Nuclear Interactions and K-Breaking Mechanisms

S. KAHANA† AND J. WENESER
Brookhaven National Laboratory, Upton, New York
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K-breaking interactions, additional to the usual Coriolis terms, are deduced from velocity-dependent interactions and shown to be of the same order as those contributed by the Coriolis force. Similar considerations, when turned towards the weak interactions, yield a term that simultaneously breaks K and mixes parity.

I. INTRODUCTION

ONE of the striking regularities of the deformed, rotational nuclei is the goodness of the K quantum number, the eigenvalue of the projection of the total angular momentum along the symmetry axis fixed in the rotating body. The small deviations from the exactness of this quantum number have been understood in terms of the Coriolis force, which, as we shall review, is a part of the kinetic energy of rotation. In the classic papers of Bohr,[1] Bohr and Mottelson,[2] and Kerman,[3] the Coriolis force is given its familiar classical representation, which couples the particles and total angular momentum:

$$-\sum_\kappa j_\kappa I_\kappa/\mathcal{I}_\kappa. \tag{1}$$

In this paper, we shall consider K-breaking mechanisms that are present in the nuclear interaction potentials, in addition to the Coriolis force from the kinetic energy. The Coriolis force, since it is obtained directly from the kinetic energy, is immediately seen to couple the velocities of the particles and of the collective core. Since the only collective variable used is the angular momentum I, time-reversal and rotational invariance dictate an $\mathbf{l}_P\cdot\mathbf{I}$ or $\boldsymbol{\sigma}_P\cdot\mathbf{I}$ dependence. A velocity dependence of the nuclear interaction clearly brings in a $\mathbf{p}_P$ dependence, and this must manifest itself in a $\mathbf{l}_P\cdot\mathbf{I}$ term. There are also more complicated forms that involve the deformations and associated angular dependences, but these are best discussed in context. Similarly a spin-dependent interaction, such as a two-particle spin-orbit force, leads to a $\boldsymbol{\sigma}_P\cdot\mathbf{I}$ dependence, as well as deformation-dependent forms. To the extent that the potential and kinetic energies are of the same order, the $\mathbf{l}_P\cdot\mathbf{I}$ and $\boldsymbol{\sigma}_P\cdot\mathbf{I}$ terms contributed by the potential energy are expected to be of the order of the corresponding Coriolis terms. To make these estimates more definite, specific model calculations are here carried through.

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† On leave from McGill University, Montreal, Canada.
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II. REVIEW OF CORIOLIS FORCE

In order to have a definite, simple model before us, let us consider the classical case of a particle outside a deformed axially symmetric core, the particle moving in a velocity-independent potential-well that has the symmetry of the core and that follows the motions of the core. We repeat here the solution of Bohr.[1]

The Hamiltonian is the sum of the rotational kinetic energy of the core and the energies of the particles,

$$H=\sum_\kappa Q_\kappa^2/2\mathcal{I}_\kappa+H_P. \tag{2}$$

Here Q_κ are the components of the core angular-momentum along the symmetry axes of the core, $\mathcal{I}_\kappa$ are the moments of inertia with respect to these same axes—for our symmetric case $\mathcal{I}_1=\mathcal{I}_2=\mathcal{I}\neq\mathcal{I}_3$. In the approximation in which the particle interaction with the core (via the deformed potential-well) is very strong, it is useful to choose the coordinates that diagonalize this interaction. To do this, the particle coordinates $\mathbf{r}_P$ should be taken relative to the core-fixed axes, and from now on we consider H_P written in terms of $\mathbf{r}_P$, and the canonically conjugate momentum $\mathbf{p}_P$. The core variables must be chosen with some care since Q_κ does not commute with $\mathbf{r}_P$, as can be seen immediately from the fact that $\mathbf{r}_P$ varies with core orientation for fixed particle position. Instead, we use the total angular momentum I_κ, which is, of course, the sum of the core and particle angular momentum,

