Algebra of Currents and the Vector-Meson–Baryon Coupling Constants in Broken $SU(3)$*

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Employing the method of current algebra and a hypothesis of partially conserved tensor current, we study the renormalization effects on the vector-meson–baryon coupling constants, due to the first-order $SU(3)$ breaking. We find that the sum rules within the couplings of one vector meson with baryons are the same as those found in the $SU(3)$ limit, and that the renormalization effects are present only when we compare the coupling of one vector meson with the couplings of different vector mesons. With the use of ω-ϕ mixing with the mixing angle $\theta=\tan^{-1}(\sqrt{\frac{1}{2}})$, the ρ and ω couplings have the same amount of renormalization effects.

RECENTLY, employing the algebra of currents[1] and the partially conserved axial-vector current (PCAC) hypothesis,[2] Bose and Hara[3] have obtained a set of sum rules for the pseudoscalar-meson–baryon coupling constants in broken $SU(3)$. It has also been proposed that the antisymmetric tensor currents, through a partial-conservation hypothesis, may have a physical content.[4,5] In this paper, we want to apply the method of current algebra and a partially conserved tensor current (PCTC) hypothesis to the study of renormalization effects for the vector-meson–baryon coupling constants, due to the first-order breaking of $SU(3)$ symmetry.

Let us consider the vector-meson–baryon vertex $B'\to B+V$. To the first-order symmetry breaking of $SU(3)$, the vector-meson–baryon vertex function can be written as[3]

$$\mathcal{G}(B'BV)=\mathcal{G}_0(B'BV)+\gamma\langle BV|S^8(0)|B'\rangle, \tag{1}$$

where γ is a constant, $S^8(0)$ is the eighth component of quark scalar current, and $g_0(B'BV)$ is the vertex function in the $SU(3)$ limit.

We shall use a PCTC hypothesis and the current-algebra method to evaluate the first-order symmetry-breaking term in Eq. (1). The PCTC hypothesis is postulated by the relation[4,5]

$$\partial_\mu \mathcal{T}_{\mu\nu}{}^j(x)=C\phi_\nu{}^{j'}(x), \tag{2}$$

where $\mathcal{T}_{\mu\nu}{}^j(x)$ is an antisymmetric tensor current, $\phi_\nu{}^j(x)$ is the Heisenberg field for the corresponding vector meson, and the PCTC constant C is

$$C=m_v^2 f, \tag{3}$$

with f being a constant independent of the mass variable m_v. If the antisymmetric tensor currents, whose space integrals are among the generators of $U(12)$ algebra,[4] can be described in terms of quark fields[5]

$$\mathcal{T}_{\mu\nu}{}^j(x)=\bar\psi(x)\sigma_{\mu\nu}\frac{\lambda^j}{2}\psi(x), \quad (j=0,\cdots,8) \tag{4}$$

we obtain the following equal-time commutation relations

$$\left[\int d^3x\, \mathcal{T}_{0\nu}{}^j(x), S^8(0)\right]=-id_{jj8}j_\nu{}^j(0), \quad (j=1,\cdots,8)$$
$$\left[\int d^3x\, \mathcal{T}_{0\nu}{}^0(x), S^8(0)\right]=-i(\sqrt{\tfrac{2}{3}})j_\nu{}^8(0), \tag{5}$$

with the quark scalar current $S^8(0)$ and the vector currents $j_\nu{}^j(x)$ being[1]

$$S^8(x)=\bar\psi(x)\frac{\lambda^8}{2}\psi(x),$$
$$j_\nu{}^j(x)=i\bar\psi(x)\gamma_\nu\frac{\lambda^j}{2}\psi(x). \tag{6}$$

Employing the reduction formula and PCTC hypothesis, the symmetry-breaking term in Eq. (1) becomes

$$(2q_0{}^j)^{1/2}\langle B_iV_j|S^8(0)|B_k'\rangle$$
$$=\frac{i}{m_j^2 f}\int d^4x\, e^{-iq^j\cdot x}(-\Box+m_j^2)\theta(x_0)$$
$$\times\langle B_i|[\partial_\mu \mathcal{T}_{\mu\nu}{}^j(x),S^8(0)]|B_k'\rangle\epsilon_\nu, \tag{7}$$

where i, j, and k are $SU(3)$ indices and m_j is the mass of the jth vector meson. Integrating by parts and continuing to $q^j\to 0$, we obtain

$$\lim_{q^j\to 0}(2q_0{}^j)^{1/2}\langle B_iV_j|S^8(0)|B_k'\rangle$$
$$=-\frac{i}{f}\langle B_i|\left[\int d^3x\, \mathcal{T}_{0a}{}^j(x), S^8(0)\right]|B_k'\rangle\epsilon_a, \tag{8}$$

