Regge Cuts and π-p Total Cross Sections*

D. L. PURSEY AND L. SERTORIO†
Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa

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The possible contribution of a cut in the angular-momentum plane to pion-proton total cross sections is investigated. Forward dispersion relations are used to derive a sum rule. The resulting predictions are compared with experimental data. Several models in which a Regge cut gives a large contribution to the total cross section are found to be indistinguishable in their predictions from the conventional model based on Regge poles.

I. INTRODUCTION

FROM the time when high-energy cross-section data were first analyzed in terms of Regge poles, the phenomenology of high-energy physics has been haunted by the fear that branch-point singularities of the scattering amplitude in the complex angular-momentum plane might have to be considered. Despite a great deal of effort, it has not so far been possible either to prove or to disprove the necessity for Regge cuts from the basic analyticity postulates of S-matrix theory. The generally accepted view,[1] however, is that there should be cuts in the angular-momentum plane; this belief is customarily tempered by a hope that the contribution from a cut might be so small as to be negligible compared to the contribution from Regge poles. Such optimism is not, of course based on any wholly conclusive theoretical argument[2]—rather it is adopted as a rationalization for the amount of effort which has been invested in ever more sophisticated fitting of the experimental data using poles alone.

Indeed the very great success which has been achieved, especially recently, in fitting high-energy cross sections with models using a limited number of Regge poles[3–6] might be taken as empirical evidence favoring the view that if cuts exist, they do not contribute significantly to the scattering amplitudes which have been analyzed so far. However, this argument is convincing only if it can be shown that similarly good fits cannot be obtained using a cut with reasonable assumptions about the discontinuity across it. The purpose of this paper is to present the results of a preliminary investigation of this point.

The effect of a cut in the angular-momentum plane on the asymptotic form of the total cross section in the crossed channel has been discussed by a number of authors.[7–10] In this paper we follow the approach of Igi[10] which leads to a slightly more general asymptotic form for the total cross section than has normally been used by other workers. The research reported here also differs from previous work by imposing a sum rule[11] derived from the forward dispersion relations.

In this exploratory investigation we confine attention to the total cross sections for pion-proton scattering, which are related to the forward scattering amplitudes through the optical theorem. This process was selected because of the existence of accurate data, the existence of well-established dispersion relations which allow the elimination of one parameter through use of a sum rule,[11] and because pion-proton scattering has already been thoroughly and successfully analyzed with a Regge-pole model.

As is well known, the total pion-proton cross sections can be fitted extremely well in the energy range from 5 to 20 GeV using a three-pole model. Two of the poles, P and P', have vacuum quantum numbers, while the third has the quantum numbers of the ρ meson. The parameters of the ρ pole appear to be very well determined; however, the parameters of the P and P' poles appear to be more sensitive to the details of the analysis. When nonforward scattering is analyzed, the trajectories of the three poles have been determined as functions of the invariant momentum transfer. When these trajectories are extrapolated into the physical region for the crossed channel, the extrapolated P' and ρ trajectories pass close to the positions of the f_0' and ρ mesons, respectively; however, the P trajectory does not pass close to any known spin-2 particle with the correct quantum numbers.[12]

In this paper we accept the empirical evidence that the ρ trajectory is truly a Regge-pole trajectory. Consequently, we consider the sum $\sigma^+ + \sigma^-$ of the total π^+p and π^-p cross sections. As is well known, the ρ trajectory does not contribute to this combination of cross sections. Since the P and P' parameters appear to be fairly

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† On leave of absence from Istituto di Fisica dell'Università, Rome, Italy.
[1] S. Mandelstam, Nuovo Cimento **30**, 1127 (1963).
[2] See, however, G. F. Chew and V. L. Teplitz, Phys. Rev. **136**, B1154 (1964).
[3] L. Sertorio and M. Toller, Phys. Letters **18**, 191 (1965).
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[8] P. G. O. Freund and R. Oehme, Phys. Letters **5**, 353 (1963).
[9] R. Oehme, in *Strong Interactions and High-Energy Physics* (Oliver and Boyd, Edinburgh, 1964).
[10] K. Igi, Nuovo Cimento **37**, 1815 (1965).
[11] K. Igi, Phys. Rev. **130**, 820 (1963).
[12] G. F. Chew, invited paper at the Washington Meeting of the American Physical Society, April, 1966 (unpublished).

flexible, and since these parameters change quite significantly if an extra pole is included, we consider that the empirical fit of the data by a model using P and P' could be merely simulating the effect of a cut. It is clear that a model based on the poles P, P' and a cut will give no reliable information concerning the existence of a cut contribution, since the data can be very adequately fitted with P and P' alone. Consequently, in our investigation we attempt to replace one of these by a cut. Since the contribution of a pole to the high-energy cross section depends on only two parameters, it is unreasonable to assume a cut contribution depending on more than two parameters—this criterion will guide our assumptions concerning the discontinuity across the cut.

In Sec. II we establish our formalism and derive a sum rule analogous to that of Igi.[11] Sections III and IV compare several models with the experimental data, while the results obtained are fully discussed in Sec. V.

