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Some Remarks Concerning Scattering in a Quark Model*

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A hierarchy of sum rules on scattering cross sections are derived under much less restriction than previously assumed. New sum rules and real parts are discussed.

Recently, sum rules on scattering cross sections[1] and electromagnetic mass differences[2] have been derived from an additive quark model and seem to agree very well with experiment. In particular, there seems to exist a "hierarchy" of scattering sum rules, i.e., certain equalities are more valid than others, although the reason for such behavior is not completely clear from their manner of derivation. For example, it is rather mysterious that, under the same assumptions, the mean of the two Johnson-Treiman relations should be more accurate than the Johnson-Treiman relations themselves. While the cross-section sum rules are concerned with the imaginary part of the forward scattering amplitudes, relatively little attention has been given to the real parts.[3] The real parts of the scattering amplitudes are of particular interest in connection with the additive quark model, since the validity of sum rules for the real parts would confirm that the cross-section relations are obtained from a consideration of scattering amplitudes and the model is *not* just a convenient method of parametrizing total cross sections. In this paper, we shall present a generalization of the additive quark model of Lipkin in which we shall focus our attention on the following features:

(a) It will be shown that, in the present model, no discrepancy in the relation between meson-baryon and baryon-baryon cross sections will arise. And, as we increase the restrictions on the quark-quark amplitudes, the sum rules that emerge are less well satisfied, so that a "hierarchy" of sum rules is formed.

(b) The present model yields predictions on the real parts of πN, NN, and $\bar{N}N$ amplitudes which can be made consistent with the present data.

The central dynamical assumption is the statement of additivity which can be written

$$S[(\cdots x_i\cdots)(\cdots x_j\cdots);(\cdots y_k\cdots)(\cdots y_l\cdots)] = \sum_{i,j,k,l} S(x_ix_j;y_ky_l)\,, \quad (1)$$

where $(\cdots x_i\cdots)$ is a state of physical particles made up of quarks or antiquarks $x_i=\mathcal{P}$, $\mathcal{N}$, or λ. By this assumption, all the meson-baryon and baryon-baryon elastic scattering amplitudes are then related to the quark amplitudes $(\mathcal{P}\mathcal{P})$, $(\mathcal{P}\mathcal{N})$, $(\mathcal{N}\mathcal{N})$, $(\lambda\mathcal{P})$, $(\lambda\mathcal{N})$, $(\bar{\mathcal{P}}\mathcal{P})$, $(\bar{\mathcal{N}}\mathcal{N})$, $(\bar{\mathcal{P}}\mathcal{N})$, $(\bar{\mathcal{N}}\mathcal{P})$, $(\bar{\lambda}\mathcal{P})$, and $(\bar{\lambda}\mathcal{N})$, where we use (AB) to denote the amplitude for the process $A+B\rightarrow A+B$.

First of all, we note that if we restrict ourselves to presently observable elastic scattering amplitudes, the assumption of additivity alone does not give any relations, since we have 11 quark amplitudes and 10 physical amplitudes, namely, (pp), $(\bar{p}p)$, $(\bar{p}n)$, (pn), (π^+p); (π^-p), (K^+p), (K^-p), (K^+n), and (K^-n). However, if we supplement additivity with $SU(2)$ symmetry for the quark amplitudes, we have the following nontrivial sum rules,[1]

$$(\bar{p}p)+(pp)=\tfrac{3}{2}[(\pi^+p)+(\pi^-p)] +\tfrac{1}{2}[(K^+p)+(K^-p)-(K^+n)-(K^-n)]\,,$$

$$\tfrac{1}{3}[(K^-p)+(\pi^+p)+(K^+n)]=\tfrac{1}{3}[(K^+p)+(\pi^-p)+(K^-n)]\,.$$

It is clear that a hierarchy of sum rules can be obtained by adding $SU(3)$ symmetry to the basic assumption. This has been investigated by Lipkin and Scheck.[1] However, examination of the above sum rules shows that while the second, i.e., the antisymmetric sum rule, is in excellent agreement with experiment, the first has a large discrepancy of about 15–20 mb.[4] This situation is rather embarrassing, since $SU(2)$ symmetry is not badly broken.