$$I_\kappa=Q_\kappa+j_\kappa.$$

I_κ does commute with $\mathbf{r}_P$, $\mathbf{p}_P$. However, Q_3 is a constant of the motion since rotation about a symmetry axis changes nothing. Eliminating Q_1, Q_2 in favor of I_1, I_2,

$$H=\frac{I_1^2+I_2^2}{2\mathcal{I}}+\frac{Q_3^2}{2\mathcal{I}_3}+\left[H_P+\frac{j_1^2+j_2^2}{2\mathcal{I}}\right]-\frac{j_1I_1+j_2I_2}{\mathcal{I}}. \tag{3}$$

The last term in Eq. (3) is the part of the Coriolis interaction that is responsible for K breaking.

If we omit this last term, the truncated Hamiltonian H_0 is easily solved.

$$H_0=\frac{I_1^2+I_2^2}{2\mathcal{I}}+\frac{(I_3-j_3)^2}{2\mathcal{I}_3}+\left[H_P+\frac{j_1^2+j_2^2}{2\mathcal{I}}\right]. \tag{4}$$

The constants of the motion include the usual total angular momentum I, and the space-fixed z component $M\hbar$. In addition there are the components of the angular momentum relative to the body-fixed symmetry axis, I_3 and j_3, each of which is a constant of the motion. The corresponding wave function has the product form

$$\psi = \mathfrak{D}_{MK}{}^{I}(\Theta)\chi_{\Omega}{}^{\epsilon P}(\mathbf{r}_P). \tag{5}$$

The Θ stands for the Eulerian angles of the core axes. The $\mathfrak{D}_{MK}{}^{I}$, which describe the rotational motion, obey the eigenvalue equations

$$\begin{aligned} I^2\mathfrak{D}_{MK}{}^{I}(\Theta) &= I(I+1)\hbar^2\mathfrak{D}_{MK}{}^{I}, \\ I_z\mathfrak{D}_{MK}{}^{I}(\Theta) &= M\hbar\mathfrak{D}_{MK}{}^{I}, \\ I_3\mathfrak{D}_{MK}{}^{I}(\Theta) &= K\hbar\mathfrak{D}_{MK}{}^{I}. \end{aligned} \tag{6}$$

The $\chi_{\Omega}{}^{\epsilon P}$ obeys the corresponding equations for the particle motion relative to the core:

$$\begin{aligned} j_3\chi_{\Omega}{}^{\epsilon P}(\mathbf{r}_P) &= \Omega\hbar\chi_{\Omega}{}^{\epsilon P}(\mathbf{r}_P), \\ \left[H_P+\frac{j_1{}^2+j_2{}^2}{2\mathcal{I}}\right]\chi_{\Omega}{}^{\epsilon P}(\mathbf{r}_P) &= \epsilon_P\chi_{\Omega}{}^{\epsilon P}(\mathbf{r}_P). \end{aligned} \tag{7}$$

The resulting eigenvalue of H_0 is

$$[I(I+1)-K^2]\frac{\hbar^2}{2\mathcal{I}}+\frac{(K-\Omega)^2\hbar^2}{2\mathcal{I}_3}+\epsilon_P. \tag{8}$$

The actual physical wave function is a linear combination of the solutions (5) and the degenerate solution obtained by the replacement $K\rightarrow -K$, $\Omega\rightarrow -\Omega$.