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[5] M. Ademollo, R. Gatto, G. Longhi, and G. Veneziano, Phys. Letters **22**, 521 (1966).

with $a=1, 2, 3$. Using the commutation relations (5), we find, for Eq. (8),

$$\lim_{q^j\to 0}(2q_0{}^j)^{1/2}\langle B_iV_j|S^8(0)|B_k'\rangle = -\frac{\gamma}{f}d_{jj8}\langle B_i|j_a{}^j(0)|B_k'\rangle\epsilon_a,$$
$$\lim_{q^0\to 0}(2q_0{}^0)^{1/2}\langle B_iV_0|S^8(0)|B_k'\rangle = -\frac{\gamma}{f}(\tfrac{2}{3})^{1/2}\langle B_i|j_a{}^8(0)|B_k'\rangle\epsilon_a. \quad (9)$$

In Eq. (9), i, k are $SU(3)$ indices, $j=1, \cdots, 8$, and V_0 stands for the unitary-singlet vector meson.

Including the first-order symmetry breaking of $SU(3)$, the vector-meson–baryon vertex functions are then given by

$$g(B_iV_jB_k')=g_0(B_iV_jB_k') - \frac{\gamma}{f}d_{jj8}\langle B_i|j_a{}^j(0)|B_k'\rangle\frac{\epsilon_a}{(2q_0{}^j)^{1/2}},$$
$$g(B_iV_0B_k')=g_0(B_iV_0B_k') - \frac{\gamma}{f}(\sqrt{\tfrac{2}{3}})\langle B_i|j_a{}^8(0)|B_k'\rangle\frac{\epsilon_a}{(2q_0{}^0)^{1/2}}, \quad (10)$$

with $j=1, \cdots, 8$. The matrix element $\langle B_i|j_a{}^j(0)|B_k'\rangle$ in Eq. (10) is to be evaluated in the $SU(3)$ limit. In dealing with the vector-meson states ω_1 (the unitary singlet) and ω_8 (the $I=Y=0$ member of the octet), we assume that the physical-meson states ω and Φ are given by the relations[6]

$$\omega=(\sqrt{\tfrac{1}{3}})\omega_8+(\sqrt{\tfrac{2}{3}})\omega_1,$$
$$\phi=-(\sqrt{\tfrac{2}{3}})\omega_8+(\sqrt{\tfrac{1}{3}})\omega_1. \quad (11)$$

Equation (10) can now be applied to obtain sum rules for various vector-meson–baryon couplings.

I. BARYON-DECUPLET–BARYON-OCTET VECTOR-MESON COUPLING

B_k', in Eq. (10), stands for the member of the $\frac{3}{2}^+$ baryon decuplet, and B_i stands for the member of the $\frac{1}{2}^+$ baryon octet in the present case. The matrix element for the baryon-decuplet–baryon-octet vector-meson coupling (apart from the usual kinematic factors) can be written as[7]

$$i\bar{u}_\mu(p')\left[g_1\delta_{\mu\nu}-i\frac{g_2}{m'+m}p_\mu\gamma_\nu + \frac{g_3}{(m'+m)^2}p_\mu p_\nu'\right]\gamma_5 u(p)\epsilon_\nu. \quad (12)$$

If the interaction is of the $M1$ type, we have

$$g=g_1=g_2,\quad g_3=0. \quad (13)$$

Applying the Wigner-Eckart theorem[8] to $g_0(B_iV_jB_k')$ and $\langle B_i|j_a{}^j(0)|B_k'\rangle$, we get a two-parameter expression for each vertex function[9]:

$$-\sqrt{2}g(N^*N\rho)=\sqrt{6}[g(Y_1{}^*\Sigma\rho)]=-2g(Y_1{}^*\Lambda\rho) =2g(\Xi^*\Xi\rho)=2\sqrt{3}g(Y_1{}^*\Sigma\omega) =2\sqrt{3}g(\Xi^*\Xi\omega)=G_0-\frac{\gamma}{f}\frac{1}{\sqrt{3}}G_1,$$
$$\sqrt{2}g(N^*\Sigma K^*)=\sqrt{6}[g(Y_1{}^*\Xi K^*)] =-\sqrt{6}[g(Y_1{}^*N\bar{K}^*)]=2g(\Xi^*\Sigma\bar{K}^*) =-2g(\Xi^*\Lambda\bar{K}^*)=g(\Omega\Xi\bar{K}^*)=G_0+\frac{\gamma}{f}\frac{1}{2\sqrt{3}}G_1, \quad (14)$$
$$2g(Y_1{}^*\Sigma\phi)=2g(\Xi^*\Xi\phi)=-\sqrt{\tfrac{2}{3}}\left[\left(G_0+\frac{\gamma}{f}\frac{2}{\sqrt{3}}G_1\right)\right],$$