There are at least two ways out of such a dilemma. The first alternative is to regard all relations between meson-baryon and baryon-baryon amplitudes as having an intrinsic discrepancy, and proceed to consider the consequences of assuming $SU(3)$ symmetry on the quark-quark amplitudes. This is generally done in the literature.[1] A second approach is to suppose that the additivity assumption alone is inadequate, and thus must be supplemented by additional dynamical assumptions. Such an approach must therefore yield a relation between meson-baryon and baryon-baryon amplitudes which is of comparable accuracy to the antisymmetric sum rule. In this paper, we shall consider a model which, although $SU(2)$ invariance is assumed, nevertheless gives rise systematically to a hierarchy of sum

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[1] E. M. Levin and L. L. Franfurt, JETP Pis'ma v Redaktsiyu **2**, 105 (1965) [English transl.: JETP Letters **2**, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966); J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42A**, 711 (1966); H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966). For a summary see H. J. Lipkin, Yalta Lecture Notes, Yalta, USSR, 1966 (unpublished).

[2] H. R. Rubinstein, Phys. Rev. Letters **17**, 41 (1966); Y. T. Chiu, EFINS Report No. 66-82 (unpublished).

[3] See J. J. J. Kokkedee and L. Van Hove in Ref. 1.

[4] All of the data on cross sections in this paper are taken from W. Galbraith, E. Jenkins, T. Kycia, B. Leontic, R. Phillips, A. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).

rules in terms of the severity of asymptotic restrictions on quark amplitudes.

The basic additional assumption introduced is that the quark amplitudes are already approaching the Pomeranchuk limit. We shall state our parametrization as follows:

$$\begin{aligned}
(\mathcal{P}\mathfrak{N})&=(\mathfrak{N}\mathcal{P})=(\mathcal{P}\mathcal{P})=(\mathfrak{N}\mathfrak{N})=P\,,\\
(\bar{\mathcal{P}}\mathfrak{N})&=(\bar{\mathfrak{N}}\mathcal{P})=P'\,,\\
(\lambda\mathcal{P})&=(\lambda\mathfrak{N})=P-S\,,\\
(\bar{\lambda}\mathcal{P})&=(\bar{\lambda}\mathfrak{N})=P'-S'\,,\\
(\bar{\mathcal{P}}\mathcal{P})&=(\bar{\mathfrak{N}}\mathfrak{N})=P'+A\,,
\end{aligned}\tag{2}$$

where P, P', S, S', and A are to be complex functions of energy. There is no obvious justification for such a parametrization, except that P is heuristically the Pomeranchuk amplitude for nonstrange quarks. S measures the correction due to $SU(3)$ breaking. A is the correction due to singlet annihilation channels that arises because the amplitudes $(\bar{\mathcal{P}}\mathcal{P})$ and $(\bar{\mathfrak{N}}\mathfrak{N})$ may not be asymptotic enough. The quantities P' and S' are analogs of P and S, except that they are for quark-antiquark scattering. Since the (qq) and $(\bar{q}q)$ channels have different baryon numbers, they are, in general, distinct, except in the asymptotic limit. Indeed, the real parts of (qq) and $(\bar{q}q)$ may behave rather differently, even in the asymptotic limit. It is interesting to note, however, that if $P=P'$ and $S=S'$, and if we apply the relations to scattering cross sections only, then we have Lipkin's additive quark model. As we shall see, the hierarchy of sum rules is partially due to distinctions between P, P', S, and S'.

Finally, since the quarks bound in mesons may behave differently from those bound in baryons, we shall add corrections B and B' to each (qq) and $(\bar{q}q)$ amplitude coming from baryon-baryon and meson-baryon scattering, respectively.

Before we continue, there are several relevant remarks to be made.

(i) From Eqs. (2), it is obvious that we have not assumed quark-quark and quark-antiquark amplitudes to approach the same Pomeranchuk limit. Physically, there is every reason to do so for moderate energies, since both singlet and triplet annihilation channels may still be important.

(ii) Because of the introduction of the Pomeranchuk limits, $SU(2)$ invariance has been introduced automatically. It is perhaps somewhat unfortunate that the supplementary assumptions, Eqs. (2), do not allow us to discuss the distinction between additivity alone and additivity plus $SU(2)$. This is not really a disadvantage, however, since the hierarchy of sum rules will arise as a result of the degree to which the quark amplitudes approach the asymptotic limit.

(iii) No $SU(3)$ symmetry is introduced initially. Note that in our model, $SU(3)$ symmetry is also treated as an asymptotic limit, namely, when the $SU(3)$-breaking terms S and S' are taken to be zero.

