The remaining case to be discussed is that of the thermal spectrum of Mn^{56}. The ratio of capture in the two-spin state at thermal energies has been measured[22] to be $\sigma_{J=3}/\sigma_{J=2}=0.96$. Since the relative contributions from both spin states is roughly equal, a high degree of correlation might be expected in this case. However, Coté and Bollinger[23] have shown that interference exists between the partial radiative cross sections of two or more resonances. It is highly possible that the observed widths at thermal energies are affected by such interference effects, thus again weakening the correlation. It can be concluded therefore, that the results observed in this work may be encompassed within the framework of a direct-capture model. A more quantitative evaluation can only be made when further information regarding the spins and stripping amplitudes of the final states are obtained.

ACKNOWLEDGMENTS

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Violation of Seniority in the Reaction $^{43}Ca(d,p)^{44}Ca$

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The reaction $^{43}Ca(d,p)^{44}Ca$ has been studied at 8.50- and 4-MeV bombarding energies. In the higher energy experiment, proton angular distributions were measured for 31 transitions, by means of the Aldermaston multiangle spectrograph. The l_n values and spectroscopic strengths were obtained from a distorted-wave analysis of the angular distributions. Energy levels were determined in the 4-MeV experiment by detecting the protons in the Copenhagen heavy-particle spectrograph. A ground-state Q value of 8920 ± 10 keV was obtained. The spectroscopic strengths were analyzed in terms of the non-energy-weighted sum rules for multipole orders $\lambda=0$, 1, and 2. Eight $f_{7/2}$ transitions were observed, compared with four predicted by the seniority-coupling scheme.

I. INTRODUCTION

THE configuration $(f_{7/2})^4$ with $T=T_z=2$ is one of the simplest cases in which the angular-momentum quantum numbers are not sufficient for characterizing the states allowed by the Pauli principle. A study of the neutron-transfer reaction $^{43}Ca(d,p)^{44}Ca$ therefore may give valuable information on the coupling-scheme situation in ^{44}Ca.[1] The only previous work on the $^{43}Ca(d,p)^{44}Ca$ reaction[2] gave information on the ^{44}Ca energy levels; in the present experiment the l_n values and spectroscopic strengths were measured in addition to level energies.

II. EXPERIMENTAL METHOD, RESULTS, AND ANALYSIS

The ^{43}Ca target was prepared in the Copenhagen isotope separator[3] to an isotopic purity of $\gtrsim 99\%$.

The angular-distribution measurements were made at the Aldermaston tandem accelerator at a bombarding energy of 8532 keV. The multiangle spectrograph of Middleton and Hinds[4] was used for momentum analysis of the reaction protons. The over-all energy resolution was 15 keV. A proton spectrum is shown in Fig. 1, and the measured angular distributions are presented in Figs. 3–6.

The cross-section scale in mb/sr was established by measuring the ratio of (d,p) yield to (d,d) yield at a bombarding energy of 8522 keV and by using the elastic-scattering cross sections of Ref. 3. It was assumed that the (d,p) cross sections at 8532 and 8522 keV were identical. The estimated uncertainty on the cross-section scale is $\pm25\%$.

A more detailed description of the experimental procedures may be found in a previous paper.[5] Energy levels in ^{44}Ca were also determined from the $^{43}Ca(d,p)$ process at about 4-MeV bombarding energy. The deuterons were accelerated in the Copenhagen 4.5-MeV

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[1] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. 32, 567 (1960).

[2] C. M. Braams, thesis, University of Utrecht, 1956 (unpublished).

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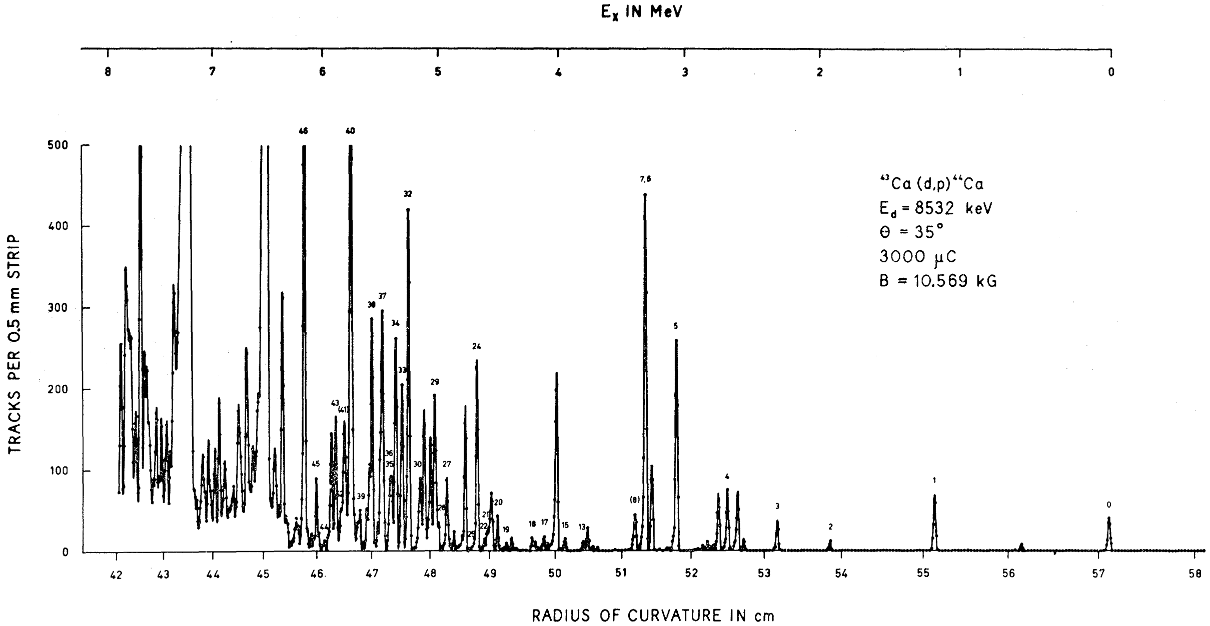