where G_0 and G_1 are the reduced matrix elements for $g_0(B_iV_jB_k')$ and $\langle B_i|j_a{}^j(0)|B_k'\rangle\epsilon_a$, respectively. We obtain also a sum rule connecting the couplings of ρ, K^* and Φ with the baryons:

$$g(N^*N\rho)-2\sqrt{3}g(Y_1{}^*N\bar{K}^*)+\sqrt{3}g(\Xi^*\Xi\phi)=0. \quad (15)$$

From Eq. (14), we see that the sum rules among the couplings of one vector meson with baryons are exactly the same as the sum rules found in the $SU(3)$ limit, and that the renormalization effects appear only when the ρ coupling with baryons is compared to the K^* coupling or the Φ coupling with baryons. We also notice that the ρ and ω couplings have the same amount of renormalization effects, due to the use of Eq. (11).

II. BARYON-SINGLET–BARYON-OCTET VECTOR-MESON COUPLING

In this case, B_k' is the $\frac{1}{2}^-$ baryon singlet $Y_0{}^*(1405)$, and B_i stands for the member of the $\frac{1}{2}^+$ baryon octet. The matrix element for the present vertex can be written as

$$\bar{u}(p')[ig_{Y_0{}^*BV}\gamma_\mu\gamma_5+g_{Y_0{}^*BV}'\gamma_5(p'-p)_\mu]u(p)\epsilon_\mu. \quad (16)$$

Using the Wigner-Eckart theorem, Eq. (10) leads to a two-parameter expression for each vertex. For the coupling constants $g_{Y_0{}^*BV}$ and $g_{Y_0{}^*BV}'$, we obtain sum

[6] S. Okubo, Phys. Letters **5**, 165 (1963).
[7] See, e.g., J. D. Jackson and H. Pilkuhn, Nuovo Cimento **33**, 906 (1964).
[8] See J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).
[9] We follow here the notations of V. Gupta and V. Singh [Phys. Rev. **135**, B1442 (1964)]. See also M. Muraskin and S. L. Glashow, *ibid.* **132**, 482 (1963); C. Becchi, E. Eberle, and G. Morpurgo, *ibid.* **136**, B808 (1964).

rules of the form

$$g_{Y_0^*\Sigma\rho}=\sqrt{3}g_{Y_0^*\Lambda\omega},$$
$$g_{Y_0^*N\bar{K}^*}=g_{Y_0^*\Xi K^*}, \quad (17)$$
$$g_{Y_0^*\Sigma\rho}-2g_{Y_0^*N\bar{K}^*}-\sqrt{\tfrac{3}{2}}(g_{Y_0^*\Lambda\phi})=0.$$

III. BARYON-OCTET–BARYON-OCTET VECTOR-MESON COUPLING

B_k' and B_i are the $\frac{1}{2}^+$ baryons in the present case. The matrix element for the BBV vertex is

$$\bar{u}(p')\left[g_{1BBV}\left(i\gamma_\mu+q_\mu\frac{m'-m}{q^2}\right)+ig_{2BBV}\frac{\sigma_{\nu\mu}q_\nu}{m'+m}\right]u(p)\epsilon_\mu, \quad (18)$$

where $q=p'-p$. Because the coupling constant $g_{1BB\rho(\omega_8)}$ is related to the charged form factor, and the coupling constant $g_{2BB\rho(\omega_8)}$ is related to the anomalous magnetic moment, we shall consider, in the following discussions, the coupling constant g_{1BBV} to be of a purely octet F-type coupling, and the coupling constant g_{2BBV} to have both F- and D-type couplings. We shall also take the coupling of a unitary-singlet meson with the octet baryons into consideration.

A. g_{1BBV} of F-Type Coupling

If the coupling constant g_{1BBV} is a F-type coupling, Eq. (10) gives a two-parameter formula for each coupling constant. The sum rules thus obtained are

$$g_{1NN\rho}=g_{1\Xi\Xi\rho}=\tfrac{1}{2}g_{1\Sigma\Sigma\rho}=g_{1NN\omega}=-g_{1NN\omega},$$
$$-g_{1N\Sigma K^*}=\frac{1}{\sqrt{3}}g_{1N\Lambda K^*}=-g_{1\Sigma\Xi K^*}=\frac{1}{\sqrt{3}}g_{1\Lambda\Xi K^*}, \quad (19)$$
$$-g_{1NN\phi}=g_{1\Xi\Xi\phi},$$
$$g_{1NN\rho}+2g_{1N\Sigma K^*}-\frac{1}{\sqrt{2}}g_{1NN\phi}=0,$$

and other coupling constants are zero. Like the previous cases, the sum rules among the couplings of one vector meson with baryons are the same as those obtained in the $SU(3)$ limit, and renormalization effects are present only when we compare the coupling of one vector meson with the couplings of different vector mesons. Also, renormalization effects are absent between the ρ and ω couplings.

