Meson Decays in Broken $SU(3)$ and Current Commutation Relations*

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Assuming that the symmetry-breaking Hamiltonian is given by the space integral of the eighth component of the scalar-quark current density, the first-order renormalization of the vector and tensor meson couplings are calculated in the formalism of the algebra of currents. The results are compared with experimental widths and the agreement is found to be good.

I. INTRODUCTION

It is well known[1,2] that $SU(3)$ symmetric calculations including octet-singlet mixing can explain the qualitative features of 1^- and 2^+ meson decay widths. On the other hand, quantitatively the disagreement between the calculation and the experimental numbers is sufficiently large as to imply that the first-order corrections to the $SU(3)$ symmetric calculations are important. In addition one notes that octet-singlet mixing itself being a symmetry-breaking phenomenon, it is only consistent that one should use broken $SU(3)$ coupling constants. Vector meson decays are of particular interest since it has been shown[3] that if one equates the vector-current coupling to the neutral vector mesons, with the electromagnetic currents, then the vector currents are unrenormalized to first order in symmetry breaking if the latter is assumed to transform as the eighth component of an octet.

There are several ways of calculating symmetry-breaking effects using different techniques of varying reliability. For example the symmetry-breaking effects for vector-meson decays have already been considered[4] in the framework of algebra of currents, in which the corrections are expressed in terms of vector-meson and pseudoscalar-meson scattering cross sections. In this paper we use the procedure of Bose and Hara[5] who assume that the symmetry-breaking Hamiltonian is given by the space integral of a scalar density which transforms as the eighth component of an octet and use equal-time commutation relations for simplifying the matrix elements. We apply this technique to calculate the decays of vector and tensor mesons. It is then found that our predictions for decays of 1^- and 2^+ mesons into two pseudoscalar mesons follow very closely those of an entirely different approach[6] which utilizes the assumption of universality of photon and graviton couplings and the dominance of their form factors by the vector and tensor-meson poles. This suggests that our approach is basically correct. We should note that we do not assume that the vector currents coupled to vector mesons and the electromagnetic current belong to the same multiplet so that the results of Gatto and Ademollo[3] are not applicable here.

II. BROKEN-SYMMETRY VERTEX

In this section we consider the first-order symmetry-breaking corrections to the vector and tensor meson decays into two pseudoscalar mesons. Assuming the symmetry-breaking interaction to be proportional to S_8, the space integral of the eighth component of quark-scalar density, we obtain for the symmetry-breaking term

$$\langle M^k|\int d^3x\, \bar{q}(\mathbf{x},t)\lambda_8 q(\mathbf{x},t)\,|p_i p_j\rangle\,, \tag{1}$$

where M^k stands for either a vector or tensor meson belonging to an octet or a singlet and P_i's for the pseudoscalar mesons belonging to an octet. We now *disperse* the P_i's in the usual manner (see, e.g., Ref. 5) and make use of the PCAC (partially conserved axial-vector current) hypothesis, namely

$$\partial_\mu J_\mu{}^k = C_k P_k\,, \tag{2}$$

where C_k is the PCAC constant. Using the equal-time commutation relation[7]

$$[F_i{}^5(t), s_j(t)] = i d_{ijk} S_k{}^5(t)\,, \tag{3}$$

where $F_i{}^5(t)$ is the space integral of the axial current and $S_k{}^5(t)$ is the space integral of the pseudoscalar current density, we finally get[5] for the broken vertex (similar to Ref. 5), in the limit of $q_i \to 0$,

$$G_{ij}{}^k = \left\{ f_0 \begin{pmatrix} 8 & 8 & r \\ i & j & k \end{pmatrix} + \lambda \begin{pmatrix} 8 & 8 & 8_s \\ 0 & i & i \end{pmatrix} \times \begin{pmatrix} 8 & 8 & r \\ i & j & k \end{pmatrix} \right\} \pm \{i \leftrightarrow j\} \cdots\,, \tag{4}$$

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TABLE I. Decay widths of the tensor mesons.