FIG. 1. Proton spectrum from the ^{43}Ca(d,p) reaction at 8532-keV incident energy. Proton groups corresponding to a residual mass of 44 are marked with level numbers in correspondence with the notation used in Table I. When an impurity group is coincident with a mass-44 group at this reaction angle, the level number is given in parentheses. Unmarked groups at proton energies higher than that of group No. 46 originate from light impurities of C, N, O, Si, S, and Cl. No analysis was attempted below group No. 46.

electrostatic generator and the reaction protons were analyzed in a heavy-particle spectrograph. The experimental set-up and procedures have been described previously.[6] The ^{210}Po α-particle energy was taken from the work of Rytz.[7] A proton spectrum is shown in Fig. 2. The ground-state Q value was measured to be 8920±10 keV, in good agreement with Ref. 2. The excitation energies determined in the present experiment are given in column 3 of Table I, whereas the values of Braams[2] are shown in column 2.

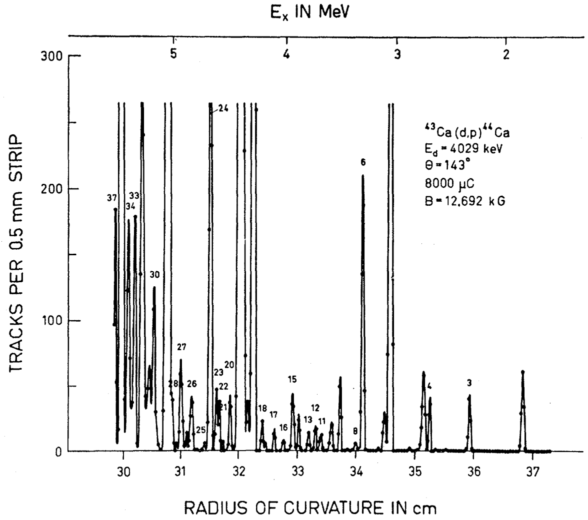

FIG. 2. Proton spectrum from the ^{43}Ca(d,p) reaction at 4029-keV incident energy. Notation as in Fig. 1.

It is clear from the spectra of Figs. 1 and 2 that many weak (d,p) transitions may have been missed in the present measurements.

The angular distributions were analyzed with distorted-wave (DW) techniques. The deuteron optical-model parameters were taken from Ref. 3. and the proton parameters from the work of Perey.[8] The captured neutron was assumed to move in a potential well of Woods-Saxon shape with a binding energy equal to the experimental separation energy ($Q_{d,p}+2.23$ MeV) and with geometrical parameters as in Ref. 5. The DW calculations[9] were made in the zero-range approximation, lower cutoffs were not employed in the radial integrals, and spin-orbit and nonlocality effects were ignored. The calculated angular distributions are shown in Figs. 3–6 in comparison with the experimental data.

Spectroscopic strengths

$$[(2J_B+1)/(2J_A+1)]\times S(J_A+j\rightarrow J_B)$$

were determined by matching the sum of the observed differential cross sections for a given transition to the corresponding sum of calculated differential cross

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[9] The DW calculations were performed at the Massachusetts Institute of Technology, Computing Center. The code JULIE originated by R. M. Drisko was used.

TABLE I. ^{44}Ca levels and spectroscopic strengths.[a]

Level No.	E_x (keV) Braams[c]	E_x (keV) This expt.	Spectroscopic strength[b] $2s/3s$	$2p$	$1f$	Comments
0	0	0	···	···	0.36	
1	1157±4	1162±10	···	0.05	0.36	
2	1885±4	1886±10	···	···	0.07	
3	2286±5	2289±10	···	0.008	0.22	
4	2658±5	2668±10	···	$\leq$0.01	0.45	
5	3047±5	3052±10	···	···	1.46	
6	3300±6	3296±10	···	···	2.45	Not resolved from level 7
7	3308±6	3302±10	···	···	···	Weak, no l_n assignment
8	3357±6	3367±10	···	···	···	Weak, no l_n assignment
9	3585±6		···	···	···	Not observed here
10	3660±6		···	···	···	Not observed here
11	3675±6	3682±10	···	···	···	In the 4-MeV expt. only
12		3729±10	···	···	···	In the 4-MeV expt. only
13		3792±10	···	···	···	Weak, no l_n assignment
14		3880±10	···	···	···	In the 4-MeV expt. only
15		3934±10	···	(0.04)	···	
16		4026±10	···	···	···	In the 4-MeV expt. only
17		4104±10	···	···	0.09	
18		4207±10	···	0.02	···	
19		4410±10	0.013/0.005	···	···	
20		4491±10	···	(0.04)	···	
21		4569±10	···	···	···	Weak, no yields
22		4598±10	···	···	···	Weak, no yields
23		4616±10	···	···	···	In the 4-MeV expt. only
24		4662±10	···	0.28	···	
25		4696±10	···	···	···	Weak, no yields
26		4826±10	···	···	···	In the 4-MeV expt. only
27		4914±10	···	0.12	···	
28		4992±10	···	0.05	···	
29		5016±10	···	0.25	···	In the 8.5-MeV expt. only
30		5143±10	···	0.12	···	
31		5172±10	···	···	···	In the 4-MeV expt. only
32		5243±10	···	0.54	···	
33		5296±10	···	0.27	···	
34		5351±10	···	0.28	···	
35		5385±10	···	0.07	···	In the 8.5-MeV expt. only
36		5405±10	···	(0.01)	···	In the 8.5-MeV expt. only
37		5468±10	···	0.33	···	
38		5558±10	···	0.40	···	Levels 38–46 In the 8.5-MeV expt. only
39		5666±10	···	···	···	No yields
40		5743±10	···	0.75	···	
41		5776±10	···	···	···	No yields
42		5832±20	···	···	···	No yields
43		5873±10	···	(0.16)	···	
44		5975±10	···	···	···	No yields
45		6050±10	···	0.08	···	
46		6156±10	···	0.46	···	
		Strength sums	0.013/0.005	4.33	5.46	