That, for the physical state, I_3, j_3—or more accurately, $I_3{}^2$, $j_3{}^2$—are constants of the motion, with quantum numbers K, Ω, is only true to the extent that

$$H_C{}' = -\frac{j_1I_1+j_2I_2}{\mathcal{I}} = -\frac{(I_1+iI_2)(j_1-ij_2)}{2\mathcal{I}} - \frac{(I_1-iI_2)(j_1+ij_2)}{2\mathcal{I}} \tag{9}$$

can be neglected. $H_C{}'$ does indeed perturb the eigenfunctions of H_0 in such a way as to admix pieces to the wave function whose I_3, j_3 quantum numbers, K', Ω', are higher or lower by one unit: $K'=K+1$, $\Omega'=\Omega+1$ or $K'=K-1$, $\Omega'=\Omega-1$. The I, K dependence of these admixture amplitudes is characteristic and determined by the rules of angular-momentum operators

$$\begin{aligned} (I_1+iI_2)\mathfrak{D}_{MK}{}^{I} &= [(I-K+1)(I+K)]^{1/2}\mathfrak{D}_{M,K-1}{}^{I}, \\ (I_1-iI_2)\mathfrak{D}_{MK}{}^{I} &= [(I+K+1)(I-K)]^{1/2}\mathfrak{D}_{M,K+1}{}^{I}. \end{aligned} \tag{10}$$

The quantitative tests of the K-breaking mechanism are really tests of this basic I, K dependence, and this in turn is only a test of the linear I_1 and I_2 dependence of the interaction and is not a test of the precise form written down in Eq. (9).[4] The additional potential terms which we adduce here will, of course, have just this same I and K dependence and to this extent are indistinguishable from the Coriolis terms.

The precise form of the K-breaking mechanism is relevant only when it comes to quantitative computation of the amplitude of the admixtures. In view of the approximate nature of the particle wave functions, only an approximate evaluation of the matrix elements of the K-breaking interactions is possible. A simple model suffices, therefore, to evaluate the contributions to the K-breaking interaction itself. To this we now turn.

III. VELOCITY-DEPENDENT EFFECTIVE INTERACTIONS

We consider here the effects of two velocity-dependent components of the effective force[5]:

$$\mathcal{V}_L = \sum_{i>j}\left[\frac{(\mathbf{p}_i-\mathbf{p}_j)^2}{4M}V_L(\mathbf{r}_i-\mathbf{r}_j) + V_L(\mathbf{r}_i-\mathbf{r}_j)\frac{(\mathbf{p}_i-\mathbf{p}_j)^2}{4M}\right], \tag{11}$$

and

$$\mathcal{V}_{L-S} = \sum_{i>j}(\mathbf{s}_i+\mathbf{s}_j)\cdot(\mathbf{r}_i-\mathbf{r}_j)\times(\mathbf{p}_i-\mathbf{p}_j)\frac{V_{L-S}(\mathbf{r}_i-\mathbf{r}_j)}{\hbar^2}. \tag{12}$$

We completely neglect exchange, and so particle-core velocity coupling can only arise from the parts of V_L and V_{L-S} that are proportional to $(\mathbf{p})_{\text{particle}}\cdot(\mathbf{p})_{\text{core}}$ or $(\mathbf{s})_{\text{particle}}\cdot(\mathbf{p})_{\text{core}}$. We neglect the possibility of terms like $(\mathbf{p})_{\text{particle}}\cdot(\mathbf{s})_{\text{core}}$ because, in the extremely simple model to be used here, core spin is assumed to average to zero. In fact, for this same reason, we do not consider

[4] A. Bohr and B. Mottelson, At. Energiya **14**, 41 (1963). [English transl: Soviet Atomic Energy **14**, 36 (1963)].

[5] S. Kahana and E. Tomusiak, Nucl. Phys. **71**, 402 (1965). Kahana and Tomusiak approximate the residual interaction in the ^1S relative state by the free reaction matrix K_F in this state. Here, let us take for the free reaction matrix:

$$K_F(^1S) = -ge^{-\mu r}+f\left[\frac{(\mathbf{p}_1-\mathbf{p}_2)^2}{4M}e^{-\nu r}+e^{-\nu r}\frac{(\mathbf{p}_1-\mathbf{p}_2)^2}{4M}\right].$$

