detail the effects of symmetry breaking. Specifically, we include effects of symmetry breaking on the purity of the SU_3 octet and decuplet representations. It turns out that the particular representation-mixing we have considered does not improve the agreement with experiment, and that therefore, with respect to this mixing, the physical baryon states are relatively pure.

In short, we obtain essentially the same results for the isospin-conserving baryon mass splittings with the boson-fermion model as have been obtained with the quark model. Other predictions, however, are different. For example, the level density of higher mass states is smaller in the two-particle model. However, we shall not treat the higher-mass states in this paper, since our primary purpose is to introduce the model and to illustrate its use by calculating baryon mass splittings.

We do not discuss the problem of saturation of the forces. The question of why a boson and two fermions are not deeply bound is analogous to the problem of why four quarks are not deeply bound, which has been discussed by Morpurgo.[4] Also, we do not discuss the mesons in any detail. The reason is that to obtain agreement with experiment, it is simplest to assume that a meson corresponds to a bound fermion-antifermion state. (The boson-antiboson states are assumed to be higher in energy.) Such a model is indistinguishable from the usual quark model in its predictions of low-energy meson states. However, if a multiplet of 27 mesons should be discovered, the situation will change. This is because such a multiplet would be contained in a four-particle state of two quarks and two antiquarks.

2. PROPERTIES OF THE BOSON AND FERMION

A boson-fermion model of baryons can be considered as inspired by the quark model. In this case the fermion triplet has the quantum numbers of a quark triplet, and the boson sextet has the quantum numbers of a bound state of two quarks. From SU_3, we obtain that

TABLE I. Quantum numbers of a boson sextet b_i and fermion triplet f_j in a quark-inspired model. Here I and I_3 are the isospin and third component, Y is the hypercharge, Q the charge, J the spin and B the baryon number.

Symbol	Mass	I	I_3	Y	Q	J	B
f_1	m_f	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
f_2	m_f	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
f_3	$m_f+\delta$	0	0	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
b_1	m_b	1	1	$\frac{2}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$
b_2	m_b	1	0	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$
b_3	m_b	1	-1	$\frac{2}{3}$	$-\frac{2}{3}$	1	$\frac{2}{3}$
b_4	$m_b+\Delta$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$
b_5	$m_b+\Delta$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	1	$\frac{2}{3}$
b_6	$m_b+2\Delta$	0	0	$-\frac{4}{3}$	$-\frac{2}{3}$	1	$\frac{2}{3}$

the nine quark-quark states reduce to the direct sum of a six- and a three-dimensional representation:

$$3\otimes 3=6\oplus\bar{3}. \quad (1)$$

If we add the assumption that in the spin-one state of the two quarks the potential is attractive in the sextet representation, we have the boson sextet. The boson triplet, belonging to the $\bar{3}$ representation, can be assumed to be somewhat higher in energy. It will therefore contribute to baryon states of higher mass, but not to the baryon octet and decuplet. The quantum numbers of the boson and fermion in the quark-inspired model are given in Table I. Note from the table that we assume that the masses of the particles have a term proportional to the hypercharge. However, because of the effects of binding, we do *not* assume that the boson mass m_b is twice the fermion mass m_f, or that the boson mass splitting parameter Δ is equal to the fermion mass splliting parameter δ.

An alternative and perhaps less-attractive description of the model is in terms of two different fields, as discussed in a different connection by Gürsey, Lee, and Nauenberg[5] and others. In this case, we do not need particles with fractional charge, hypercharge, and baryon number. Possible quantum numbers of the boson and fermion in this model are given in Table II. Here the quantum numbers of the fermion triplet, except for masses, are taken to be the same as those of the proton, neutron, and Λ. The quantum numbers of the first five members of the boson sextet, except for the masses, are taken to be the same as those of the ρ and $\bar{K}^*$ mesons, but there is no known meson with the quantum numbers of b_6 ($Y=-2, I=0$),

Table II gives only one of many possibilities for the quantum numbers of the boson and fermion in a two-field model. In general, if the boson has baryon number B, the fermion has baryon number $1-B$. Likewise, if the isospin triplet of the boson has hypercharge Y, the boson isospin doublet and singlet have hypercharge

[4] G. Morpurgo, Phys. Letters **20**, 684 (1966).

