Alpha Reduced Widths of Even-Mass Polonium Nuclei

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An attempt is made to obtain the alpha decay rates of even-mass polonium nuclei as a product of two factors: (a) the probability of occurrence of an alpha cluster in a surface well, and (b) the decay rate when an alpha cluster is already formed there. From a comparison with the empirical values, it seems that our method for calculating the decay rates is valid. It is shown that the break in alpha reduced width on going from Po^{210} to Po^{212} can be interpreted as due to a break in the well width. It is suggested that the nuclear surface of Pb^{206} is considerably more diffuse than that of doubly magic Pb^{208}. It is also pointed out that our method gives a larger probability of finding an alpha cluster for Po^{210} than for Po^{212}, whereas the Mang theory leads to the opposite result—a larger probability for Po^{212}.

I. INTRODUCTION

THE theory of alpha decay proposed by Mang[1] is capable of explaining the ratios of alpha reduced widths; however, the calculated decay rates are too small by a factor between 10 and 1000. In a previous work[2] we have shown that the decay rate of Po^{212} can be obtained in terms of two factors: (a) the probability of occurrence of an alpha cluster in a surface well, and (b) the decay rate when an alpha cluster is already formed there. We now extend the investigation to other even-mass polonium nuclei, and show that our method for calculating the decay rates seems to be valid.

II. ALPHA REDUCED WIDTH

First we assume that the probability of occurrence of an alpha cluster in a (parent) nucleus is given by $f_v|A_0|^2$, where f_v is the ratio[3] of the volume of a surface well[4] to the nuclear volume, and A_0 is the overlap of the wave function of the parent nucleus and that of the daughter cluster plus the alpha cluster. When an alpha cluster is already formed, it is assumed to be moving in the surface well before penetrating the Coulomb barrier or dissolving. Following the Bethe treatment[5] of the one-body model, we find that the decay rate in the absence of a potential barrier is given by

$$\lambda'=f_v|A_0|^2\left[\frac{16\pi(E-U)^{3/2}}{h(2k_1\Delta R-\sin 2k_1\Delta R)(B-E)^{1/2}}\right], \quad (1)$$

with

$$\tan k_1\Delta R=-\left(\frac{E-U}{B-E}\right)^{1/2}. \quad (2)$$

Here B is the barrier height; h, Planck's constant; E, the total energy of the daughter cluster plus the alpha cluster; ΔR, the width of the surface well; U, the potential energy when the alpha cluster is inside the well; and $k_1=2\pi[2M(E-U)]^{1/2}/h$, M being the reduced mass.

It is noted that λ' is proportional to the alpha reduced width δ^2 of Rasmussen[6] ($\delta^2=h\lambda'$). Rasmussen has computed the values of δ^2 from measured decay rates and with the use of Igo's optical-model potential; his values will be referred to as the empirical values.

III. COMPARISON BETWEEN CALCULATED AND EMPIRICAL REDUCED WIDTHS

A. Reduced Widths of Po^{210} and Po^{212}

Taking into account configuration mixing, the value of $|A_0|^2$ for Po^{212} was previously[2] calculated to be 0.00412. The enhancement due to configuration mixing is of a factor of 7.1. In the same way, with the configurations[7]

Pb^{206}: protons: closed shell,
neutrons: $0.866(2p_{1/2})^2+0.316(1f_{5/2})^2+0.387(2p_{3/2})^2$,
Po^{210}: protons: $0.975(0h_{9/2})^2+0.224(1f_{7/2})^2$,
neutrons: closed shell,

the value of $|A_0|^2$ for Po^{210} was found to be 0.0319. The enhancement in this case is of a factor of 4.3.

The outer radius of the surface well may be interpreted as the sum of the "radii" of an alpha particle and the daughter nucleus. Since no experiment has suggested a break in nuclear radius on going from Pb^{206} to Pb^{208}, we used the same value of the outer radius for both Po^{210} and Po^{212} decay; for convenience a value of 10 F was chosen. In the evaluation of f_v the nuclear radius was taken to be equal to the outer radius.

By assuming that the calculated and empirical values of the ratio[8] $\delta^2(Po^{210})/\delta^2(Po^{212})$ agree, we can find, from Eqs. (1) and (2), the ratio $\Delta R(Po^{210})/\Delta R(Po^{212})$.[9] The results for ground-state transitions are given in Table I.

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[1] H. J. Mang, Ann. Rev. Nucl. Sci. **14**, 1 (1964).
[2] F. C. Chang, Phys. Rev. **141**, 1136 (1966).
[3] The factor f_v is intended to take account of the fact that clustering is possible only in the nuclear surface.
[4] G. H. Winslow, Phys. Rev. **96**, 1032 (1954).
[5] H. A. Bethe, Rev. Mod. Phys. **9**, 161 (1937).
[6] J. O. Rasmussen, Phys. Rev. **113**, 1593 (1959).
[7] The configurations chosen here are similar to those used in Harada's calculations [Progr. Theoret. Phys. (Kyoto) **26**, 667 (1961)].
[8] N. K. Glendenning and K. Harada, Nucl. Phys. **72**, 481 (1965). In a detailed shell-model calculation they obtained the ratio $\delta^2(Po^{210})/\delta^2(Po^{212})$ for ground-state transitions, in satisfactory agreement with experiment.
[9] Winslow obtained a value of $\Delta R=0.6$ F for Po^{210} decay, and a larger value of $\Delta R=1.2$ F for Po^{212} decay, but he did not explain why such a difference in ΔR should occur.

TABLE I. Ratio of the well width for Po^{210} decay to that for Po^{212} decay for different well depths.