In comparing an asymptotic cross-section formula with experimental data, we are faced with the vexed question of guessing an energy beyond which the asymptotic form may be assumed to be reliable. Somewhat related to this is the choice of experimental data. While a full statistical analysis is desirable, we feel this is unnecessary to an exploratory investigation such as that reported here. Our judgment in this respect is fully supported by the discussion to be presented in Sec. V. We choose to consider only the data of Citron *et al.*[13] in the momentum region from 2.6 to 6.8 BeV/c and the data of Galbraith *et al.*[14] from 8 to 20 BeV/c. These are selected because the quoted errors are smaller than in most other experiments, and because the two sets of data are in reasonable agreement where they overlap in the neighborhood of 6 BeV/c. The two sets of data do not appear to be completely consistent with each other, however, since the most natural extrapolation of the points of Citron *et al.* would fall below the points obtained by Galbraith *et al.* It is also worth noting that cross sections obtained by other workers tend to lie higher than either of the sets of data we have used in the energy range between 5 and 8 BeV. With these comments in mind, it appears to us most reasonable to assume that the onset of the asymptotic region occurs somewhere between 5 and 6 BeV/c.

Since the Pomeranchuk trajectory is so thoroughly entrenched in the high-energy folklore, our first attempt to fit the cross-section data assumed a Pomeranchuk pole together with a cut. In Sec. III it is shown that this is possible only if the Pomeranchuk pole is completely dominated by the cut contribution, and then only if the asymptotic region does not begin until $E\approx 6$ BeV. This suggests that if one believes in the Pomeranchuk trajectory, then one must also believe in the P' trajectory and reject the cut as being unimportant. However, this conclusion depends strongly on the assumptions we have made about the discontinuity across the cut, and cannot therefore be regarded as completely certain.

Another possibility is to replace the Pomeranchuk pole with a cut. Although this would conflict with currently accepted opinion, and although many strong arguments have been advanced for believing in the Pomeranchuk pole, we do not know of any completely compelling reason for retaining it. If it should be possible to replace the Pomeranchuk trajectory by a cut, one would at least have an explanation for the absence of any spin-2 boson associated with this trajectory. In Sec. IV we show that it is possible to obtain an acceptable fit to the data assuming a model using P' and a cut, for quite a wide range of the cut and pole parameters. Of course this conclusion must be regarded as very tentative—it may not be possible to fit other data such as nonforward scattering and polarization effects with the model, or to fit data for other processes using a cut with the same parameters. Our results are discussed more fully in Sec. V.

II. BASIC FORMALISM

We shall deal with amplitudes $T_{\pm}(E)$ related to the π^+p and π^-p cross sections $\sigma^{\pm}$ by the optical theorem

$$\sigma^{\pm}(E)=(4\pi/k)\,\mathrm{Im}T_{\pm}(E)\,, \tag{1}$$

where E is the total energy of the pion in the laboratory frame and $k=(E^2-\mu^2)^{1/2}$, $\mu=$pion mass in energy units.

It is more convenient to use the amplitudes

$$\begin{aligned}T_1(E)&=\tfrac{1}{2}[T_-(E)+T_+(E)]\,,\\ T_2(E)&=\tfrac{1}{2}[T_-(E)-T_+(E)]\,.\end{aligned} \tag{2}$$

$T_1(E)$ and $T_2(E)$ correspond to $I=0$ and $I=1$ in the crossed channel, respectively. Each amplitude is split into real and imaginary parts by

$$T_j(E)=D_j(E)+iA_j(E)\,,\quad j=1,2,+,-\,. \tag{3}$$

With these notations, the forward dispersion relations are

$$D_1(E)=D_1(\mu)+\frac{f^2}{M}\frac{k^2}{E^2-(\mu/2M)^2}+\frac{k^2}{\pi}\int_0^\infty A_1(E')\frac{dk'^2}{k'^2(k'^2-k^2)} \tag{4}$$

and

$$\frac{D_2(E)}{E}=\frac{D_2(\mu)}{\mu}-2\frac{f^2}{\mu^2}\frac{k^2}{E^2-(\mu^2/2M)^2}+\frac{k^2}{\pi}\int_0^\infty \frac{A_2(E')}{E'}\frac{dk'^2}{k'^2(k'^2-k^2)}\,. \tag{5}$$

[13] A. Citron, W. Galbriath, T. F. Kycia, B. A. Leontic, R. H. Phillips, and A. Rousset, Phys. Rev. Letters **13**, 205 (1964).

[14] W. Galbriath, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).

By using these dispersion relations in conjunction with a Regge-pole model, Sertorio and Toller[3] were able to derive a simple form for the sum rule. Since in our model we shall assume that T_2 is dominated by the ρ trajectory, we shall add nothing to their results for this amplitude. For T_1, however, we shall derive a sum rule assuming that at sufficiently large energies T_1 is dominated by a series of Regge poles together with a Regge cut.

For $T_1(E)$ we take the form

$$T_1(E)=\sum_{j=1}^{n} a_j[i-\cot(\pi\alpha_j/2)]\left(\frac{E}{\mu}\right)^{\alpha_j}+\int_{-\beta}^{1} d\alpha[i-\cot(\pi\alpha/2)]\beta(\alpha)\left(\frac{E}{\mu}\right)^{\alpha}+O(E^{-\beta})+O(E^{-1/2}). \quad (6)$$

The signature factors are an essential consequence of the relativistic treatment of the problem.[9] The cut is assumed to be along the real axis up to 1. For the derivation of the sum rule to be valid, it is necessary to take the lower limit of the cut integral to be negative; for convenience we shall take it to be $-\frac{1}{2}$ in what follows although this is not strictly necessary.