[5] F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964).

$Y-1$ and $Y-2$, respectively. Then the fermion isospin doublet has hypercharge $-Y+1$ and the isospin singlet has hypercharge $-Y$. Here B and Y are arbitrary. The charge of any boson or fermion state is given by the usual Gell-Mann–Nishijima formula $Q=I_3+Y/2$.

In all subsequent considerations of this paper, it is irrelevant whether we start from the quark-inspired or the two-field model. In either case, the baryons will have the same quantum numbers.

With six bosons and three fermions, we have 18 states, which are just sufficient to treat the baryon octet and decuplet. The SU_3 numerology is

$$6\otimes 3=10\oplus 8. \tag{2}$$

We shall treat higher-energy baryon states, arising either from $6\otimes 3$ or $\bar{3}\otimes 3$, in a subsequent paper.

3. THE POTENTIAL MODEL

In our model we assume that the potential between boson and fermion has a large term which is SU_3-invariant plus a small term which is proportional to the hypercharge. The large term $V_{2J}{}^n$ is allowed to depend on both the total spin J of the system and the particular representation n of SU_3 in which the particles find themselves. We write the small term $Yv_{2J}{}^n$ to show explicitly that it is proportional to the hypercharge. It also depends on J and n, and is the same for all states with a given J, n, and Y. Such a potential is most easily treated using eigenstates of SU_3. We denote these states by $X^n(I,I_3,Y)$.

However, the masses and kinetic energies of the boson and fermion are not necessarily diagonal in a representation in which the states are eigenstates of SU_3. Rather, they are diagonal in the product states, which we denote by b_if_j ($i=1,2,\cdots,6$; $j=1,2,3$). If we use the states b_if_j, the potential energy is not diagonal.

We find it preferable to consider the problem with the mass and kinetic energy terms diagonal and the potential not diagonal, rather than the other way around. Therefore, in those cases in which the kinetic energy is not diagonal in the SU_3 eigenstates, we expand our wave functions in the states b_if_j, or rather in linear combinations of such states corresponding to definite values of the boson mass, the fermion mass, and the isospin. We denote these states by $\chi(I_b,I_f,I,I_3,Y)$. The relations between the product states b_if_j, the SU_3 eigenstates $X^n(I,I_3,Y)$, and the isospin eigenstates $\chi(I_b,I_f,I,I_3,Y)$ are given in the Appendix.

Since angular momentum is conserved, there is no coupling between the members of the baryon octet ($J=\frac{1}{2}$) and members of the decuplet ($J=\frac{3}{2}$). However, for a given J there may be coupling between the members of an 8 and a 10 representation. Thus, for example, the physical baryons may not be purely members of an octet, but may have some small admixture of a spin-$\frac{1}{2}$ decuplet which lies much higher in energy.

TABLE II. Possible quantum numbers of a boson sextent b_i and fermion triplet f_i in a two-field model.

Symbol	Mass	I	I_3	Y	Q	J	B
f_1	m_f	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	1
f_2	m_f	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	1
f_3	$m_f+\delta$	0	0	0	0	$\frac{1}{2}$	1
b_1	m_b	1	1	0	1	1	0
b_2	m_b	1	0	0	0	1	0
b_3	m_b	1	-1	0	-1	1	0
b_4	$m_b+\Delta$	$\frac{1}{2}$	$\frac{1}{2}$	-1	0	1	0
b_5	$m_b+\Delta$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	1	0
b_6	$m_b+2\Delta$	0	0	-2	-1	1	0

With our assumptions, we can write down the potential between a boson and fermion as follows:

$$V=(V_{2J}{}^{10}+Yv_{2J}{}^{10})P^{10}+(V_{2J}{}^{8}+Yv_{2J}{}^{8})P^{8}, \tag{3}$$

where P^{10} and P^8 are projection operators on the ten- and eight-dimensional representations, respectively.