[a] The energies of Ref. 2 have been corrected to the more recent value for the energy of ^{210}Po α particles (see Ref. 7). The E_x of the next column is the average value of the 4- and 8.5-MeV data. The strengths are given in single-hole units. No l_n values other than 0, 1, and 3 were found. The uncertainty on the strengths is 25% from the uncertainty on the cross-section scale alone.

[b] If a strength is given in parentheses, the l_n assignment is tentative.

[c] See Ref. 2.

sections. In cases of mixed l_n values ($l_n=1$ and 3), the ratio between the two contributions was obtained by comparing experimental and calculated intensities at angles around 20° (where $l_n=1$ distributions have their maximum) and around 35° (where $l_n=3$ transitions are at maximum). The $1f$ strengths obtained as described above were multiplied by a factor of 1.45 in order to simulate the neglected effects of finite range, spin-orbit coupling, and nonlocality (see, e.g., the discussion of Bjerregaard *et al.*[5] and Lee *et al.*[10]). No corrections were applied to the $2p$ strengths. The final strengths are presented in Table I. The $l_n=1$ angular distributions (Figs. 3–6) exhibit no evidence for a sharp back-angle minimum as observed for even targets and $2p_{1/2}$ transitions in this region.[11]

III. DISCUSSION

A. Level Scheme

The low-lying part of the ^{44}Ca level scheme is shown in Fig. 7. The spins and parities of the states at 0,

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[11] L. L. Lee and J. P. Schiffer, Phys. Rev. **136**, B405 (1964).