The discontinuity $\beta(\alpha)$ across the cut is of course unknown. However, in order that the cut integral should converge near $\alpha=0$, we assume that $\beta(\alpha)$ has a factor α. Otherwise, we expect the high-energy amplitude to be dominated by the contribution of the cut near $\alpha=1$, together with the poles. Following Igi,[10] we write $x=1-\alpha$ and

$$\beta(\alpha)=c\alpha x^{\gamma}\psi(x)=c(1-x)x^{\gamma}\psi(x), \quad \gamma>-1 \quad (7)$$

where $\psi(0)=1$. We assume that $\psi(x)$ can be expanded as a power series convergent for $0\leq x\leq\frac{3}{2}$. Since we want a two-parameter contribution from the cut, and since we expect the cut contribution to be dominated by $x=0$ (i.e., $\alpha=1$) at high energies, we make the further approximation of retaining only the first term in the power series for $\psi(x)$, and take

$$\beta(\alpha)=c(1-x)x^{\gamma}, \quad x=1-\alpha, \quad \gamma>-1. \quad (8)$$

The cut integral may now be written

$$\begin{aligned}T_{\text{cut}}(E)&=\int_{-1/2}^{1}\beta(\alpha)[i-\cot(\pi\alpha/2)]\left(\frac{E}{\mu}\right)^{\alpha}d\alpha\\&=\frac{E}{\mu}\int_{0}^{3/2}c(1-x)x^{\gamma}e^{-x\ln(E/\mu)}[i-\tan(\pi x/2)]dx\\&=ic\frac{E}{\mu}\frac{\Gamma(\gamma+1)}{[\ln(E/\mu)]^{\gamma+1}}\left(1-\frac{\gamma+1}{\ln(E/\mu)}\right)\\&\quad-c\frac{E}{\mu}\int_{0}^{3/2}x^{\gamma}(1-x)e^{-x\ln(E/\mu)}\tan(\pi x/2)dx+O(E^{-1/2}). \quad (9)\end{aligned}$$

Hence the real and imaginary parts of $T_1(E)$, according to our hypotheses, are

$$D_1(E)=-\sum_{j=1}^{n}a_j\cot(\pi\alpha_j/2)\left(\frac{E}{\mu}\right)^{\alpha_j}-c\frac{E}{\mu}\int_{0}^{3/2}x^{\gamma}(1-x)e^{-x\ln(E/\mu)}\tan(\pi x/2)dx+O(E^{-1/2}), \quad (10)$$

$$A_1(E)=\sum_{j=1}^{n}a_j\left(\frac{E}{\mu}\right)^{\alpha_j}+c\frac{E}{\mu}\frac{\Gamma(\gamma+1)}{[\ln(E/\mu)]^{\alpha+1}}\left(1-\frac{\gamma+1}{\ln(E/\mu)}\right)+O(E^{-1/2}). \quad (11)$$

For sufficiently large energies, the second term in the cut contribution to A_1 may be ignored, while $(1-x)\tan(\pi x/2)$ may be approximated by $\frac{1}{2}\pi x$ to yield

$$D_1(E)\approx-\sum_{j=1}^{n}a_j\cot(\pi\alpha_j/2)\left(\frac{E}{\mu}\right)^{\alpha_j}-\tfrac{1}{2}\pi c\left(\frac{E}{\mu}\right)\frac{\Gamma(\gamma+2)}{[\ln(E/\mu)]^{\gamma+2}}. \quad (12)$$

In this limit, our results agree with these obtained by Igi.[10]

The next step is to insert (6) into the dispersion relation (4). The dispersion integral is divided into two ranges, from 0 to $\bar{K}$ and from $\bar{K}$ to ∞. For $E>\bar{E}$, the asymptotic form (6) is assumed valid, while for $E<\bar{E}$ experimental data will be used. We obtain

$$\begin{aligned}D_1(E)=D_1(\mu)&+\frac{f^2}{M}\frac{k^2}{E^2-(\mu^2/2M)^2}+\frac{k^2}{\pi}\int_0^{\bar{K}^2}A_1(E')\frac{dk'^2}{k'^2(k'^2-k^2)}+\frac{k^2}{\pi}\sum_{j=1}^{n}a_jP\int_{\bar{K}^2}^{\infty}\left(\frac{E'}{\mu}\right)^{\alpha_j}\frac{dk'^2}{k'^2(k'^2-k^2)}\\&+\frac{k^2}{\pi}\int_{-1/2}^{1}\beta(\alpha)d\alpha\,P\int_{\bar{K}}^{\infty}\left(\frac{E'}{\mu}\right)^{\alpha}\frac{dk'^2}{k'^2(k'^2-k^2)}+O(E^{-1/2}). \quad (13)\end{aligned}$$