Decay mode	Widths[a] (MeV)	Exptl. widths[b] (MeV)
$f^0 \to \pi\pi$	110 (input)	118 ± 20
$f^0 \to K\bar{K}$	8.5	2 ± 2
$f^0 \to \eta\eta$	0.5	?
$A_2 \to K\bar{K}$	4.5 (input)	4.5 ± 1.5
$A_2 \to \pi\eta$	9.7	3 ± 2
$K^{**} \to K\pi$	40 (input)	39 ± 20
$K^{**} \to K\eta$	1.0	2 ± 2
$f' \to \pi\pi$	0.3	0 ± 6
$f' \to K\bar{K}$	40 (input)	51 ± 25
$f' \to \eta\eta$	11	?

[a] The parameters are $f_0'=0.76$, $\lambda'=0.27$, $g_0=-1.92$, and $\delta=1.08$.
[b] From Ref. 2.

where f_0 is the strength of the $SU(3)$-symmetric coupling, λ a constant proportional to the strength of the symmetry-breaking interaction,[8] r the $SU(3)$ representation to which M belongs ($r=8_s$ for tensor mesons and 8_a for vector mesons); the upper or lower sign is taken for tensor- or vector-meson decays, respectively. This comes from the use of Bose statistics for the final pseudoscalar mesons. We have a similar expression for the decay of the singlet tensor meson with f_0 and λ replaced by g_0 and δ. The corresponding amplitude for the decay of the singlet vector meson is zero. The mixing angle for the octet and the singlet in both the cases is obtained from the experimental masses.[9] Thus we are able to describe the decays of nine tensor mesons into two pseudoscalars in terms of four parameters and the decays of nine vectors into two pseudoscalars, in terms of two parameters. The parameters are determined by using some of the experimental widths and then the other widths are predicted.

III. DECAY WIDTHS

We now apply formula (4) to the decays of vector and tensor mesons.

A. Vector-Meson Decays

The physical ω and ϕ are defined in the usual way;

$$\begin{aligned}\omega&=\omega_0\cos\theta+\omega_8\sin\theta,\\ \phi&=-\omega_0\sin\theta+\omega_8\cos\theta,\end{aligned}\tag{5}$$

where θ is the mixing angle between the singlet and the octet[9] and is taken to be 40°. Then the use of formula (4) of the last section combined with the phase space of p^3/m^2 (p is the c.m. decay momentum and m is the mass of the decaying meson) yields the following relations for the decay widths of the vector mesons into two pseudoscalars;

$$\begin{aligned}\Gamma(\rho\to\pi\pi)&=(f_0+\lambda)^2,\\ \Gamma(K^*\to K\pi)&=0.29(f_0+0.25\lambda)^2,\\ \Gamma(\phi\to K\bar{K})&=0.019(f_0-0.50\lambda)^2.\end{aligned}\tag{6}$$

Taking $\Gamma(K^*\to K\pi)=50$ MeV and $\Gamma(\phi\to K\bar{K})=3.0$ MeV, we get

$$\Gamma(\rho\to\pi\pi)=120\text{ MeV}\tag{7}$$

which is in good agreement with the experimental width in contrast to the 170-MeV width one gets from unbroken $SU(3)$.

B. Tensor-Meson Decays

The interaction Lagrangian for even-parity tensor mesons coupling to two pseudoscalar octets is

$$\mathfrak{L}_I=G_{ij}{}^kT_k{}^{ij},\tag{8}$$

where

$$T_k{}^{ij}=M_k{}^{\mu\nu}\{\tfrac{1}{2}(P^iP^j+P_\mu{}^iP_\nu{}^j)-g_{\mu\nu}\tfrac{1}{2}[(P^i)^\lambda P_\lambda{}^j+\tfrac{1}{2}(m_i^2+m_j^2)P^iP^j]\},\tag{9}$$

where $G_{ij}{}^k$ is defined in (4) with f^0 and λ replaced by f_0' and λ' for $r=8_s$ and by g_0 and δ for $r=1$; $T_k{}^{ij}$ is the form of the coupling vertex between a tensor and the generalized energy momentum tensor for two pseudoscalars i and j; $P_\mu{}^i=\partial_\mu P^i$, P^i, and $M_k{}^{\mu\nu}$ are the pseudoscalar and tensor fields. The phase space coming from such a vertex, for a tensor meson decay into two pseudoscalars, is p^5/m^2 (p is the c.m. decay momentum and m is the mass of the decaying tensor meson). The physical particles f and f' are given by

$$\begin{aligned}f&=f_1\cos\theta+f_8\sin\theta,\\ f'&=-f_1\sin\theta+f_8\cos\theta,\end{aligned}\tag{10}$$

and $\theta\simeq 30°$ as obtained[1] from the physical masses of the tensors. Then using (4), (9), and (10) the decay widths of the tensor mesons into two pseudoscalars are given in Table I.