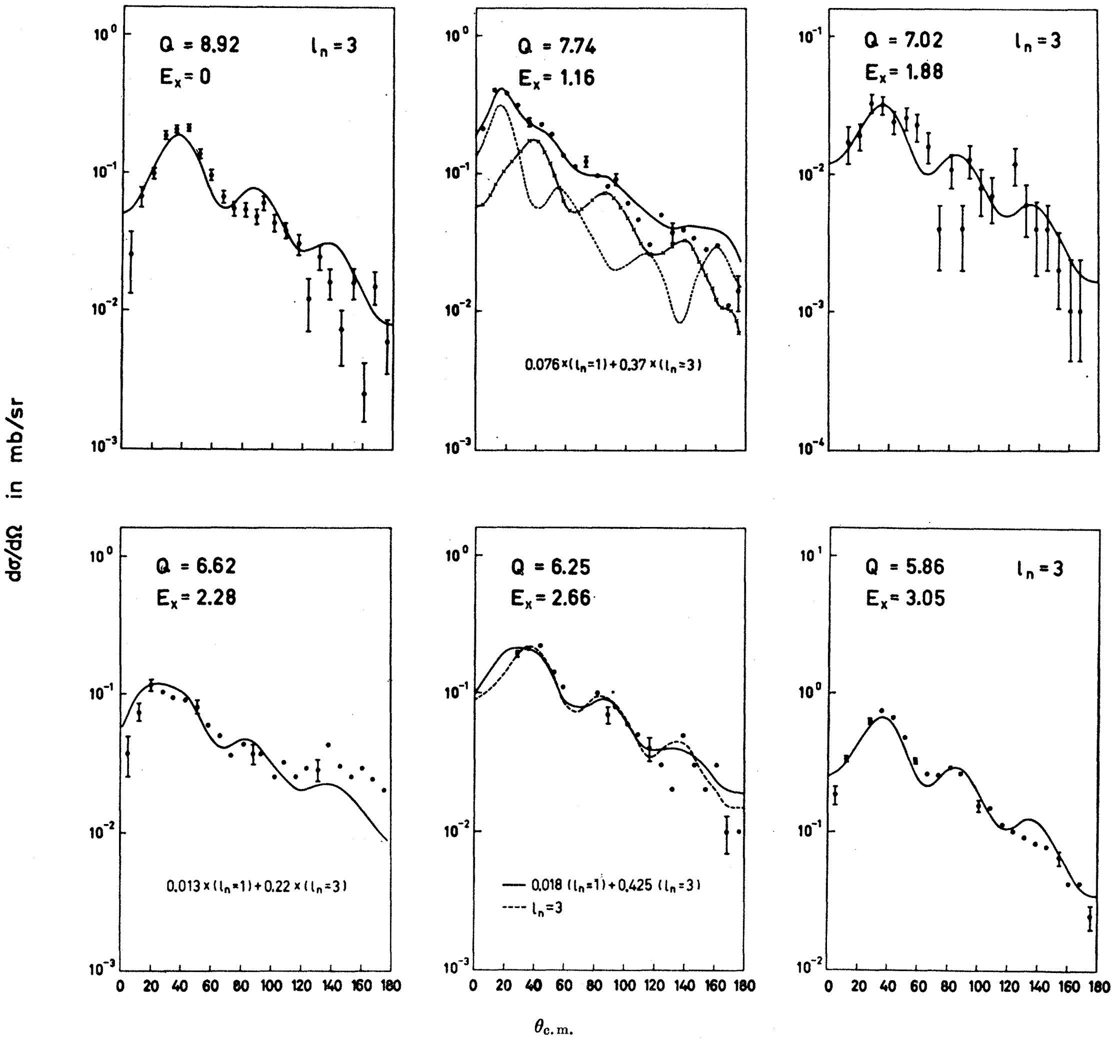

FIG. 3. Angular distributions from the ^{43}Ca(d,p) reaction at 8.5-MeV bombarding energy. The experimental points are marked with filled circles. The error flags indicate statistical and scanning errors. The cross-section scale has an additional uncertainty of ±25%. The curves are DW predictions (see the text). In one case the decomposition into $l_n=1$ and $l_n=3$ contributions is shown. Each distribution is identified by the Q value and excitation energy. Level No. 7 was not resolved from level No. 6, but the yield from level No. 7 is less than 10% of the yield from level No. 6 as estimated from observations at angles with particularly good energy resolution.

1162, 1886, and 2668 keV are known,[12,13] whereas the spin assignments for the 2289-, 3052-, and 3296-keV states are tentative and supported by the present data only.

A 6^+ state of the $(f_{7/2})^4$ configuration is expected around 3-MeV excitation by analogy to ^{42}Ca. Such a state should be excited in (d,p) with a pure $l_n=3$ transition and with a strength of 2.17, in good agreement with the data (Table I) for the 3296-keV state. The 4^+ assignments for the levels at 2289 and at 3052 keV were made because the sum of the $l_n=3$ strengths to these two states equals the $(f_{7/2})^3$, $J=\frac{7}{2}\rightarrow(f_{7/2})^4$, $J=4$ strength allowed by the shell model. (See also below.) There is, however, a conflict in the 4^+ assignment for the 2289-keV level. The $l_n=1+3$ character of the (d,p) transition limits the spin to $2\leq J\leq 5$ and the decay-scheme data[12] lead to $J=1$ or 2.

B. Strength Function and Sum-Rule Analysis

The ^{43}Ca strength function is shown in Fig. 8. The main portion of the $l_n=3$ strength is located below 3.3 MeV of excitation with only one small fragment at 4.1 MeV. Thus, the $l_n=3$ strength probably is due to $1f_{7/2}$ transfers and most of the available strength presumably has been observed. The analysis of the ^{43}Ca(d,t) experiment[3] was consistent with a pre-

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[13] S. M. Matin, D. J. Church, R. Horoshko, and G. E. Mitchell, Phys. Letters **15**, 51 (1965); G. E. Mitchell (private communication).