P denotes a principal-value integration. The change in order of integration is readily justified using our assumed form for $\beta(\alpha)$. The integrals over k' may be performed explicitly, but since we are ignoring terms of order $E^{-1/2}$, we may simplify them first by identifying E' with k', E with k. We next use the result[15]

$$P\int_X^\infty \frac{x^\alpha}{x-y}dx = -\pi y^\alpha \cot\pi\alpha + \frac{X^{\alpha+1}}{(\alpha+1)y}F(1, 1+\alpha, 2+\alpha; X/y)$$

to obtain

$$P\int_{\bar{K}^2}^\infty \left(\frac{E'}{\mu}\right)^\alpha \frac{dk'^2}{k'^2(k'^2-k^2)} \simeq P\int_{\bar{K}^2}^\infty \mu^{-\alpha}(k'^2)^{\alpha/2-1}\frac{dk'^2}{k'^2-k^2} = -\pi\left(\frac{k}{\mu}\right)^\alpha k^{-2}\cot(\pi\alpha/2) + \frac{2}{\alpha k^2}\left(\frac{\bar{K}}{\mu}\right)^\alpha F\left(1, \tfrac{1}{2}\alpha, 1+\tfrac{1}{2}\alpha; \frac{\bar{K}}{k}\right). \quad (14)$$

Finally we insert (14) into (13) and expand in decreasing powers of E to obtain

$$D_1(E) = D_1(\mu) + \frac{f^2}{M} - \frac{2}{\pi}\int_0^{\bar{K}} A_1(E')\frac{dk'}{k'} - \sum_{j=1}^n a_j\left(\frac{E}{\mu}\right)^{\alpha_j}\cot(\pi\alpha_j/2) + \frac{2}{\pi}\sum_{j=1}^n a_j\left(\frac{\bar{E}}{\mu}\right)^{\alpha_j}\frac{1}{\alpha_j}$$
$$+\int_{-1/2}^1 \beta(\alpha)d\alpha\left\{-\left(\frac{E}{\mu}\right)^\alpha\cot(\pi\alpha/2) + \frac{2}{\pi\alpha}\left(\frac{\bar{E}}{\mu}\right)^\alpha F(1, \tfrac{1}{2}\alpha, 1+\tfrac{1}{2}\alpha; \bar{K}/k)\right\} + O(E^{-1/2}). \quad (15)$$

The cut integral may be evaluated as before to obtain the final result

$$D_1(E) = D_1(\mu) + \frac{f^2}{M} - \frac{2}{\pi}\int_0^{\bar{K}} A_1(E')\frac{dk'}{k'} - \sum_{j=1}^n a_j\left(\frac{E}{\mu}\right)^{\alpha_j}\cot(\pi\alpha_j/2) + \frac{2}{\pi}\sum_{j=1}^n \frac{a_j}{\alpha_j}\left(\frac{\bar{E}}{\mu}\right)^{\alpha_j}$$
$$-\frac{\pi}{2}c\frac{\Gamma(\gamma+2)}{[\ln(E/\mu)]^{\gamma+2}}\frac{E}{\mu} + \frac{2}{\pi}\frac{\bar{E}}{\mu}c\frac{\Gamma(\gamma+1)}{[\ln(\bar{E}/\mu)]^{\gamma+1}} - c\left(\frac{E}{\mu}\right)\int_0^{3/2} x^\gamma(1-x)e^{-x\ln(E/\mu)}\tan(\pi x/2)dx + O(E^{-1/2}). \quad (16)$$

Comparison of (16) with (10) now yields the sum rule

$$D_1(\mu) + \frac{f^2}{M} - \frac{2}{\pi}\int_0^{\bar{K}} A_1(E')\frac{dk'}{k'} + \frac{2}{\pi}\sum_{j=1}^n \frac{a_j}{\alpha_j}\left(\frac{\bar{E}}{\mu}\right)^{\alpha_j} + \frac{2}{\pi}\frac{\bar{E}}{\mu}c\frac{\Gamma(\gamma+1)}{[\ln(\bar{E}/\mu)]^{\gamma+1}} = 0; \quad (17)$$

or since by hypothesis the asymptotic form (11) is already valid at energy $\bar{E}$,

$$D_1(\mu) + \frac{f^2}{M} - \frac{2}{\pi}\int_0^{\bar{K}} A_1(E')\frac{dk'}{k'} + \frac{2}{\pi}A_1(\bar{E}) + \frac{2}{\pi}\sum_{j=1}^n a_j\frac{1-\alpha_j}{\alpha_j}\left(\frac{\bar{E}}{\mu}\right)^{\alpha_j} + \frac{2}{\pi}\frac{\bar{E}}{\mu}c\frac{\Gamma(\gamma+2)}{[\ln(\bar{E}/\mu)]^{\gamma+2}} = 0. \quad (18)$$

As a consistency check on the sum rule, we may differentiate (18) by $\bar{K}$: this yields a first-order differential equation for $A_1(E)$ which is easily found to be satisfied by the form (11).

In the sections following, the sum rule will be used to express the coefficient of the cut contribution in terms of the pole parameters and the single remaining cut parameter γ. The imaginary part of the scattering amplitude $A_1(E)$ will also be related to the total π-p cross sections by the optical theorem. It is convenient to define

$$EB = \int_0^{\bar{K}}(\sigma_+ + \sigma_-)dk - 4\pi^2(D_1(\mu) + f^2/M) - \bar{E}[\sigma_+(\bar{E}) + \sigma_-(\bar{E})], \quad (19)$$

$$b_j = (8\pi a_j/\mu)(\bar{E}/\mu)^{\alpha_j - 1}. \quad (20)$$

[15] A. Erdelyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Tables of Integral Transformations* (McGraw-Hill Book Company, Inc., New York, 1954), Vol. 2, formula (15.2.34).