The parameters in this equation are fixed by noting that the on-the-energy shell matrix element of $K_F(^1S)$ is proportional to the tangent of the ^1S phase shift δ. In fact,

$$\tan\delta = -\frac{Mk}{\hbar^2}\int_0^{\infty} r^2 j_0(kr)K_F(r,p)j_0(kr)dr,$$

where $\hbar k$ is the relative momentum of a pair of nucleons. The first term in $K_F(^1S)$ alone provides a fit to the low-energy scattering data while the second term is required if the high-energy data are also to be fitted. A choice of parameters which gives a good over-all fit is

$$\begin{aligned} g &= 23.8 \text{ MeV}, \\ \mu^{-1} &= 2.47 \text{ F}, \\ f &= 1.12, \\ \nu^{-1} &= 0.63 \text{ F}, \end{aligned}$$

and then there results $c_L = 3.5$ F^3 as quoted above. One should perhaps also notice that the velocity-dependent term in $K_F(^3S)$ is very much like that in $K_F(^1S)$, and we assume it to be identical. The spin-orbit term in the free reaction matrix is deduced by considering the $^3P_{0,1,2}$ states and has the form $V_{L-S}(r) = -1528e^{-r/(0.36\text{ F})}$ MeV.

any forces that only contribute by involving the core spin. We omit also the analogue of the core kinetic energy terms $(\mathbf{p})_{\text{core}}\cdot(\mathbf{p})_{\text{core}}$ from $\mathcal{V}_L$, since they can be seen to be of order $\mathcal{I}_{\text{single-particle}}/\mathcal{I}_{\text{core}}$ relative to the terms we retain. Thus, the portions of $\mathcal{V}_L$ and $\mathcal{V}_{L-S}$ that are actually employed are

$$\mathcal{V}_L \rightarrow \sum_c \frac{1}{2M}[-\mathbf{p}_P\cdot\mathbf{p}_c V_L - V_L\mathbf{p}_P\cdot\mathbf{p}_c], \quad (11a)$$

and

$$\mathcal{V}_{L-S} \rightarrow \sum_c \mathbf{s}_P\cdot(\mathbf{r}_c-\mathbf{r}_p)\times\mathbf{p}_c\frac{V_{L-S}}{\hbar^2}. \quad (12a)$$

The motion of the core particles will be treated semiclassically and, therefore, we ascribe to the core an over-all density and a velocity-density. We shall consider the two extreme cases of rigid rotation and irrotational flow. The effective interactions obtained from $\mathcal{V}_L$ and $\mathcal{V}_{L-S}$, on carrying out the integration over the core densities, are H_L' and H_{L-S}':

$$H_L' = -\frac{\mathbf{p}_P}{M}\cdot\int\rho_c(\mathbf{r}_c)\mathbf{p}_c V_L(\mathbf{r}_c-\mathbf{r}_p)d\tau_c, \quad (13)$$

$$H_{L-S}' = +\mathbf{s}_P\cdot\int\rho_c(\mathbf{r}_c)(\mathbf{r}_c-\mathbf{r}_P)\times\mathbf{p}_c V_{L-S}(\mathbf{r}_c-\mathbf{r}_P)d\tau_c. \quad (14)$$

The core-fluid velocity $\mathbf{v}_c$ and the density, ρ_c must be given to specify the model. The fluid density is assumed to be constant inside a spheroidal volume. Rigid motion is simply specified by

$$\rho_c\mathbf{p}_c = \rho_c M\mathbf{v}_c = \rho_c M\boldsymbol{\omega}\times\mathbf{r}_c, \quad (15)$$

where ω is the angular velocity of the core. In fact, the components of the angular velocity can be related to the components of the angular momentum,

$$\omega_\kappa = Q_\kappa/\mathcal{I}_\kappa, \quad (16)$$

all components referred to the body-fixed axes. If, alternatively, we describe the core as a fluid undergoing irrotational motion, we can take over the description of Bohr.[1] The velocity is given in terms of the gradient of a scalar field:

$$\mathbf{v}_c = \nabla(\tfrac{1}{2}r^2\dot{\alpha}_\mu Y_{2\mu}), \quad (17)$$

where the α_μ describe the quadrupolar distortion of the surface of the incompressible nuclear fluid with respect to space-fixed axes,

$$R = R_0(1+\alpha_\mu Y_{2\mu}), \quad (18a)$$

or with respect to the body-fixed symmetry axes,

$$R = R_0(1+\beta Y_{20}). \quad (18b)$$

The $\dot{\alpha}_\mu$ can be expressed as the sum of the time rate of change of the shape parameters viewed with respect to symmetry axes rotating with the body plus the time rate of change due to rotation of the instantaneously fixed shape. The first term describes the motion due to "internal shape vibrations" which do not concern us here. Ignoring these completely and sticking to the requirement of pure axial symmetry, the Bohr result becomes

$$\dot{\alpha}_\mu \rightarrow -i\beta\mathfrak{D}_{\mu m}^{(2)*}(M_\kappa)_{m0}Q_\kappa/\mathcal{I}_\kappa, \quad (19)$$

and

$$\rho_c\mathbf{p}_c = M\rho_c\mathbf{v}_c \rightarrow -iM\rho_c\nabla(\tfrac{1}{2}r^2Y_{2m})(M_\kappa)_{m0}^*\beta Q_\kappa/\mathcal{I}_\kappa, \quad (20)$$

where $(M_\kappa)_{m0}$ is the matrix element,

$$\int Y_{2m}^*((1/i)\mathbf{r}\times\nabla)_\kappa Y_{20},$$

of the rotation generators.

The functions V_L and V_{L-S} are taken from the reaction-matrix determined by Kahana and Tomusiak.[5] Since the range of these potential functions, V, is much shorter than any core or orbit dimension we shall use effective δ functions instead, since the zero-range form can be more simply compared with the Coriolis interaction.

We begin by evaluating H_L' for the case of a rigidly rotating core. Inserting the expressions (15) and (16) for the core velocity into (13), and using the zero-range approximation to $V_L(r)$,

$$V_L(r) \rightarrow C_L\delta(r),\ C_L = \int V_L(\mathbf{r})d\tau \sim 3.5(\text{F})^3, \quad (21)$$

a rather transparent form for H_L' results:

$$H_L' = -\sum_\kappa C_L\rho_c(\mathbf{r}_P)(\mathbf{r}_P\times\mathbf{p}_P)_\kappa Q_\kappa/\mathcal{I}_\kappa. \quad (22)$$

The K-breaking part, obtained from (22) by the replacement of $Q_\kappa \rightarrow I_\kappa$ and confining the sum over components to $\kappa = 1, 2$, is

$$-\sum_{\kappa=1,2} C_L\rho_c(\mathbf{r}_P)l_{P\kappa}I_\kappa/\mathcal{I}_\kappa. \quad (23)$$

This can be directly compared with the Coriolis interaction, Eq. (9). If we take for $\rho_c(\mathbf{r}_P)$ the usual

$$\rho_c(\mathbf{r}_P) = A/(\tfrac{4}{3}\pi r_0^3 A), \quad r < r_0A^{1/2}, \quad r_0 = 1.2\ \text{F}, \quad (24)$$

Eq. (23) becomes

$$-0.5l_{P\kappa}I_\kappa/\mathcal{I}_\kappa. \quad r < r_0A^{1/3}. \quad (25)$$

This seems to indicate that a moderate-sized contribution can be expected from V_L that is neither large nor negligible compared to the orbital part of the Coriolis interaction.