4. EQUATIONS FOR THE BARYON MASSES

We assume that the baryon masses are given by solutions to a Schrödinger equation with the potentials described in the previous section. Although later we shall assume that the kinetic energy is small, we shall use the Klein-Gordon equation to describe the motion.[6] Then the Hamiltonian H_0 without interaction is given by

$$\begin{aligned}H_0=&[(p^2+m_b{}^2)^{1/2}+(p^2+m_f{}^2)^{1/2}]P_1\\&+[(p^2+(m_b+\Delta)^2)^{1/2}+(p^2+m_f{}^2)^{1/2}]P_2\\&+[(p^2+(m_b+2\Delta)^2)^{1/2}+(p^2+m_f{}^2)^{1/2}]P_3\\&+[(p^2+m_b{}^2)^{1/2}+(p^2+(m_f+\delta)^2)^{1/2}]P_4\\&+[(p^2+(m_b+\Delta)^2)^{1/2}+(p^2+(m_f+\delta)^2)^{1/2}]P_5\\&+[(p^2+(m_b+2\Delta)^2)^{1/2}+(p^2+(m_f+\delta)^2)^{1/2}]P_6,\end{aligned} \tag{4}$$

where $\mathbf{p}$ is the relative momentum of the boson and fermion and the $P_i(i=1,\cdots,6)$ are projection operators. The projector P_1 is for the states b_if_j ($i=1, 2, 3$; $j=1,2$) or any linear combination of such states; likewise P_2 is the projector for any combination of the states ($i=4,5$; $j=1,2$); P_3 for ($i=6$; $j=1,2$); P_4 for ($i=1,2,3$; $j=3$); P_5 for ($i=4,5$; $j=3$); and P_6 for ($i=6$, $j=3$). The states b_if_j and $\chi(I_b,I_f,I,I_3,Y)$ are always eigenstates of H_0. If a state $X^n(I,I_3,Y)$ happens also to be an eigenstate of H_0, we treat the problem with this state, but if not, we expand the wave function in the states $\chi(I_b,I_f,I,I_3,Y)$.

We let the space wave function of the system be denoted by the symbol ψ. Then, if $X^n(I,I_3,Y)$ is an eigenstate of H_0, the total wave function Ψ, including unitary spin indices, is given by

$$\Psi=\psi X^n(I,I_3,Y). \tag{5}$$

[6] L. J. Tassie and D. B. Lichtenberg, Australian J. Phys. 19, 599 (1966).

On the other hand, if $X^n(I,I_3,Y)$ is not an eigenstate of H_0, the problem is not diagonal in either the states $X^n(I,I_3,Y)$ or the states $\chi(I_b,I_f,I,I_3,Y)$. We then expand in the states $\chi(I_b,I_f,I,I_3,Y)$, obtaining for Ψ

$$\Psi=\sum_{\alpha=1}^{2}\psi_\alpha\chi(I_{b\alpha},I_{f\alpha},I,I_3,Y). \tag{6}$$

We have written only two terms, since it turns out that is the maximum number needed in our considerations.

The eigenvalue problem to be solved is

$$(H_0+V)\Psi=B\Psi, \tag{7}$$

where the eigenvalue B is the mass of the appropriate baryon. (We use the symbol for a baryon to denote its mass.)

It can be seen from the wave functions in the Appendix and the mass values of the boson and fermion (Table I or II) that H_0 is diagonal in the $X^n(I,I_3,Y)$ representation for N, Λ, N^*, and Ω.

Substituting (3), (4), and (5) in (7), and taking the scalar product of the result with the appropriate $X^n(I,I_3,Y)$, we obtain

$$[(p^2+m_b^2)^{1/2}+(p^2+m_f^2)^{1/2}+V_1^8+v_1^8]\psi=N\psi, \tag{8}$$

$$[(p^2+(m_b+\Delta)^2)^{1/2}+(p^2+m_f^2)^{1/2}+V_1^8]\psi=\Lambda\psi, \tag{9}$$

$$[(p^2+m_b^2)^{1/2}+(p^2+m_f^2)^{1/2}+V_3^{10}+v_3^{10}]\psi=N^*\psi, \tag{10}$$

$$[(p^2+(m_b+2\Delta)^2)^{1/2}+(p^2+(m_f+\delta)^2)^{1/2}+V_3^{10}-2v_3^{10}]\psi=\Omega\psi. \tag{11}$$

Equations (8), (9), (10), and (11) must be solved to give the masses of N, Λ, N^*, and Ω, respectively.