Then the sum rule yields

$$\frac{8\pi}{\mu}c\frac{\Gamma(\gamma+2)}{[\ln(\bar{E}/\mu)]^{\gamma+2}} = B - \sum_{j=1}^n b_j\frac{1-\alpha_j}{\alpha_j}, \quad (21)$$

and

$$\sigma(E) \equiv \sigma_+(E) + \sigma_-(E) = \sum_{j=1}^n b_j\left(\frac{E}{\bar{E}}\right)^{\alpha_j - 1}$$
$$+\left[B - \sum_{j=1}^n \frac{1-\alpha_j}{\alpha_j}b_j\right]\left[\frac{\ln(\bar{E}/\mu)}{\ln(E/\mu)}\right]^{\gamma+2}\left[\frac{\ln(E/\mu)}{\gamma+1} - 1\right]. \quad (22)$$

III. ATTEMPTED FIT WITH P+CUT

In this section we discuss the possibility of replacing the second vacuum pole P' by a cut. We therefore assume a single vacuum pole with $\alpha = 1$ together with a cut and obtain the cross-section formula

$$\sigma(E)\equiv\sigma_+(E)+\sigma_-(E)$$
$$=b+B\left[\frac{\ln(\bar{E}/\mu)}{\ln(E/\mu)}\right]^{\gamma+2}\left[\frac{\ln(E/\mu)}{\gamma+1}-1\right], \quad (23)$$

which involves only two free parameters, b and γ. It should therefore be possible to evaluate the two parameters by a two-point fit to the data. For convenience we choose one of these points to be at energy $\bar{E}$. Hence, we obtain

$$\bar{\sigma}\equiv\sigma(\bar{E})=b+B\left[\frac{\ln(\bar{E}/\mu)}{\gamma+1}-1\right]; \quad (24)$$

since $b\geq 0$, the only acceptable values of γ must be such that

$$B\left[\frac{\ln(\bar{E}/\mu)}{\gamma+1}-1\right]\leq\bar{\sigma} \quad (25)$$

or

$$\gamma+1\geq\frac{B\ln(\bar{E}/\mu)}{\bar{\sigma}+B}. \quad (26)$$

We shall consider both $\bar{K}=5$ BeV/c and $\bar{K}=6$ BeV/c. From previous numerical work on fitting a pole model to the same data,[3,4] we obtain

$$B=10.51\pm0.06 \text{ mb} \quad (27)$$

for both values of $\bar{K}$. For the total cross sections at the two energies we take

$$\bar{\sigma}\equiv\bar{\sigma}_+ +\bar{\sigma}_- =55.98\ \pm0.01, \quad (\bar{K}=5 \text{ BeV}/c)$$
$$=54.056\pm0.01, \quad (\bar{K}=6 \text{ BeV}/c). \quad (28)$$

Hence the lower limit on $\gamma+1$ is

$$\gamma+1\geq0.562, \quad (\bar{K}=5 \text{ BeV}/c)$$
$$\geq0.609, \quad (\bar{K}=6 \text{ BeV}/c). \quad (29)$$

At the same time, we obtain from (24) and (23)

$$Z(E)\equiv\frac{\bar{\sigma}-\sigma(E)}{B}=\frac{\ln(\bar{E}/\mu)}{\gamma+1}-1$$
$$-\left[\frac{\ln(\bar{E}/\mu)^{\gamma+2}}{\ln(E/\mu)}\right]\left[\frac{\ln(E/\mu)}{\gamma+1}-1\right], \quad (30)$$

or

$$\left[\frac{\ln(\bar{E}/\mu)}{\ln(E/\mu)}\right]^{\gamma+2}=\frac{\ln(\bar{E}/\mu)-[Z(E)+1](\gamma+1)}{\ln(E/\mu)-(\gamma+1)}. \quad (31)$$

The obvious solution $\gamma+1=0$ must be rejected since by hypothesis $\gamma+1>0$. Since $Z(E)>0$, it is easily seen that there is no solution for $(\gamma+1)>\ln(E/\mu)$. If we choose $K=18$ BeV/c, where K is the momentum corresponding to energy E, and take $\sigma(E)=48.5\pm0.36$ mb,[8] we find that for $\bar{K}=5$ BeV/c, Eq. (31) has a second solution in the vicinity of $\gamma+1=0.30$. This conflicts with the inequality (29): Clearly the best we can do to fit the data is to choose $\gamma+1=0.562$, in which case the coefficient b of the Pomeranchuk pole contribution to (23) vanishes. If $\bar{K}=6$ BeV/c, the second solution is close to $\gamma+1=0.609$, in which case the residue of the Pomeranchuk pole again vanishes.