Now, with the use of (2), we parametrize the meson-baryon and baryon-baryon amplitudes as follows:

$$\begin{aligned}
(pp)&=(np)=9P+9B\,,\\
(\bar{p}n)&=9P'+4A+9B\,,\\
(\bar{p}p)&=9P'+5A+9B\,,\\
(\pi^+p)&=3P+3P'+A+6B'\,,\\
(\pi^-p)&=3P+3P'+2A+6B'\,,\\
(K^+p)&=(K^+n)=3P+3P'-3S'+6B'\,,\\
(K^-n)&=3P+3P'-3S+A+6B'\,,\\
(K^-p)&=3P+3P'-3S+2A+6B'\,.
\end{aligned}\tag{3}$$

In what follows, we shall discuss our hierarchy of sum rules, decreasing in accuracy, which arise systematically as we impose progressively more severe asymptotic restrictions on the quark amplitudes, namely, additivity plus the conditions numbered 1 to 5 below.

1. The parametrization of Eqs. (2): From these we can read off the sum rules

$$(pp)=(np)\,,\tag{4a}$$

$$(K^+p)=(K^+n)\,,\tag{4b}$$

$$\tfrac{1}{3}[(K^-p)+(\pi^+p)+(K^+n)]=\tfrac{1}{3}[(K^+p)+(\pi^-p)+(K^-n)]\,,\tag{4c}$$

$$\tfrac{1}{3}[(\bar{p}p)+(np)+(\pi^+p)]=\tfrac{1}{3}[(pp)+(\bar{p}n)+(\pi^-p)]\,.\tag{4d}$$

Since these sum rules are derived with the least restrictive assumptions, we should expect them to be very well satisfied. This is indeed the case. The relations (4b) and (4c) have been discussed by Lipkin, and are found to be satisfied within the small errors of meson-baryon scattering cross sections. Any discrepancy is less than 2%. The relation (4a) is also satisfied well within the experimental errors.[4]

Relation (4d) is *new*, and is particularly interesting in that it directly relates baryon-baryon and meson-baryon cross sections. Table I shows that agreement with experiment[4] is indeed excellent—again all discrepancy is well within the error limits of about 3%, while all previous sum rules of this type have systematic errors of 15–20%.

TABLE I. Test of relation (4d).

Momentum (BeV/c)	Left-hand side (mb)	Right-hand side (mb)
6	42.7±0.68	42.8±1.3
8	41.2±0.33	41.6±1.3
12	38.8±0.33	39.7±1.25
14	38.2±0.34	39.3±1.25
16	37.6±0.33	38.8±1.25
18	37.7±1.7	36.1±3.0

The remarkable accuracy of the set of sum rules (4) is rather significant in that, irrespective of whether the present quark model possesses meaning, sum rules of such accuracy, similar to the Gell-Mann–Okubo sum rule, are probably not accidental. Relations (4a) and (4b) are also obtainable from Regge-pole theory with exchange degeneracy.[5] Relations (4c) and (4d) are so far obtainable from the additive quark model only, and thus constitute support for the additivity assumption as a dynamical principle. Furthermore, we note that the Johnson-Treiman relations of $SU(3)$ and the Freund relations of $U(6,6)$, being less accurate, are not obtainable in the present model unless further restrictions are placed on the quark amplitudes. It may bear repeating here that no bad relations are obtained in the present model, in contrast to previous models.

2. The $SU(3)$-breaking parameters in the quark amplitudes are equal, i.e., $S=S'$. Note that $SU(3)$ symmetry is not yet imposed.

If we require $S=S'$ with all the rest of the parameters unchanged, then the "antisymmetric sum rule" (4c) *splits* into the two Johnson-Treiman relations[6]:

$$\tfrac{1}{3}[(K^-p)+2(\pi^+p)]=\tfrac{1}{3}[(K^+p)+2(\pi^-p)], \quad (5a)$$

$$\tfrac{1}{3}[(K^-p)+2(K^+n)]=\tfrac{1}{3}[(K^+p)+2(K^-n)]. \quad (5b)$$

3. Triplet annihilation channels are unimportant, i.e., $P=P'$. On the other hand, if we require $P=P'$ with all the rest of the parameters unchanged, the relation (4d) *splits* also into the two Freund relations[7]:

$$[(\bar{p}p)-(pp)]=5[(\pi^-p)-(\pi^+p)] =5/4[(\bar{p}n)-(pn)]. \quad (5c)$$

Since the Johnson-Treiman relations have a systematic error of 3% and the Freund relations have a systematic error of 15%, we have obtained a hierarchy of sum rules which decrease in accuracy as we increase the restrictive assumptions. It is also interesting to note that the 15% discrepancy seen in the Freund relations is not due to the fact that they relate baryon-baryon to meson-baryon cross-sections, but is due to the distinction between nonstrange quark-quark and quark-antiquark amplitudes.