To obtain the equations for Σ, Ξ, Y^*, and Ξ^*, we substitute (3), (4), and (6), in (7). The states $\chi(I_b,I_f,I,I_3,Y)$ are eigenfunctions of the kinetic energy projectors $P_i(i=1,\cdots,6)$. To carry out the projection operations in the expression for the potential, we write the $\chi(I_{b\alpha},I_{f\alpha},I,I_3,Y)$ states in terms of the states $X^n(I,I_3,Y)$ which are eigenstates of the potential projectors P^8 and P^{10}. After operating with these projection operators, we re-express the $X^n(I,I_3,Y)$ in terms of the $\chi(I_{b\alpha},I_{f\alpha},I,I_3,Y)$. Finally we take the scalar product of the result with the $\chi(I_{b\beta},I_{f\beta},I,I_3,Y)$, where $\beta=1$, 2, in turn. We obtain two coupled equations each for Σ, Ξ, Y^*, and Ξ^*.

For Σ the equations are

$$[(p^2+m_b^2)^{1/2}+(p^2+(m_f+\delta)^2)^{1/2}+\tfrac{1}{3}V_1^{10}+\tfrac{2}{3}V_1^8]\psi_\alpha+\tfrac{1}{3}\sqrt{2}(V_1^{10}-V_1^8)\psi_\beta=\Sigma\psi_\alpha, \tag{12a}$$

$$[(p^2+(m_b+\Delta)^2)^{1/2}+(p^2+m_f^2)^{1/2}+\tfrac{2}{3}V_1^{10}+\tfrac{1}{3}V^8]\psi_\beta+\tfrac{1}{3}\sqrt{2}(V_1^{10}-V_1^8)\psi\alpha=\Sigma\psi_\beta. \tag{12b}$$

The same equations hold for Y^* with $\Sigma\to Y^*$ and

$$V_1^{10}\to V_3^{10},\quad V_1^8\to V_3^8. \tag{13}$$

The equations for Ξ are

$$[(p^2+(m_b+\Delta)^2)^{1/2}+(p^2+(m_f+\delta)^2)^{1/2}+\tfrac{2}{3}V_1^{10}+\tfrac{1}{3}V_1^8-\tfrac{2}{3}v_1^{10}-\tfrac{1}{3}v_1^8]\psi_\alpha+\tfrac{1}{3}\sqrt{2}(V_1^{10}-V_1^8-v_1^{10}+v_1^8)\psi_\beta=\Xi\psi_\alpha, \tag{14a}$$

$$[(p^2+(m_b+2\Delta)^2)^{1/2}+(p^2+m_f^2)^{1/2}+\tfrac{1}{3}V_1^{10}+\tfrac{2}{3}V_1^8-\tfrac{1}{3}v_1^{10}-\tfrac{2}{3}v_1^8]\psi_\beta+\tfrac{1}{3}\sqrt{2}(V_1^{10}-V_1^8-v_1^{10}+v_1^8)\psi_\alpha=\Xi\psi_\beta. \tag{14b}$$

These equations also hold for Ξ^* with $\Xi\to\Xi^*$ and

$$V_1^{10}\to V_3^{10},\quad V_1^8\to V_3^8,\quad v_1^{10}\to v_3^{10},\quad v_1^8\to v_3^8. \tag{15}$$

5. APPROXIMATE SOLUTIONS

In principle, Eqs. (8)–(15) can be solved for the eight different baryon masses once m_b, m_f, Δ, δ, V_1^{10}, V_1^8, V_3^{10}, V_3^8, v_1^{10}, v_1^8, v_3^{10}, and v_3^8 are given. However, such a solution would not be meaningful, since there are more parameters than masses. What we shall do, therefore, is to obtain approximate solutions for the mass differences which are relatively independent of most of the parameters.

We assume that all potentials are square wells of radius a. Furthermore, we assume that the masses m_b and m_f and the potentials V_{2j}^n are large compared with the baryon masses and compared with Δ, δ, and v_{2j}^n. Finally, we assume that the conditions of the problem are such that it is a good approximation to assume that the wave function vanishes at $r=a$. This approximation is a good one if $a\gg 1/m_b$ and $a\gg 1/m_f$. This condition also allows one to use a nonrelativistic expression for the kinetic energy, but we do not find it necessary to do so.