We notice that by virtue of the sum rule the model based on a cut alone involves just one free parameter γ which is completely determined by fitting the theory to one experimental point. Figure 1 shows the cut prediction fitted to the Citron data at 5 BeV/c and at 6 BeV/c. For comparison, Fig. 2 shows a conventional two-pole model fitted at 5 BeV/c with $\alpha_{p'}=0.69$ [5] and fitted at 6 BeV/c with $\alpha_{p'}=0.615$.[4] Clearly the cut model fitted at 5 BeV/c is quite unacceptable; however, the cut model fitted at 6 BeV/c fits the Galbraith data quite as well as the two-pole model.

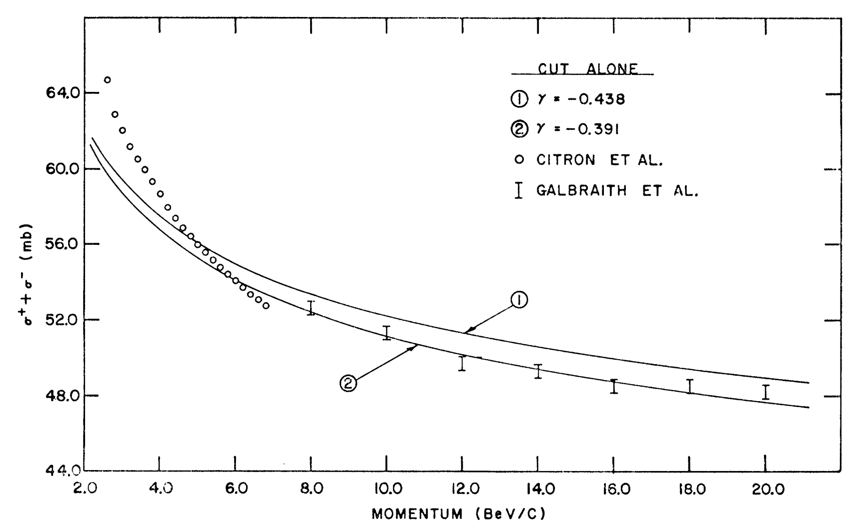

FIG. 1. Predictions of the model using a cut alone. The values of γ for curves 1 and 2 are determined by fitting the curves to the data of Citron *et al.* at 5 and 6 BeV/c, respectively. The error bars for the data of Citron *et al.* are not larger than the circles used to denote their points.

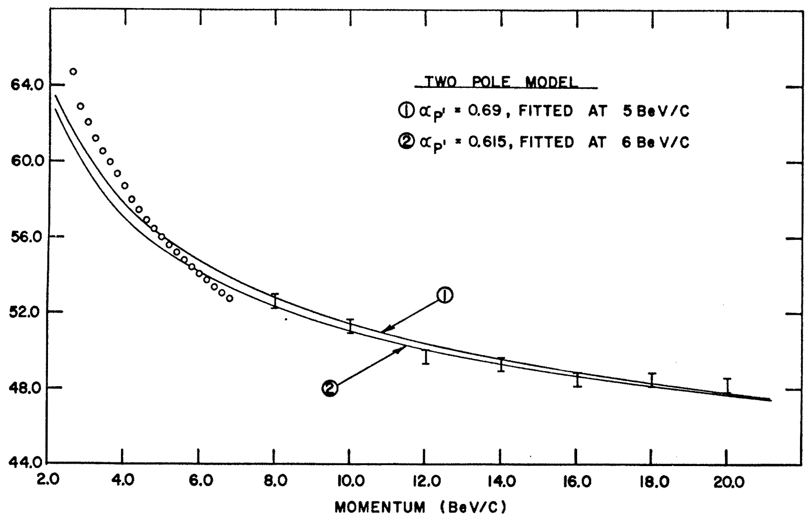

FIG. 2. Predictions of the conventional Regge-pole model. Curve 1, with $\alpha=0.69$, is fitted to the data of Citron *et al.* at 5 BeV/c, while curve 2, with $\alpha=0.615$, is fitted to the data at 6 BeV/c.

It is not too surprising that the two-pole model can represent the data reasonably well whether the theory is fitted at 5 BeV/c or at 6 BeV/c, whereas the cut model gives an adequate representation only when fitted at 6 BeV/c, since the two-pole model contains two free parameters whereas the cut model has only one.

Further discussion of these results is postponed until Sec. V.

IV. ATTEMPTED FIT WITH $P'+$CUT

For the reasons stated in the Introduction it is interesting to attempt to fit the data with a model in which the Pomeranchuk pole is replaced by a cut. We are further encouraged to try this by the results of the previous section, in which we saw that the attempt to replace the second vacuum pole by a cut led to the elimination of the Pomeranchuk pole.

The cross-section formula is now

$$\sigma(E)=b\left(\frac{E}{\bar{E}}\right)^{\alpha-1}+\left[B-b\frac{1-\alpha}{\alpha}\right]\left[\frac{\ln(\bar{E}/\mu)}{\ln(E/\mu)}\right]^{\gamma+2}\left[\frac{\ln(E/\mu)}{\gamma+1}-1\right]. \quad (32)$$

It is convenient to determine b by fitting (32) to the experimental data at energy $\bar{E}$. In this way we obtain the formula

$$\sigma(E)=\frac{1}{\lambda-y}\left\{\alpha[B-y(\bar{\sigma}+B)]\left(\frac{E}{\bar{E}}\right)^{\alpha-1}+[(\bar{\sigma}+B)\lambda-B]\left[\frac{\ln(E/\mu)}{\ln(E/\mu)}\right]^{\gamma+1}\left[1-\frac{\gamma+1}{\ln(E/\mu)}\right]\right\}, \quad (33)$$