4. The symmetry of $SU(3)$, i.e., $S=S'=0$: Next let us take the $SU(3)$-breaking terms S and S' equal to zero. We obtain

$$(\pi^+p)=(K^-n), \quad (6a)$$

$$(\pi^-p)=(K^-p), \quad (6b)$$

$$\tfrac{1}{4}[(\pi^+p)+(\pi^-p)+(K^-n)+(K^+n)] =\tfrac{1}{2}[(K^-p)+(K^+p)], \quad (6c)$$

which have a systematic discrepancy of 12–15%, as is expected in relations which require $SU(3)$.

[5] R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).
[6] K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).
[7] P. G. O. Freund, Phys. Rev. Letters 15, 929 (1965).

TABLE II. Test of relation (7).

Momentum (BeV/c)	$(pp)+(\bar{p}p)$ (mb)	$2(\pi^-p)+(\pi^+p)$ (mb)	$18(B-B')$ (mb)
6	99.9±1.3	83.2±0.5	16.7
8	96.4±1.0	80.1±0.5	16.3
12	91.1±1.0	76.0±0.5	15.1
14	89.8±1.1	74.7±0.5	15.1
16	87.9±1.0	73.6±0.5	14.3
18	89.0±4.1	73.5±0.5	15.5

5. Neglect of corrections due to difference of binding for quarks in mesons and baryons. For this purpose, we obtain the relation

$$(pp)+(\bar{p}p)=2(\pi^-p)+(\pi^+p)+18(B-B'). \quad (7)$$

Table II shows the cross sections $(pp)+(\bar{p}p)$, $2(\pi^-p)+(\pi^+p)$, and $18(B-B')$. Since these cross sections have very small errors, and relation (7) is obtained under the same assumptions as relations (4), the difference $(B-B')$ of about 1 mb is probably reliable. Therefore, even under the assumption of additivity, the effects of binding are *not* negligible. It is interesting that relations (4), (5), and (6) are all obtained with $B\neq B'$. Furthermore, $(B-B')$ seems to be independent of energy, so that the uninteresting limit, i.e., that baryon-baryon cross sections approach $\frac{3}{2}$ of meson-baryon cross sections, may not be reached until extremely high energies. It is interesting also that (7), with $B=B'$, is obtained in all previous models. Such an inaccurate relation cannot possibly be put in the same class as (4), even though they both were previously obtained under the same assumptions.

Finally, we can ignore the problem of spin wave functions, as in the case of quark models for electromagnetic mass splitting,[2] and obtain the following relation:

$$(\rho^0p)=(\pi^0p)=\tfrac{1}{2}[(\pi^+p)+(\pi^-p)]. \quad (5d)$$

For 6–20 BeV/c, this predicts that the ρ^0p cross section is between 24 and 26 mb. This is to be compared with 50±5 mb obtained from diffraction dissociation models[8] and 58 mb obtained from absorption-model fits to the $\pi^-p\rightarrow\rho^0n$ process.[9] The discrepancy may be due to the role of spin in the quark wave functions.

Let us now turn to the real parts of the scattering amplitudes. Under the same assumptions (3) from which the imaginary part relations (4) are obtained, we have:

$$\mathrm{Re}(pp)=\mathrm{Re}(pn), \quad (8a)$$

$$\mathrm{Re}(K^+p)=\mathrm{Re}(K^+n), \quad (8b)$$

$$\tfrac{1}{2}[\mathrm{Re}(K^-p)+\mathrm{Re}(\pi^+p)]=\tfrac{1}{2}[\mathrm{Re}(\pi^-p)+\mathrm{Re}(K^-n)]. \quad (8c)$$

[8] M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966); S. D. Drell and J. S. Trefil, Phys. Rev. Letters 16, 552 (1966); 16, 832E (1966).
[9] J. D. Jackson *et al.*, Phys. Rev. 139, B428 (1965).