The vanishing of the wave function at $r=a$ means that the momentum p is a constant which depends on the radius a ($p=\pi/a$ in an S state), but is independent of all other parameters in the problem.

Under these conditions, Eqs. (8)–(15) can be treated as algebraic equations. In particular, assuming an S-wave bound state, the mass of the nucleon is given by

$$N=(p^2+m_b^2)^{1/2}+(p^2+m_f^2)^{1/2}+V_1^8+v_1^8. \tag{16}$$

The mass differences Λ-N, N^*-N, and Ω-N^* are then obtained from (9), (10), (11), and (16). We expand the square roots in these equations to first order in the parameters x and y defined by

$$2x=m_f\delta(p^2+m_f^2)^{-1/2}+m_b\Delta(p^2+m_b^2)^{-1/2},$$
$$2y=m_f\delta(p^2+m_f^2)^{-1/2}-m_b\Delta(p^2+m_b^2)^{-1/2}. \tag{17}$$

We obtain

$$\Lambda-N=x-y-v_1^8, \tag{18}$$

$$N^*-N=V_3^{10}-V_1^8+v_3^{10}-v_1^8, \tag{19}$$

$$\Omega-N^*=3x-y-3v_3^{10}. \tag{20}$$

The masses of the Σ, Ξ, Y^*, and Ξ^* are each obtained by diagonalizing a pair of coupled equations. Each set of two coupled equations has two roots. The lower one is taken to be the mass of the baryon. We introduce the notation

$$U_1=\tfrac{1}{2}(V_1{}^{10}-V_1{}^8),\quad u_1=\tfrac{1}{2}(v_1{}^{10}-v_1{}^8),$$
$$U_3=\tfrac{1}{2}(V_3{}^8-V_3{}^{10}),\quad u_3=\tfrac{1}{2}(v_3{}^8-v_3{}^{10}). \tag{21}$$

The experimental fact that the lowest $J=\frac{1}{2}$ representation is an octet and the lowest $J=\frac{3}{2}$ representation is a decuplet shows that $V_1{}^8$ and $V_3{}^{10}$ are deeply attractive (negative). Therefore, we take $U_1>0$ and $U_3>0$. With this notation, we obtain the following expressions for the mass differences:

$$\Sigma-\Lambda=y+U_1-U_1\{1-\tfrac{2}{3}(y/U_1)+(y/U_1)^2\}^{1/2}, \tag{22}$$

$$\Xi-N=2x-y+U_1-u_1-2v_1{}^8-(U_1-u_1)\times\{1+\tfrac{2}{3}[y/(U_1-u_1)]+[y/(U_1-u_1)]^2\}^{1/2}, \tag{23}$$

$$Y^*-N^*=x+U_3-v_3{}^{10}-U_3\{1+\tfrac{2}{3}(y/U_3)+(y/U_3)^2\}^{1/2}, \tag{24}$$

$$\Xi^*-N^*=2x-y+U_3-u_3-2v_3{}^{10}-(U_3-u_3)\times\{1-\tfrac{2}{3}[y/(U_3-u_3)]+[y/(U_3-u_3)]^2\}^{1/2}. \tag{25}$$

It is interesting to see whether the expressions (18)–(20) and (21)–(25) can give agreement with the observed baryon mass splittings in the absence of a symmetry-breaking potential. We find this is not the case, as can be seen by the following argument. If we put $v_{2J}{}^n=0$ in the expressions for the mass differences, we obtain the following equations (among others which we do not need):

$$\Lambda-N=x-y, \tag{26}$$

$$\Omega-N^*=3x-y, \tag{27}$$

$$\Sigma-\Lambda=y+U_1-U_1\{1-\tfrac{2}{3}(y/U_1)+y^2/U_1{}^2\}^{1/2}. \tag{28}$$

Using the experimental mass differences ($\Lambda-N=176$ MeV, $\Omega-N^*=440$ MeV), we find from (26) and (27) that

$$x=132\text{ MeV},\quad y=-44\text{ MeV}.$$

Then, using $\Sigma-\Lambda=78$ MeV, we obtain from Eq. (28)

$$122\text{ MeV}=U_1-U_1\{1+\tfrac{2}{3}|y|/U_1+(y/U_1)^2\}^{1/2}. \tag{29}$$

But this equation cannot be satisfied for any positive U_1, since the square root on the right-hand side is greater than 1. But if U_1 is negative, a spin-$\frac{1}{2}$ baryon *decuplet* will bind more deeply than the baryon octet, in contradiction to experiment. Thus, representation mixing in the absence of a symmetry-breaking interaction is not sufficient in this model to lead to predictions in agreement with experiment.