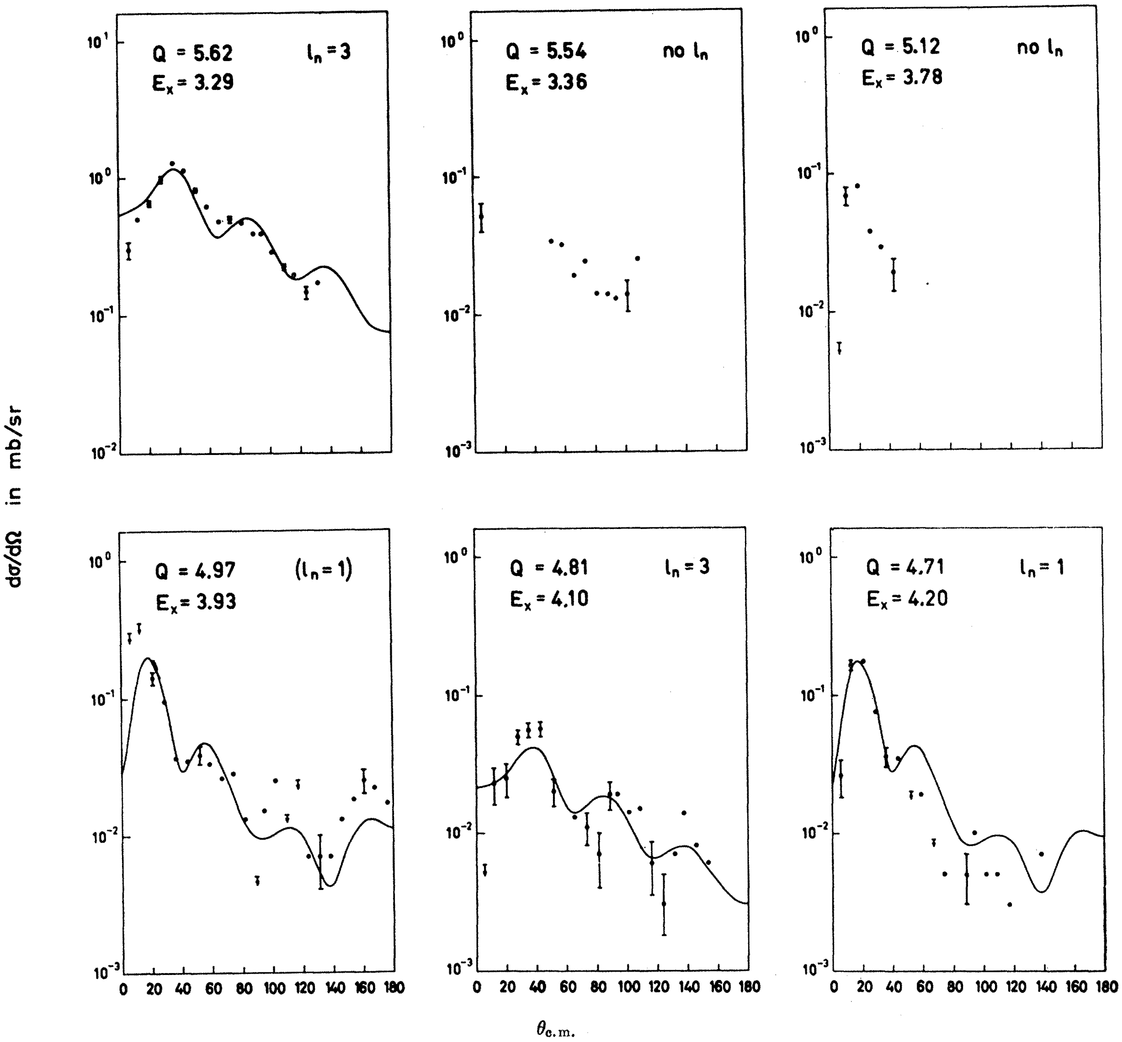

FIG. 4. Angular distributions from the ^{43}Ca(d,p) reaction at 8.5-MeV bombarding energy.

dominating $(1f_{7/2})^3$, $J=\frac{7}{2}$ character of ^{43}Ca(0); hence a (d,p) strength of ≈ 5 neutron holes is expected, in good agreement with the value of 5.4±25% found here (Table I).

The shape of the $2p$ strength function would suggest that only part of the total $2p$ strength has been found. The $2p$ strength sum is 4.3 (Table I) compared to the expected value of 6 neutron holes.

The conclusion from the (d,t) experiment[3] that an $(f_{7/2})^3$, $J=\frac{7}{2}$ configuration is probably predominant in ^{43}Ca(0) has further implications. For example, the (d,p) strength sums for definite final spin J_B should equal 0.50, 0.83, 1.50 and 2.17 for $J_B=0$, 2, 4 and 6, respectively. This is fulfilled for $J_B=0$ and 2 and, as shown above, spins of 4 and 6 may be assigned so that the corresponding (d,p) strengths obey this rule.

More information on the ^{43}Ca ground state can be obtained from the higher-order multipole sum rules of French.[14] The dipole sum rule for a neutron-capture reaction is

$$\frac{\langle J_AJ_A|J_0(j)|J_AJ_A\rangle}{J_A}=\sum_{i,J_B}\mathcal{L}(J_A,j,J_B)\times\frac{2J_B+1}{2J_A+1}S_i(J_A+j\rightarrow J_B). \quad (1)$$

The left-hand side of Eq. (1) is the relative contribution to the ground-state spin J_A coming from neutrons in the j orbital (here $j=\frac{7}{2}$). The factor $\mathcal{L}(J_A,j,J_B)$ is

$$\mathcal{L}(J_A,j,J_B)=\frac{J_A(J_A+1)+j(j+1)-J_B(J_B+1)}{2J_A(J_A+1)}, \quad (2)$$

and J_0 designates the z component of the spin operator.

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[14] J. B. French, Phys. Letters 13, 249 (1964).