Trial	Nucleus	$\|A_0\|^2$	f_v	U (MeV)	ΔR (F)	$\Delta R(Po^{210})/\Delta R(Po^{212})$
1	Po^{210}	0.0319	0.803	3.06	4.19	10
	Po^{212}	0.00412	0.115	−112	0.400	
2	Po^{210}	0.0319	0.994	4.74	8.18	8.2
	Po^{212}	0.00412	0.271	−17.0	1.00	
3	Po^{210}	0.0319	1.00	4.96	10.0	7.2
	Po^{212}	0.00412	0.359	−6.39	1.38	

Although ΔR is a parameter describing the nuclear interaction between an alpha cluster and a daughter cluster, it seems plausible, as clustering is possible only in the nuclear surface,[10] that a larger value of ΔR is associated with a more diffuse surface. Hence, the values of $\Delta R(Po^{210})/\Delta R(Po^{212})$ in Table I may indicate that the surface of Pb^{206} is considerably more diffuse than that of doubly magic Pb^{208}.

According to Eqs. (1) and (2) δ^2 is a decreasing function of ΔR. We calculated the values of δ^2 for Po^{210} and Po^{212}, with $\Delta R = 10.0$ and 1.38 F, respectively. In both cases the calculated value is larger than the empirical one by a factor of about 2.9.

B. Relative Values of Reduced Widths

Using the pure configurations assumed by Mang[1] and choosing, somewhat arbitrarily, four values for ΔR, we calculated the relative values of δ^2 for even-mass polonium nuclei. The results, together with the values of $|A_0|^2$, are given in Tables II and III. The relative values of δ^2 for Po^{210} and Po^{212} are taken to be 1.00 and 10.6, respectively, so that the calculated and empirical values of $\delta^2(Po^{210})/\delta^2(Po^{212})$ agree.

TABLE II. Values of $|A_0|^2$ and relative values of δ^2 for even-mass polonium nuclei with mass number $\leqslant 210$.

		δ^2		
			Calculated	
Nucleus	$\|A_0\|^2$	Empirical	$\Delta R = 4.18$ F	$\Delta R = 10.0$ F
Po^{202}	0.00122	3.70	0.166	0.166
Po^{204}	0.00390	4.42	0.529	0.529
Po^{206}	0.00326	2.44	0.441	0.441
Po^{208}	0.00275	1.54	0.371	0.371
Po^{210}	0.00738	1.00	1.00	1.00

TABLE III. Values of $|A_0|^2$ and relative values of δ^2 for even-mass polonium nuclei with mass number >210.

		δ^2		
			Calculated	
Nucleus	$\|A_0\|^2$	Empirical	$\Delta R = 0.400$ F	$\Delta R = 1.38$ F
Po^{212}	0.000577	10.6	10.6	10.6
Po^{214}	0.000882	16.4	15.7	15.8
Po^{216}	0.000978	16.1	17.1	17.3
Po^{218}	0.000882	17.8	15.1	15.3

[10] See, for instance, D. R. Inglis, Rev. Mod. Phys. 34, 169 (1962).

In view of the crudeness of our calculation method, it is noteworthy that, for Po^{212}, Po^{214}, Po^{216}, and Po^{218}, there is reasonable agreement between the calculated and empirical relative values of δ^2. For polonium nuclei with mass number $\leqslant 210$ the discrepancy between the calculated and empirical relative values of δ^2 is considerable. Still there is some similarity in trend, when Po^{210} is excluded. The discrepancy can be lessened, if it is assumed that, for these nuclei, configuration mixing becomes larger as the mass number decreases.

IV. DISCUSSION

It is of interest to point out that our calculation method gives a larger probability of finding an alpha cluster for Po^{210} than for Po^{212}, whereas the Mang theory leads to the opposite result— a larger probability for Po^{212}. The disagreement can be attributed to one essential difference between the two approaches. The Mang theory derives the probability amplitude that an alpha cluster is present near the nuclear surface, by projecting from the wave function of the parent nucleus (near the nuclear surface) a component describing the daughter nucleus plus an alpha particle. On the other hand, our method obtains the probability amplitude that an alpha cluster is present in the nuclear surface, from the overlap (over the entire nuclear volume) of the wave function of the parent nucleus and that of the daughter cluster plus an alpha cluster.

Finally we discuss briefly some of the uncertainties in our calculations. First, the use of harmonic oscillator wave functions is questionable. However, such wave functions are not expected to cause appreciable errors in the ratios of alpha reduced widths. Furthermore, the wave function of Po^{212} was constructed neglecting the neutron-proton force. The effect of the neutron-proton force on alpha reduced width has been studied by several authors.[8,11] Their studies show that, in the case of Po^{212}, the neutron-proton force contributes an enhancement factor of about 1.2. When this additional enhancement is taken into account, the values of $\Delta R(Po^{210})/\Delta R(Po^{212})$ in Table I are lowered by less than 10%. Second, our knowledge of the interaction between an alpha particle and a nucleus is inadequate. In particular, it is not known whether or not the potential barrier in Po^{210} decay is, as Rasmussen has assumed in his computation of δ^2, the same as that in Po^{212} decay. Of course any modification of the barrier will affect the empirical values of δ^2. Lastly, it is seen from Table I that the values of f_v for Po^{210} are probably too large. By lowering the value of f_v for Po^{210},[12] we can bring closer agreement between the calculated and empirical reduced widths.

[11] J. O. Rasmussen, Nucl. Phys. 44, 93 (1963).

[12] For any meaningful reduction in the value of f_v for Po^{210}, it can be shown that $\Delta R(Po^{210})$ remains to be several times larger than $\Delta R(Po^{212})$.