We next explore the consequences of the assumption that U_1 and U_3 are large compared to the symmetry-breaking terms x, y, u_1, and u_3. This means that the potentials $V_1{}^{10}$ and $V_3{}^8$ are not deeply attractive, a reasonable assumption in view of the experimental fact that no low-mass baryon spin-$\frac{1}{2}$ decuplet or spin-$\frac{3}{2}$ octet has been seen. Under these conditions, we expand the square roots in (22), (23), (24), and (25) to first order in y/U_1, $y/(U_1-u_1)$, y/U_3, and $y/(U_3-u_3)$, respectively. We obtain

$$\Sigma-\Lambda=\tfrac{4}{3}y, \tag{30}$$

$$\Xi-N=2x-\tfrac{4}{3}y-2v_1{}^8, \tag{31}$$

$$Y^*-N^*=x-\tfrac{1}{3}y-v_3{}^{10}, \tag{32}$$

$$\Xi^*-N^*=2x-\tfrac{2}{3}y-2v_3{}^{10}. \tag{33}$$

Equations (18)–(20) and (30)–(33) give the seven mass differences in terms of four parameters: namely, y and the following three linear combinations of the parameters introduced previously:

$$\xi=x-\tfrac{1}{3}y-v_3{}^{10},$$
$$\eta=x-v_1{}^8, \tag{34}$$
$$\zeta=V_3{}^{10}-V_1{}^8+v_3{}^{10}-v_1{}^8.$$

It is apparent from Eqs. (18)–(20) and (30)–(33) that the decuplet masses satisfy the equal spacing rule and the octet masses satisfy the Gell-Mann–Okubo relation.

Our result obtained by expanding the square root is the same as the one we would have obtained if we had done the calculation allowing no representation mixing from the start. This is simply the perturbation theory result.

6. DISCUSSION

Our two-particle model is somewhat more flexible than the quark model in that it contains two mass splitting parameters rather than one. For this reason it gives an intrinsic $\Sigma-\Lambda$ mass splitting even in the absence of a symmetry-breaking potential. However, in the absence of a symmetry-breaking potential, the model leads to Eq. (29), which cannot be satisfied with reasonable parameters. In fact, our model calculated in perturbation theory without symmetry breaking leads to the following relation between the decuplet and octet spacings:

$$\Omega-N^*=\tfrac{3}{2}(\Xi-N)+\tfrac{3}{4}(\Sigma-\Lambda). \tag{35}$$

This result is analogous to the quark-model prediction,

$$\Omega-N^*=\tfrac{3}{2}(\Xi-N), \tag{36}$$

because in the quark model without symmetry breaking, we have $\Sigma-\Lambda=0$. The boson-fermion model is able to give a positive $\Sigma-\Lambda$ mass difference, but the resulting Eq. (35) disagrees with experiment even more violently than Eq. (36). Thus, both the boson-fermion model and the quark model appear to require a symmetry-breaking interaction. The symmetry-breaking potential which destroys this unwanted relation can depend just on the hypercharge in the two-particle boson-fermion model, rather than also on the isospin, as it must in the quark model to give a $\Sigma-\Lambda$ mass splitting.

We have introduced our model partly in order to be able to calculate the effects of representation mixing in a simple way. However, one of the results of the calculation is negative: Under the assumptions of the model, no amount of representation mixing can give the experimental baryon mass differences in the absence of a symmetry-breaking interaction. However, we have neglected other possible types of mixing. For example, we have assumed that a baryon is a bound S state of a boson and fermion. It could equally well be a bound D state or, more likely, a mixture of the two.

Our final result, that with a symmetry-breaking interaction proportional to the hypercharge, the mass splittings are given in terms of four parameters, is not startling. We get the familiar decuplet equal spacing rule and the Gell-Mann–Okubo octet mass formula, but nothing more. The same results could have been derived from the model with considerably less work using perturbation theory plus group transformation properties. But we would not have been able to explore in detail the consequences of higher-order effects.