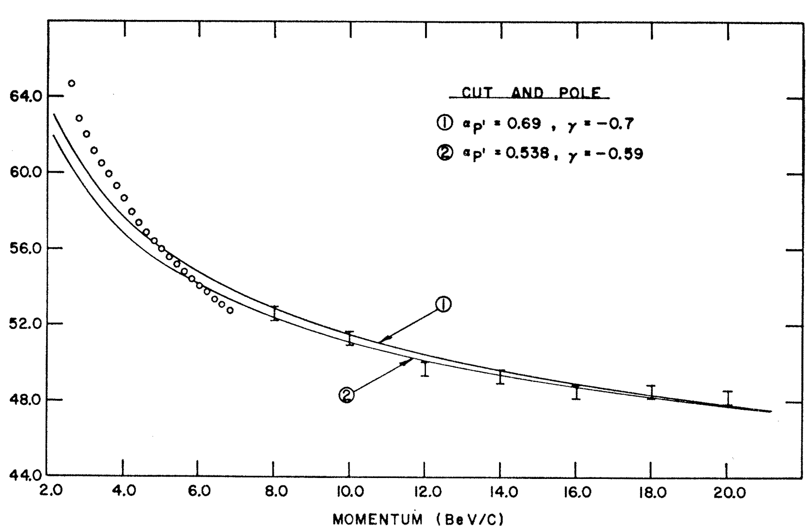

FIG. 3. Predictions of the mode using P' and a cut, for two sets of cut parameters. In curve 1, the residue at the pole is determined by fitting the data of Citron *et al.* at 5 BeV/c, while in curve 2 this residue is found by fitting at 6 BeV/c. Equally good fits to the data may be found using a wide range of values of the parameters.

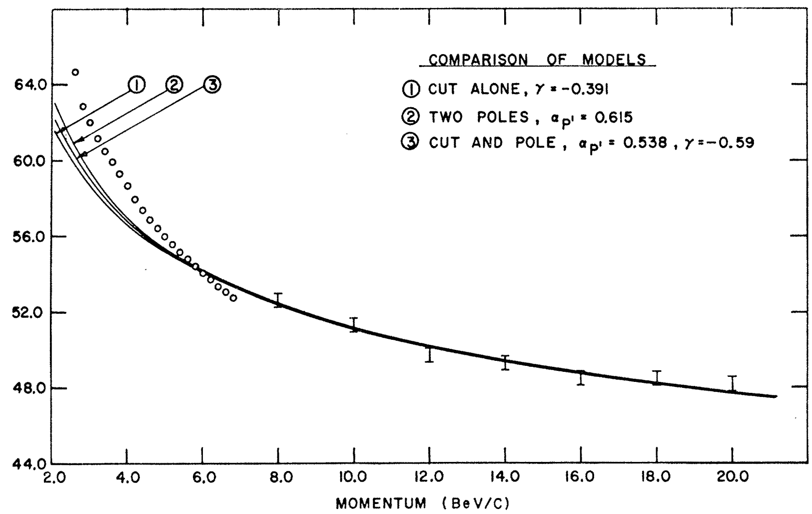

FIG. 4. Comparison of the model based on P' and a cut, for two different sets of parameters, with the conventional two-pole model. All curves are fitted to the data at 5 BeV/c.

where

$$\lambda = 1-\alpha \tag{34}$$

and

$$y=(\gamma+1)/\ln(\bar{E}/\mu). \tag{35}$$

If desired, $\bar{\sigma}$ may be treated as a parameter to be fitted along with α and γ by a least-squares procedure, instead of being taken from experimental data.

It proves in fact quite easy to fit the experimental data within the experimental accuracy for quite a wide range of the parameters. Two typical curves are shown in Fig. 3. The possibility of such a fit is not too surprising since the model under consideration has three free parameters and excellent fits have already been found in the last section and in previous work[3–6] using only one or two free parameters. It appears in fact that provided the theoretical curve is chosen to fit adequately at the ends of the 5–20 BeV/c interval it is almost certain to fit reasonably well between these extremes. The theoretical curves illustrated both have $\alpha_{p'}$ reasonably close to the values obtained[3–6] from the two-pole model; however, other fits are possible in which $\alpha_{p'}$ is very different from the two-pole value, and it is even possible to find fits for which the residue b of the pole is negative. Since the trajectory of the P', as obtained from the two-pole model, passes close to the position of the f_0', we are biased in favor of fits such as those illustrated in which $\alpha_{p'}$ is not too different from the two-pole value.

V. CONCLUSION

In the previous two sections, we have shown that a model involving a Regge cut can fit the experimental data for the sum of π^+p and π^-p total cross sections in the momentum range from about 6 to 20 BeV/c. It remains to discuss the significance of these results.