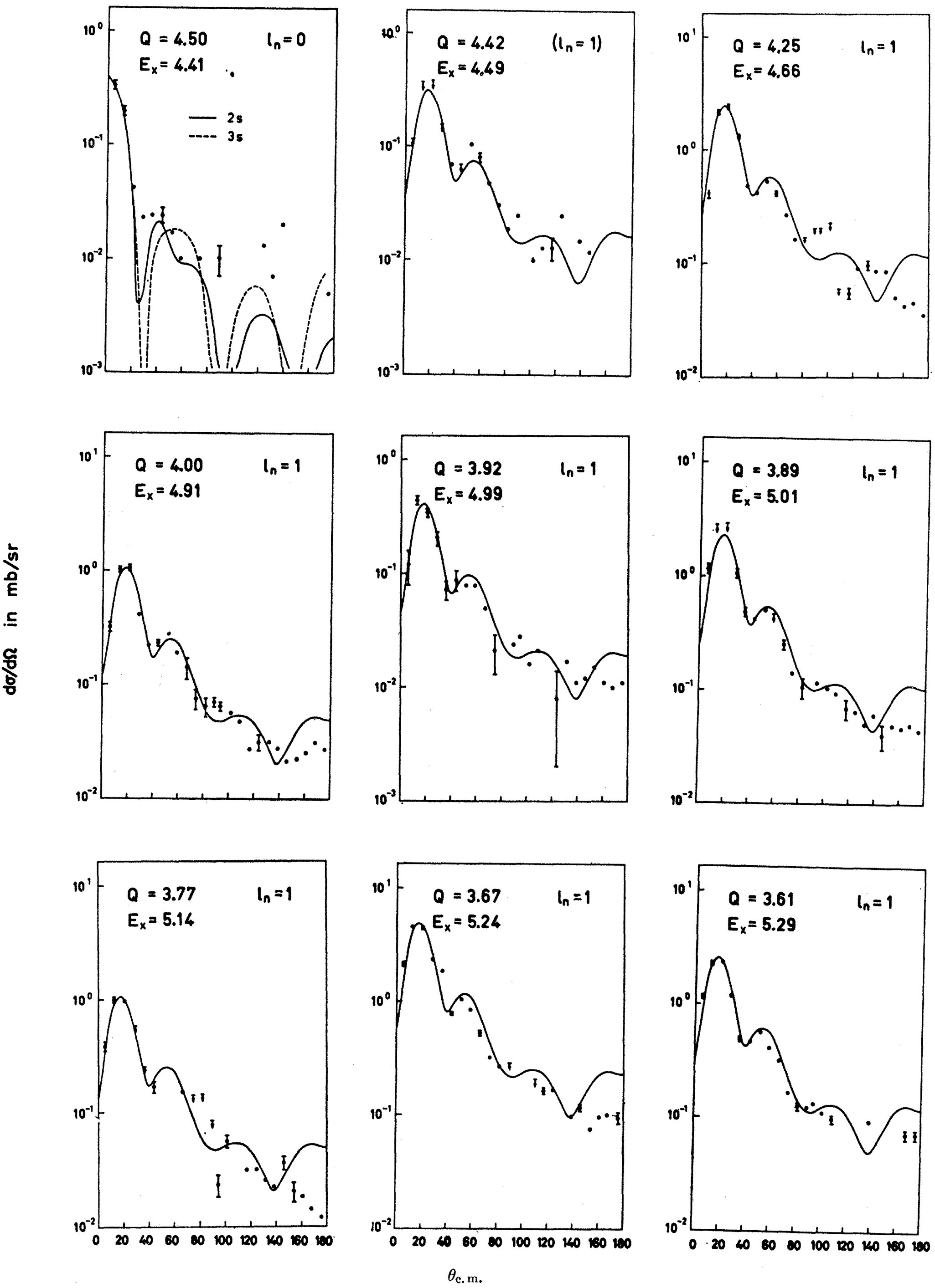

FIG. 5. Angular distributions from the ^{43}Ca(d,p) reaction at 8.5-MeV bombarding energy.

The corresponding sum rule for neutron pickup between nuclei of maximum isospin is

$$\frac{\langle J_AJ_A|J_0(j)|J_AJ_A\rangle}{J_A}=\sum_{i,J_C}\mathcal{L}(J_A,j,J_C)S_i(J_A\rightarrow j+J_C)\,. \quad (3)$$

The $f_{7/2}$ strengths of Table I combined with the spin assignments of Sec. IIIA lead to a value of the left-hand side of Eq. (1) of 0.9±0.2 and the (d,t) data[3] yield 1.0±0.2 for the same quantity [using Eq. (3)]. Thus, the $1f_{7/2}$ neutrons line up their spins in the ^{43}Ca ground state to produce an expectation value of $\frac{7}{2}$. The expectation value for the resultant spin of all other configu-