B. g_{1BBV} of F-Type and Singlet Couplings

The coupling constant g_{1BBV} now has three parameters. Since the unitary-singlet coupling contributes only to the ω and Φ couplings, the coupling constants $g_{1BB\rho}$ and g_{1BBK^*} remain the same as those in the case of Sec. III A. On the other hand, the sum rules involving the ω and Φ mesons become

$$2g_{1NN\rho}=g_{1NN\omega}-g_{1\Xi\Xi\omega},$$
$$g_{1NN\rho}+2g_{1\Xi\Xi\omega}-\frac{1}{2\sqrt{2}}(g_{1NN\phi}-g_{1\Xi\Xi\phi})=0, \quad (20)$$
$$\tfrac{1}{2}(g_{1NN\omega}+g_{1\Xi\Xi\omega})=\frac{1}{\sqrt{2}}(g_{1NN\phi}+g_{1\Xi\Xi\phi})=g_{1\Sigma\Sigma\omega}=g_{1\Lambda\Lambda\omega}=\sqrt{2}g_{1\Sigma\Sigma\phi}=\sqrt{2}g_{1\Lambda\Lambda\phi}.$$

C. g_{2BBV} of Both F- and D-Type Couplings

Since the coupling constant g_{2BBV} has both F- and D-type couplings, we obtain a four-parameter formula for each coupling constant. The sum rules thus obtained are

$$-g_{2\Sigma\Sigma\rho}+g_{2NN\rho}+g_{2\Xi\Xi\rho}=0,$$
$$-\sqrt{3}g_{2\Sigma\Lambda\rho}+g_{2NN\rho}-g_{2\Xi\Xi\rho}=0, \quad (21)$$
$$\sqrt{3}g_{2N\Lambda K^*}-g_{2N\Sigma K^*}-2g_{2\Xi\Sigma K^*}=0,$$
$$2g_{2N\Sigma K^*}+\sqrt{3}g_{2\Xi\Lambda K^*}+g_{2\Xi\Sigma K^*}=0,$$

and

$$g_{2NN\omega}-g_{2\Xi\Xi\omega}=2g_{2\Sigma\Sigma\rho},$$
$$-g_{2\Lambda\Lambda\omega}=g_{2\Sigma\Sigma\omega}=\sqrt{3}g_{2\Sigma\Lambda\rho},$$
$$g_{2NN\omega}+g_{2\Xi\Xi\omega}+g_{2\Sigma\Sigma\omega}=0,$$
$$g_{2\Lambda\Lambda\phi}=-g_{2\Sigma\Sigma\phi}, \quad (22)$$
$$g_{2NN\phi}+g_{2\Xi\Xi\phi}+g_{2\Sigma\Sigma\phi}=0,$$
$$g_{2\Sigma\Lambda\rho}-\frac{2}{\sqrt{3}}(g_{2N\Sigma K^*}-g_{2\Xi\Sigma K^*})-\sqrt{\tfrac{3}{2}}g_{2\Sigma\Sigma\phi}=0,$$
$$g_{2\Sigma\Sigma\rho}+\frac{2}{\sqrt{3}}(g_{2N\Lambda K^*}-g_{2\Xi\Lambda K^*})-\frac{1}{\sqrt{2}}(g_{2NN\phi}-g_{2\Xi\Xi\phi})=0.$$

Here, the renormalization effects are present only in the last two sum rules of Eq. (20), while other sum rules are the same as those found in the $SU(3)$ limit.