Finally, we wish to stress that we attach no fundamental significance to the boson and fermion of our model. If any bound-state model of baryons makes sense at all, the quark model appears to be preferred because it is conceptually the simplest. From this point of view, the predictions of the boson-fermion model result from the quark model with the added dynamical assumption that it is a good approximation to consider the interaction of a quark with a bound state of two quarks. Such might be the case if a bound state of two quarks has a smaller mass than that of a single quark. This is in contrast to the usual approximations made in treating a three-quark system.

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APPENDIX

We give here the relationship between the SU_3 eigenstates $X^n(I,I_3,Y)$, the states $\chi(I_b,I_f,I,I_3,Y)$, and the states b_if_j. Neglecting representation mixing due to symmetry breaking, the states $X^n(I,I_3,Y)$ correspond to the members of the baryon octet and decuplet. This correspondence is also given.

$$N^{*++}=X^{10}(\tfrac{3}{2}\;\tfrac{3}{2}\;1)=b_1f_1=\chi(1\;\tfrac{1}{2}\;\tfrac{3}{2}\;\tfrac{3}{2}\;1)\,,$$
$$N^{*+}=X^{10}(\tfrac{3}{2}\;\tfrac{1}{2}\;1)=(\sqrt{\tfrac{1}{3}})b_1f_2+(\sqrt{\tfrac{2}{3}})b_2f_1=\chi(1\;\tfrac{1}{2}\;\tfrac{3}{2}\;\tfrac{1}{2}\;1)\,,$$
$$N^{*0}=X^{10}(\tfrac{3}{2}\;-\tfrac{1}{2}\;1)=(\sqrt{\tfrac{2}{3}})b_2f_2+(\sqrt{\tfrac{1}{3}})b_3f_1=\chi(1\;\tfrac{1}{2}\;\tfrac{3}{2}\;-\tfrac{1}{2}\;1)\,,$$
$$N^{*-}=X^{10}(\tfrac{3}{2}\;-\tfrac{3}{2}\;1)=b_3f_2=\chi(1\;\tfrac{1}{2}\;\tfrac{3}{2}\;-\tfrac{3}{2}\;1)\,,$$
$$Y^{*+}=X^{10}(1\;1\;0)=(\sqrt{\tfrac{1}{3}})b_1f_3+(\sqrt{\tfrac{2}{3}})b_4f_1=(\sqrt{\tfrac{1}{3}})\chi(1\;0\;1\;1\;0)+(\sqrt{\tfrac{2}{3}})\chi(\tfrac{1}{2}\;\tfrac{1}{2}\;1\;1\;0)\,,$$
$$Y^{*0}=X^{10}(1\;0\;0)=(\sqrt{\tfrac{1}{3}})(b_2f_3+b_4f_2+b_5f_1)=(\sqrt{\tfrac{1}{3}})\chi(1\;0\;1\;0\;0)+(\sqrt{\tfrac{2}{3}})\chi(\tfrac{1}{2}\;\tfrac{1}{2}\;1\;0\;0)\,,$$
$$Y^{*-}=X^{10}(1\;-1\;0)=(\sqrt{\tfrac{1}{3}})b_3f_3+(\sqrt{\tfrac{2}{3}})b_5f_2=(\sqrt{\tfrac{1}{3}})\chi(1\;0\;1\;-1\;0)+(\sqrt{\tfrac{2}{3}})\chi(\tfrac{1}{2}\;\tfrac{1}{2}\;1\;-1\;0)\,,$$