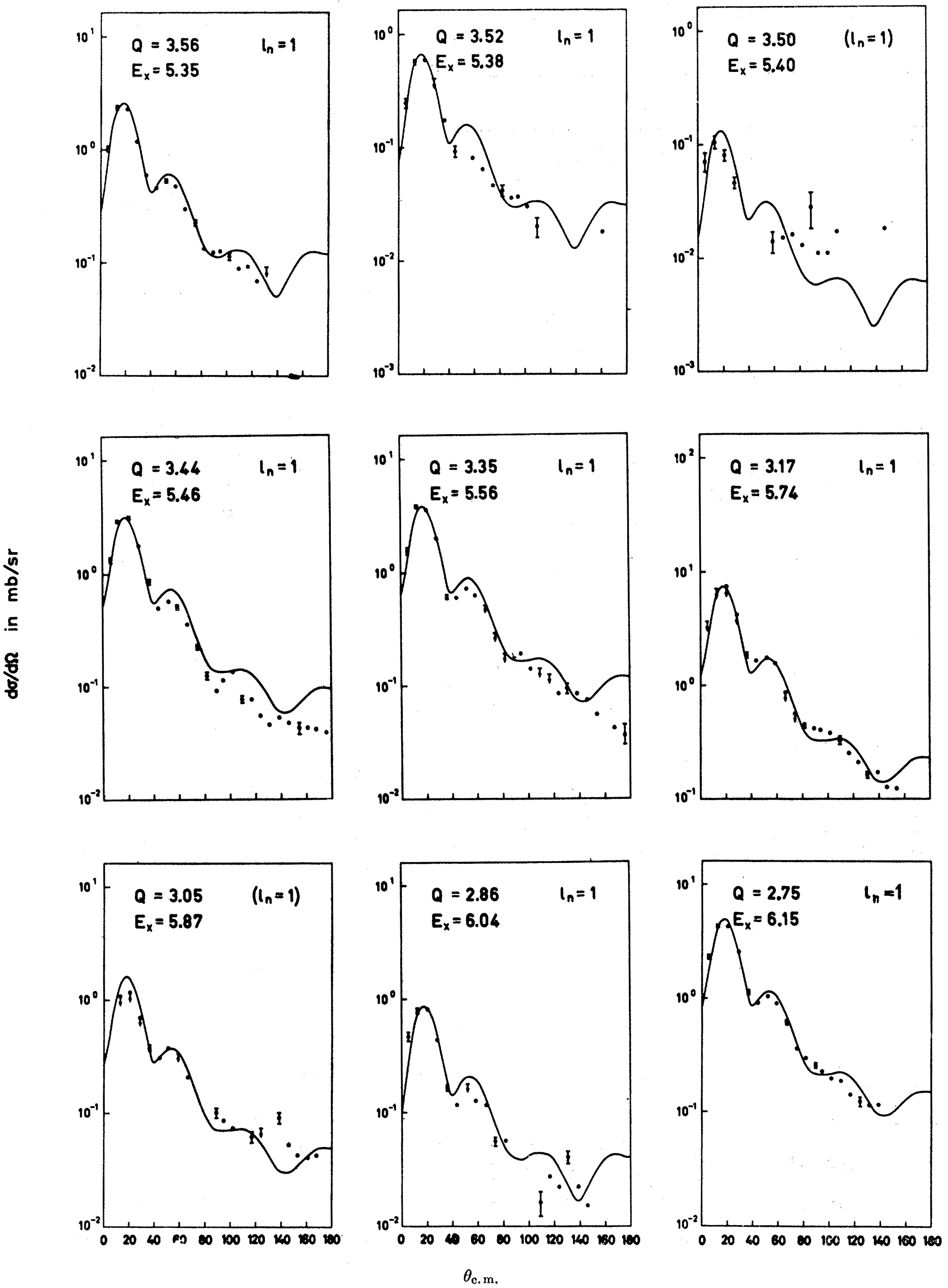

FIG. 6. Angular distributions from the ^{43}Ca(d,p) reaction at 8.5-MeV bombarding energy.

rations in ^{43}Ca(0) must vanish. A pure $(f_{7/2})^3$, $J=\frac{7}{2}$ configuration gives the value unity for Eqs. (1) or (3).

The magnetic moment generated by the $f_{7/2}$ neutrons in the ground state of ^{43}Ca should then be close to the Schmidt value and, provided the state is pure $(f_{7/2})^3$, $J=\frac{7}{2}$, it should also be close to the total magnetic moment of ^{43}Ca(0). This latter moment is 82% of the Schmidt value, but whether this deviation is caused by small configuration admixtures, or by a change in the free neutron magnetic moment cannot be distinguished with the present accuracy of determining spectroscopic factors.

The quadrupole sum rule for a (d,p) reaction may be written

$$\frac{Q(j)}{\langle r^2\rangle_j}=-\frac{[\frac{3}{4}-j(j+1)][3J_A{}^2-J_A(J_A+1)]}{(2j-1)j(j+1)(2j+3)(2J_A-1)J_A(J_A+1)(2J_A+3)}\times\sum_{i,J_B}[3X(X-1)-4j(j+1)J_A(J_A+1)]\frac{2J_B+1}{2J_A+1}S_i(J_A+j\rightarrow J_B)\,,\quad(4)$$

$$X=j(j+1)+J_A(J_A+1)-J_B(J_B+1)\,.$$

$Q(j)$ is the expectation value for the mass quadrupole operator, operating on the j neutrons taken for the ground state with $M=J_A$, and $\langle r^2\rangle_j$ is the mean squared radius of the j neutrons. Taking $\langle r^2\rangle_{7/2}=17.0$ F^2 (see Ref. 3) and using the present (d,p) data, we arrive at a value of $Q(\frac{7}{2})=-0.05\pm0.03$ b. The total ^{43}Ca quadrupole moment is not known. The numerical value of $Q(\frac{7}{2})$ for ^{43}Ca is smaller than the $Q(\frac{7}{2})$ of ^{47,49}Ti that were found, from (d,p) data, to be +0.09 and +0.15 b, respectively.[15] The ^{43}Ca $Q(\frac{7}{2})$ is consistent with an $(f_{7/2})^3$, $J=\frac{7}{2}$ configuration in a spherical shell model j-j coupling scheme. A comparison between $Q(\frac{7}{2})$ and the total ^{43}Ca quadrupole moment would be interesting, since it would indicate the degree of core polarization.

C. Nuclear-Structure Discussion

Detailed shell-model calculations on the Ca isotopes, which take into account $1f$ and $2p$ orbitals, have been published recently.[16,17] Since the appropriate wave functions are not given in Refs. 16 and 17, a detailed comparison with the present data is not possible. The present data on energy levels do not change the basis for the level scheme discussions of Refs. 16 and 17.

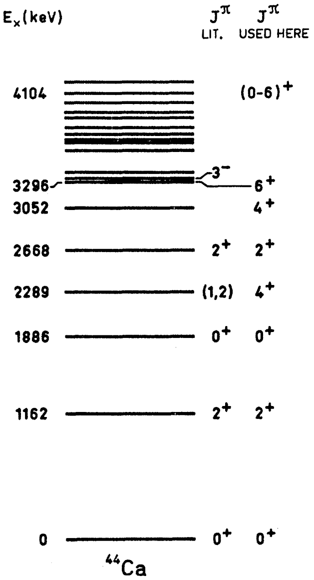

FIG. 7. Low-lying levels in ^{44}Ca. Energies and spin parity assignments are shown for levels excited by $l_n=3$, (d,p) transitions. The tentative J^π assignments from the present data are discussed in the text.