The interesting thing to note about the predictions for both vector and tensor decays into two pseudoscalars is that they very closely follow the predictions following[6] from the assumption of universality of photon and graviton couplings and the domination of their form factors by the vector and tensor mesons, respectively. The number of parameters in both our approach and that of Ref. 6 is also the same. This seems to indicate that both these approaches are essentially correct.

C. Tensor Decays into a Pseudoscalar and a Vector Meson

The procedure for the decay of a tensor meson k into a vector meson j and a pseudoscalar meson i is very similar to that developed in Sec. II. The final formula is

[8] By assumption f_0 and λ are taken real. For both vector and tensor mesons the same form (4) is applicable but different constants. We will use f_0 and λ for vector mesons and f_0' and λ' for tensor mesons.
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similar to (4) and gives the coupling as[10]

$$h_{rs}\begin{pmatrix}8 & s & r\\ i & j & k\end{pmatrix}+\gamma_{rs}\begin{pmatrix}8 & 8 & 8_s\\ 0 & i & i\end{pmatrix}\begin{pmatrix}8 & s & r\\ i & j & k\end{pmatrix}. \tag{11}$$

If both the tensor meson and the vector meson belong to octets, then C conservation requires that only the antisymmetric coupling is present. We then see that keeping i fixed but varying j and k, the coupling constant (11) transforms like an $SU(3)$ symmetric quantity. Thus there is no renormalization for the ratio[11] of coupling constants

$$\frac{g(A_2\rho\pi)}{g(K^{**}K^*\pi)},\quad \frac{g(K^{**}\rho K)}{g(K^{**}\omega_8 K)}. \tag{12}$$

[10] Note that there is only one pion to disperse.

[11] This result is similar to the one obtained in Ref. 5 for meson couplings with two baryons.

Therefore the ratios of decay widths,

$$\frac{\Gamma(A_2\to\rho\pi)}{\Gamma(K^{**}\to K^*\pi)},\quad \frac{\Gamma(K^{**}\to\rho K)}{\Gamma(K^{**}\to\omega K)} \tag{13}$$

are the same as those calculated by Glashow and Socolow,[1] even after including first-order breaking in the coupling constants. The experimental information about the second ratio is scanty but the first ratio is in good agreement[1] with the experimental number.

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Baryon Mass Splitting in a Boson-Fermion Model*

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A model is proposed in which a baryon consists of a boson and fermion deeply bound in a potential. The fermion can be regarded as a quark and the boson as a bound state of two quarks. In the model, the mass splitting of the different isospin multiplets among the baryons arises partly from mass splittings within the boson and fermion multiplets and partly from a symmetry-breaking interaction. The isospin-conserving mass splittings within the boson and fermion multiplets, as well as the symmetry-breaking interaction, are assumed to be proportional to the hypercharge. Under the assumption that these mass splittings and symmetry-breaking interactions are small, it is found that the seven baryon mass splittings which conserve isospin are given in terms of four parameters, and that the Gell-Mann–Okubo mass formula holds. Some effects of representation mixing are considered.

1. INTRODUCTION

A NUMBER of authors have considered models in which a baryon is assumed to be a bound state of three quarks. In particular, Morpurgo[1] discussed the possibility that the quark-quark interaction might be described by a nonrelativistic potential, even though the binding energies are comparable to the quark masses. At first glance, it is attractive to add to this hypothesis the assumption that the mass differences among the baryon isospin multiplets arise solely from an intrinsic mass splitting of the quark masses themselves, and that the quark-quark interaction is invariant under SU_3. However, this point of view leads to two predictions in contradiction to experiment, as remarked by Dalitz[2] and others. The first of these predictions is that the Σ-Λ mass splitting is zero, and the second is that the Ω-N^* mass difference is $\frac{3}{2}$ times the Ξ-N mass difference. Dalitz,[2] Federman, Rubinstein, and Talmi,[3] and others therefore assumed that the quark-quark interaction must break the symmetry.

In this paper, we propose an alternative model which gives the seven mass splittings (neglecting electromagnetic effects) of the baryon octet and decuplet in terms of four parameters. In this model a baryon of the octet or decuplet is a bound state of a boson of spin 1

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