Relation (8a) has been verified at 19 BeV/c.[10] Relations (8b) and (8c) can be considered to be predictions of the model. Since the relations (4) are almost exact, relations (8) will therefore be a crucial test of the model. By making further restrictions we may arrive at other sum rules, but we shall not enumerate them further except to point out the following interesting speculation. To this end, we recall the following features concerning the real parts of πn, pp, and $\bar{p}p$ amplitudes:

(a) $\mathrm{Re}(pp)/\mathrm{Im}(pp)$ is constant with energy[11] in the range 7–26 BeV/c. The relation

$$\frac{\mathrm{Re}(\bar{p}p)}{\mathrm{Im}(\bar{p}p)} \simeq -\frac{\mathrm{Re}(pp)}{\mathrm{Im}(pp)} \quad \text{for} \quad s\rightarrow\infty \tag{8'}$$

is consistent with forward dispersion relations and data on the imaginary parts.[12] Sakurai[13] has recently speculated that the constancy with energy and (8′) follow from vector-meson exchange with absorption.

(b) It is well known that experiment[14] on $\mathrm{Re}(\pi^{\pm}p)$ seems to be in direct contradiction with the predictions of forward dispersion relations with Regge-pole asymptotic behavior[15]; namely, $\mathrm{Re}(\pi^-p)/\mathrm{Re}(\pi^+p)>1$ and rapidly varying with energy instead of $\mathrm{Re}(\pi^-p)/\mathrm{Re}(\pi^+p)\approx 0.74$ for $\alpha(0)=0.5$ and slowly varying with energy.

It is interesting to note that, given the relation (8′) and the constancy of $\mathrm{Re}(pp)$ and $\mathrm{Re}(\bar{p}p)$, our predictions on the real parts of $\pi^{\pm}p$ amplitudes become consistent with experiment if $\mathrm{Re}(B')$ is small. This can be seen from Eqs. (3). Since A approaches zero in the asymptotic limit, the statement $\mathrm{Re}(pp)=-\mathrm{Re}(\bar{p}p)$ is equivalent to $\mathrm{Re}P=-\mathrm{Re}P'$. Applying this relation to $\mathrm{Re}(\pi^{\pm}p)$, we immediately have the relation

$$\mathrm{Re}(\pi^-p)\approx 2\,\mathrm{Re}(\pi^+p)\rightarrow 0 \quad \text{as} \quad S\rightarrow\infty. \tag{9}$$

TABLE III. Test of relation (9). The values are taken directly from Ref. 11. The upper and lower values correspond to the upper and lower bounds given by the systematic errors quoted in the reference. In obtaining the ratio of the real parts, $\sigma_T(\pi^+p)/\sigma_T(\pi^-p)$ is assumed approximately equal to 1.

Momentum (BeV/c)	$\dfrac{\mathrm{Re}(\pi^+p)}{\mathrm{Im}(\pi^+p)}$	$\dfrac{\mathrm{Re}(\pi^-p)}{\mathrm{Im}(\pi^-p)}$	$\dfrac{\mathrm{Re}(\pi^-p)}{\mathrm{Re}(\pi^+p)}$
10	$-0.109\begin{Bmatrix}-0.200\\-0.074\end{Bmatrix}$	$-0.300\begin{Bmatrix}-0.338\\-0.206\end{Bmatrix}$	$2.75\begin{Bmatrix}2.8\\1.7\end{Bmatrix}$
12	$-0.132\begin{Bmatrix}-0.225\\-0.095\end{Bmatrix}$	$-0.408\begin{Bmatrix}-0.4280\\-0.0339\end{Bmatrix}$	$3.10\begin{Bmatrix}3.6\\1.9\end{Bmatrix}$

The relation (9) is tested against the available data in Table III and is in much better agreement with experiment than is $\mathrm{Re}(\pi^-p)\simeq 0.8\,\mathrm{Re}(\pi^+p)$.[16] Finally, we note that our prediction on the real parts of $(\pi^{\pm}p)$ hinges on (8′). If (8′) does not hold, a different relation may be obtained. Violation of forward dispersion relations is unlikely; however, the input to their derivation, i.e., the Regge-pole asymptotic behavior, may need reexamination. On the other hand, better experimental data are needed to confirm the relation (9).

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[14] K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. Yuan, Phys. Rev. Letters **14**, 862 (1965).

[15] G. Höhler, G. Ebel, and J. Giesecke, Z. Physik **180**, 430 (1964).

[16] The question of whether $\mathrm{Re}(\pi^-p)$ is greater or less than $\mathrm{Re}(\pi^+p)$ is important, in that it has direct bearing on the sign of the ratio of the real to the imaginary part of the charge-exchange amplitude. Unfortunately, no direct measurement of this sign exists at high energies.