The seniority coupling scheme allows all together four (d,p) transitions to the lowest 0, 2, 4, and 6 + states. Since eight are observed, the seniority-two and -four states mix strongly; in fact, the main part of the $J_B=2$ and 4 seniority-two strength is found in the second 2+ and 4+ states.

As pointed out in Ref. 17, the 0+ state at 1.89 MeV does not belong to the f–p degrees of freedom, but it should rather be considered as a candidate for a six-particle two-hole assignment of $(f_{7/2})^6(d_{3/2})^{-2}$ or $(f_{7/2})^6(s_{1/2})^{-2}$ composition. Particle-hole admixtures of the $(f_{7/2})^{n+2}(s,d)^{-2}$ type have been found in the ground states of all even Ca isotopes below ^{48}Ca,[18] so some admixtures of $(f_{7/2})^5(s,d)^{-2}$ into ^{43}Ca(0) would be expected. Such a component would allow a (d,p) transition to proceed to a predominantly $(f_{7/2})^6(s,d)^{-2}$ 0+ state in ^{44}Ca. The experimental strength for this transition is 20% of that of the ground state. A quantitative evaluation of the percentage of $(f_{7/2})^3(s,d)^{-2}$ configuration in the ^{43}Ca ground state cannot be made before one knows the isospin of the particle components of the $(f_{7/2})^{n+2}(s,d)^{-2}$ configurations of ^{43,44}Ca as well as the degree of admixture of $(f_{7/2})^4$, 0+ into the 1.89-MeV state in ^{44}Ca.

The collective 3$^-$ state at 3.302-MeV excitation[19] (level No. 7) presumably is not excited strongly in the present experiment (see captions of Fig. 2 and Table I). Another odd-parity state at 4.410 MeV is observed with a weak $l_n=0$ transition. The nature of this state is not known otherwise.

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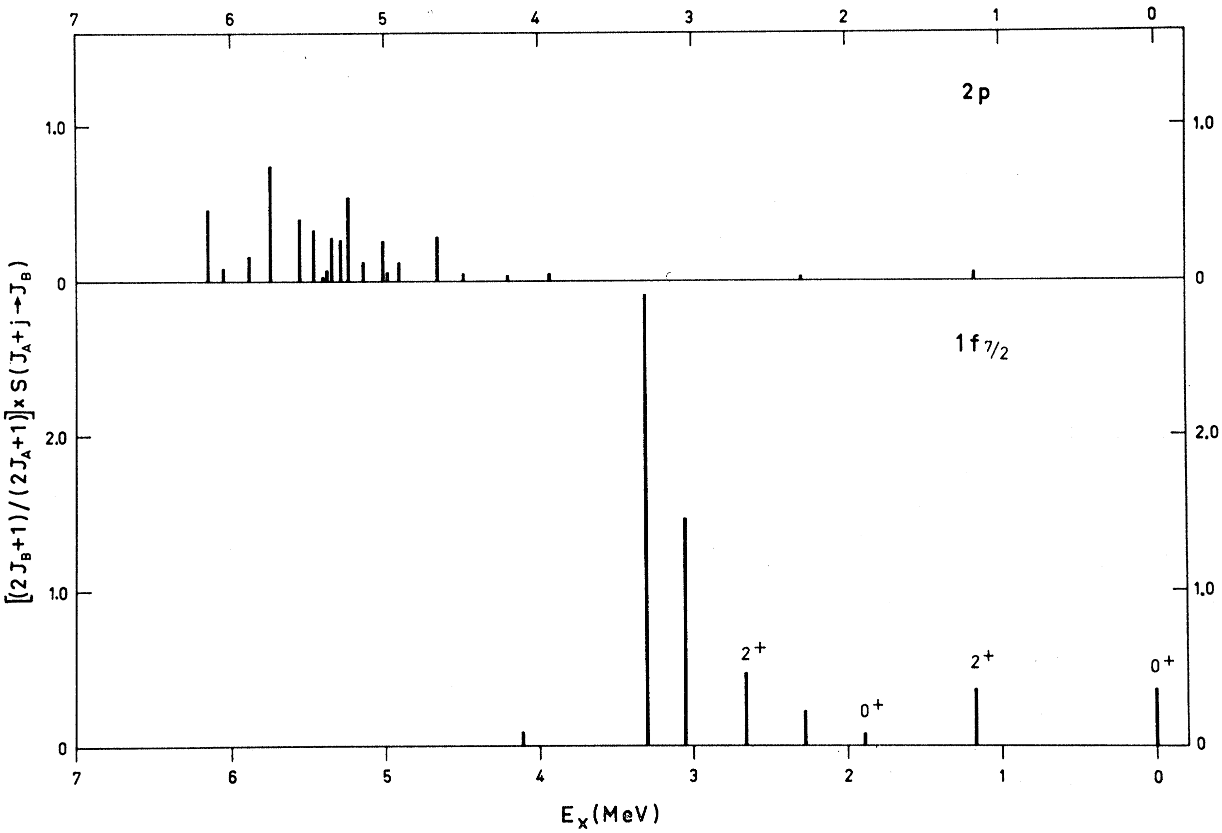

FIG. 8. Strength functions. The measured strengths $[(2J_B+1)/(2J_A+1)]\times S(J_A+j\rightarrow J_B)$ are plotted versus excitation energy for $l_n=1$ and for $l_n=3$ transitions. The figure is discussed in the text.

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