D. g_{2BBV} of F-type, D-type, and Singlet Couplings

In this case, each coupling constant has five parameters. Because the singlet coupling does not contribute to the couplings involving ρ and K^* mesons, the sum rules (19) remain unaffected. For coupling constants involving ω or Φ mesons, we have, instead of Eq. (20), the following sum rules.

$$g_{2NN\phi}+g_{2\Xi\Xi\phi}=2g_{2\Lambda\Lambda\phi},$$
$$g_{2NN\omega}+g_{2\Xi\Xi\omega}=2g_{2\Lambda\Lambda\omega},$$
$$g_{2NN\omega}-g_{2\Xi\Xi\omega}=2g_{2\Sigma\Sigma\rho},$$
$$g_{2\Sigma\Sigma\omega}-g_{2\Lambda\Lambda\omega}=2\sqrt{3}g_{2\Sigma\Lambda\rho},$$
$$g_{2\Sigma\Sigma\omega}+g_{2\Lambda\Lambda\omega}=\sqrt{2}(g_{2\Sigma\Sigma\phi}+g_{2\Lambda\Lambda\phi}), \quad (23)$$
$$g_{2\Sigma\Lambda\rho}-\frac{2}{\sqrt{3}}(g_{2N\Sigma K^*}-g_{2\Xi\Sigma K^*})-\tfrac{1}{2}\sqrt{\tfrac{3}{2}}(g_{2\Sigma\Sigma\phi}-g_{2\Lambda\Lambda\phi})=0,$$
$$g_{2\Sigma\Sigma\rho}+\frac{2}{\sqrt{3}}(g_{2N\Lambda K^*}-g_{2\Xi\Lambda K^*})-\frac{1}{\sqrt{2}}(g_{2NN\phi}-g_{2\Xi\Xi\phi})=0.$$

In Eq. (21), the renormalization effects appear only in the last two sum rules.

The sum rules obtained in the present note are much stronger than the sum rules obtained from the pure group-theory methods.[9] In employing the current-algebra method, our sum rules are subject to the off-mass-shell corrections. They become exact only when our use of the limit $q \to 0$ is justified. However, we notice that the vector-meson–baryon coupling constants considered in this note can only be determined indirectly from the information of vertex factors in certain pole-dominant reaction processes. If the coupling constant $g_{B'BV}$ is to be defined as the corresponding vertex factor $g_{B'BV}(q^2 \approx 0)$ at relatively small momentum transfer in the pole-dominant reaction process, and if the form factor $g_{B'BV}(q^2)$ is a slowly varying function of momentum transfer q^2, then our sum rules will be quite satisfactory, and the use of the limit $q \to 0$ is justifiable. Of course, the final justification of our sum rules depends on the postulate of the partially conserved tensor currents.

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Phenomenological Analysis of the Photoproduction of Neutral Vector Mesons and Strange Particles*

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Branching ratios for photoproduction of vector mesons and strange particles are discussed. The one-pion-exchange mechanism cannot explain the observed ratio between ρ and ω photoproduction cross sections. Various versions of pseudoelastic mechanisms are studied and it is shown that although they correctly predict the large $\rho:\omega$ production ratio, they cannot account for the extremely small preliminary cross section for φ production. It is shown that no combination of one-pion exchange and the diffraction mechanism with exact or broken $SU(3)$ can explain the low φ production rate. The multiperipheral model may explain the low φ production, but predicts the wrong $\rho:\omega$ production ratio. Various possible sources of this discrepancy are studied and experimental tests are discussed which can distinguish between the different proposed theories. A large number of new predictions based on exact or broken $SU(3)$ symmetry are derived and compared with experiment.

I. INTRODUCTION

RECENT counter and bubble-chamber experiments at the Cambridge Electron Accelerator and DESY have yielded a large amount of information on the photoproduction of meson and baryon resonances at intermediate photon energies of 1–6 GeV. This has provided for the first time a possibility of testing some theoretical ideas which had been proposed in the last few years in order to explain the production mechanisms of these resonances and the branching ratios among the various competing channels.

Some particular aspects which have recently attracted wide attention are the phenomenology of the photoproduction of neutral vector mesons at forward angles and the production rates of strange particles. These reactions are of great experimental and theoretical importance. Experimentally, they may serve as the main sources of future secondary π and K beams in high-energy electron accelerators. Theoretically, they provide a convenient testing ground for ideas such as $SU(3)$ symmetry and its breaking, vector-meson pole dominance of the electromagnetic current and the mechanisms which are responsible for pseudoelastic scattering processes.

Our purpose in this paper is to study the general problem of the relative intensities of various competing photoproduction reactions and to derive predictions for the relevant production rates using, as input, various possible dynamical assumptions, broken and unbroken $SU(3)$ symmetry, and coupling constants which are either known or can be independently determined from vector-meson decay rates. In a few cases, we will briefly mention the predictions of some more speculative theories such as $SU(6)_W$ and the quark model.

We first discuss processes of the type

$$\gamma+p \to V^0+p, \tag{1}$$

where V^0 is a neutral vector meson (ρ^0, ω or φ). Our particular interest in the reaction (1) stems from two

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