$$\Xi^{*0}=X^{10}(\tfrac{1}{2}\;\tfrac{1}{2}\;-1)=(\sqrt{\tfrac{2}{3}})b_4f_3+(\sqrt{\tfrac{1}{3}})b_6f_1=(\sqrt{\tfrac{2}{3}})\chi(\tfrac{1}{2}\;0\;\tfrac{1}{2}\;\tfrac{1}{2}\;-1)+(\sqrt{\tfrac{1}{3}})\chi(0\;\tfrac{1}{2}\;\tfrac{1}{2}\;\tfrac{1}{2}\;-1)\,,$$
$$\Xi^{*-}=X^{10}(\tfrac{1}{2}\;-\tfrac{1}{2}\;-1)=(\sqrt{\tfrac{2}{3}})b_5f_3+(\sqrt{\tfrac{1}{3}})b_6f_2=(\sqrt{\tfrac{2}{3}})\chi(\tfrac{1}{2}\;0\;\tfrac{1}{2}\;-\tfrac{1}{2}\;-1)+(\sqrt{\tfrac{1}{3}})\chi(0\;\tfrac{1}{2}\;\tfrac{1}{2}\;-\tfrac{1}{2}\;-1)\,,$$
$$\Omega^{-}=X^{10}(0\;0\;-2)=b_6f_3=\chi(0\;0\;0\;0\;-2)\,,$$
$$p=X^{8}(\tfrac{1}{2}\;\tfrac{1}{2}\;1)=(\sqrt{\tfrac{2}{3}})b_1f_2-(\sqrt{\tfrac{1}{3}})b_2f_1=\chi(1\;\tfrac{1}{2}\;\tfrac{1}{2}\;\tfrac{1}{2}\;1)\,,$$
$$n=X^{8}(\tfrac{1}{2}\;-\tfrac{1}{2}\;1)=(\sqrt{\tfrac{1}{3}})b_2f_2-(\sqrt{\tfrac{2}{3}})b_3f_1=\chi(1\;\tfrac{1}{2}\;\tfrac{1}{2}\;-\tfrac{1}{2}\;1)\,,$$
$$\Lambda^{0}=X^{8}(0\;0\;0)=(\sqrt{\tfrac{1}{2}})(b_4f_2-b_5f_1)=\chi(\tfrac{1}{2}\;\tfrac{1}{2}\;0\;0\;0)\,,$$
$$\Sigma^{+}=X^{8}(1\;1\;0)=(\sqrt{\tfrac{2}{3}})b_1f_3-(\sqrt{\tfrac{1}{3}})b_4f_1=(\sqrt{\tfrac{2}{3}})\chi(1\;0\;1\;1\;0)-(\sqrt{\tfrac{1}{3}})\chi(\tfrac{1}{2}\;\tfrac{1}{2}\;1\;1\;0)\,,$$
$$\Sigma^{0}=X^{8}(1\;0\;0)=(\sqrt{\tfrac{1}{6}})(2b_2f_3-b_4f_2-b_5f_1)=(\sqrt{\tfrac{2}{3}})\chi(1\;0\;1\;0\;0)-(\sqrt{\tfrac{1}{3}})\chi(\tfrac{1}{2}\;\tfrac{1}{2}\;1\;0\;0)\,,$$
$$\Sigma^{-}=X^{8}(1\;-1\;0)=(\sqrt{\tfrac{2}{3}})b_3f_3-(\sqrt{\tfrac{1}{3}})b_5f_2=(\sqrt{\tfrac{2}{3}})\chi(1\;0\;1\;-1\;0)-(\sqrt{\tfrac{1}{3}})\chi(\tfrac{1}{2}\;\tfrac{1}{2}\;1\;-1\;0)\,,$$
$$\Xi^{0}=X^{8}(\tfrac{1}{2}\;\tfrac{1}{2}\;-1)=(\sqrt{\tfrac{1}{3}})b_4f_3-(\sqrt{\tfrac{2}{3}})b_6f_1=(\sqrt{\tfrac{1}{3}})\chi(\tfrac{1}{2}\;0\;\tfrac{1}{2}\;\tfrac{1}{2}\;-1)-(\sqrt{\tfrac{2}{3}})\chi(0\;\tfrac{1}{2}\;\tfrac{1}{2}\;\tfrac{1}{2}\;-1)\,,$$
$$\Xi^{-}=X^{8}(\tfrac{1}{2}\;-\tfrac{1}{2}\;-1)=(\sqrt{\tfrac{1}{3}})b_5f_3-(\sqrt{\tfrac{2}{3}})b_6f_2=(\sqrt{\tfrac{1}{3}})\chi(\tfrac{1}{2}\;0\;\tfrac{1}{2}\;-\tfrac{1}{2}\;-1)-(\sqrt{\tfrac{2}{3}})\chi(0\;\tfrac{1}{2}\;\tfrac{1}{2}\;-\tfrac{1}{2}\;-1)\,.$$