First, we should ask whether the two-pole model may not give a significantly better representation of the data. Indeed, the sum of squares of the deviations between

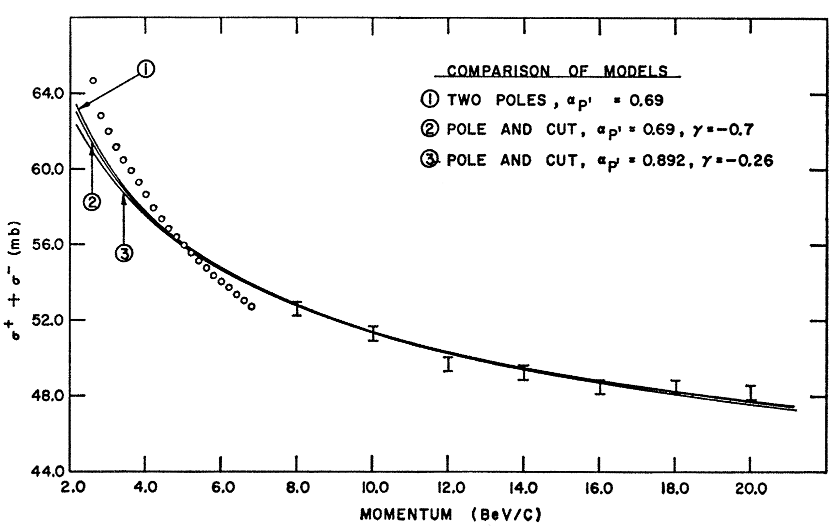

FIG. 5. Comparison of the predictions of the models based on a cut alone, on P' and a cut, and on two poles. All the curves are fitted to the data at 6 BeV/c.

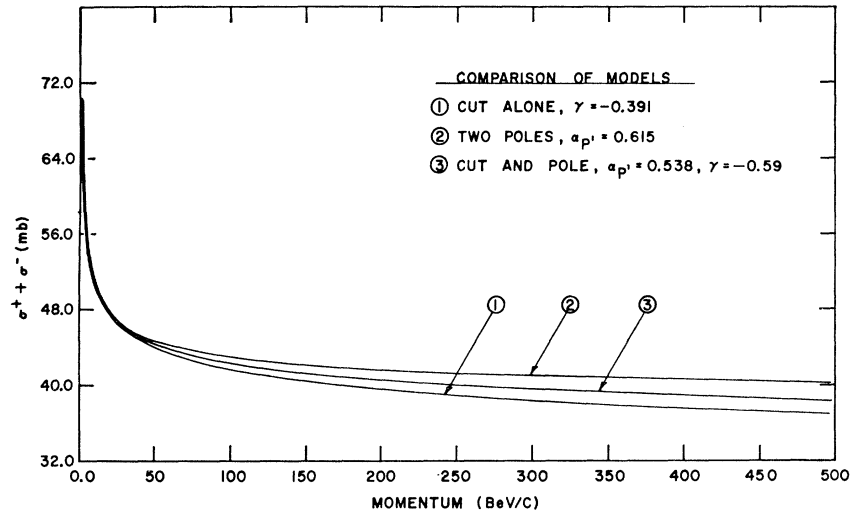

FIG. 6. Comparison of the same models illustrated in Fig. 5, in the momentum range 0–500 BeV/c.

the theoretical predictions and the experimental points is smaller using a two-pole model than with any of the models using a cut. This, however, is not significant, as is clearly shown in Figs. 4 and 5. In these, the predictions of several models are plotted on the same graph, and in each figure the theoretical models clearly cannot be distinguished from each other in the significant energy region.

These two plots also justify our rather arbitrary selection of experimental data, since they show that whatever may be the finally accepted best fit to the data using a two-pole model, we can obtain a whole family of essentially indistinguishable fits using the (P'+cut) model. To resolve this question experimentally with data at energies less than 20 BeV, it would be necessary to obtain π^+p and π^-p total cross sections with an accuracy considerably better than 0.05 mb for the whole energy range up to 20 BeV.

One of the most striking consequences of eliminating the Pomeranchuk pole in a model using a cut would be that total cross sections should tend asymptotically to zero rather than to a constant at very high energies. This gives a method in principle of distinguishing various models. Figure 6 shows the same curves as in Fig. 5 extrapolated to 450 BeV/c. A measurement of $\sigma^++\sigma^-$ at 100 BeV/c would have to have an error no greater than ± 0.8 mb in order to distinguish between these three curves. At 500 BeV/c the three curves illustrated could just be distinguished by a measurement of $\sigma^++\sigma^-$ accurate to 2 mb. Even a single experimental point at 500 BeV/c with this accuracy would not decide the question, however, since existing least-squares fits to the data for $K<20$ BeV/c [3–6] lead to predictions for the infinite energy limit of $\sigma^++\sigma^-$ which differ by millibarns.

In conclusion, we see very little hope of distinguishing between the various models we consider here by total cross-section measurements on pion-proton scattering. This preliminary investigation suggests that a model in which a major part of the total π-p cross section at high energies is contributed by a Regge cut in the crossed channel must be considered to be a live option. Of course we cannot reject the usual Regge-pole model which has had so much success. We must seek other types of data which are capable of distinguishing between the different models. The most hopeful possibilities appear to us to be the ratio of the real and imaginary parts of the forward scattering amplitude, since the real part is sensitive to the infinite energy limit of the imaginary part through the forward dispersion relation, and polarization effects in pion-proton scattering which in the usual theory depends on $\alpha_P-\alpha_{P'}$.