We complete the examination of V_L contributions to K breaking by examining H_L' in the case of irrotational flow. The insertion of expression (20) for the core velocity results in what is, at first sight, a rather

cumbersome form for H_L':

$$H_L'=\tfrac{1}{2}ic_L\beta\rho_c(\mathbf{r}_P)\sum_m\sum_{\kappa=1,2}\nabla(r_P{}^2Y_{2m})\cdot\mathbf{p}_P(M_\kappa)_{m0}{}^*I_\kappa/\mathcal{I}_\kappa$$

$$=\frac{i(6)^{1/2}}{4}\beta c_L\rho_c(\mathbf{r}_P)$$

$$\times\sum_{\mu=+1,-1}\left\{(\nabla r_P{}^2Y_{2-\mu})\cdot\mathbf{p}_P\left(\frac{I_1+i\mu I_2}{\mathcal{I}}\right)\right\}. \quad (26)$$

It will now be recognized that this effective interaction is just of the form of an electromagnetic $E2$ interaction. The Coriolis interaction, Eq. (9), with which we want to compare, is of the form of an electromagnetic $M1$ interaction. Clearly such a comparison cannot be carried through except in the framework of a definite particle model. Rather than attempt this here, we note that there is, actually, more of a resemblance than is usually professed between magnetic-dipole and electric-quadrupole matrix elements. To make this more definite, we note that the important ingredient in (26) is

$$i(8\pi/15)^{1/2}\nabla(r^2Y_{2,\pm1})\cdot\mathbf{p}=z(-p_y\pm ip_x)+p_z(y\mp i_x), \quad (27)$$

while the orbital part of the Coriolis operator is just l_μ, and

$$l_{\pm1}=z(-p_y+ip_x)-p_z(y\mp i_x). \quad (28)$$

(As an aside, it might be noted that the $M1$ and $E2$ multipoles arise from the same term in the retardation expansion.) If we boldly put aside the fact that we are comparing different irreducible tensors, and assert that the expressions in (27) and (28) are about equal, H_L' is weighted relative to the Coriolis by approximately

$$(45/16\pi)^{1/2}C_L\beta\rho_c\sim0.5\beta. \quad (29)$$

For $\beta\sim0.4$, this contribution to the K breaking is about a fifth the Coriolis contribution. This actually may be an underestimate since the irrotational moment of inertia appearing in (26) is much smaller than the experimental value used in the estimate. The situation appears, again, to be that the interaction contribution, H_L', is neither large nor negligible—whether calculated on the assumption of rigid rotation or irrotational flow.

The similar evaluation for H_{L-S}', Eq. (14), is somewhat more complicated. Because the spin-orbit interaction v_{L-S} is a p-wave interaction, the zero-range expansion must be carried one stage further. Thus, for the case of rigid rotation we evaluate

$$H_{L-S}'=M\mathbf{s}_P\cdot\int\rho_c(\mathbf{r}_c)(\mathbf{r}_c-\mathbf{r}_P)[\boldsymbol{\omega}\times\mathbf{r}_c]V_{LS}(\mathbf{r}_c-\mathbf{r}_P), \quad (30)$$

by using the expansion

$$\rho_c(\mathbf{r}_c)\mathbf{r}_c=\rho_c(\mathbf{r}_P)\mathbf{r}_P+[(\mathbf{r}_c-\mathbf{r}_P)\rho_c(\mathbf{r}_P)$$
$$+\mathbf{r}_P(\mathbf{r}_c-\mathbf{r}_P)\cdot\nabla\rho_c(\mathbf{r})_P], \quad (31)$$

and obtain

$$H_{LS}'=\tfrac{1}{3}M\mathbf{s}_P\cdot\{2\boldsymbol{\omega}\rho_c(\mathbf{r}_P)-(\boldsymbol{\omega}\times\mathbf{r}_P)\times\nabla\rho_c(\mathbf{r}_P)\}$$
$$\times\left(\int r^2\frac{V_{L-S}}{\hbar^2}d\tau\right). \quad (32)$$

On the usual assumption that $\rho_c(\mathbf{r}_P)$ is uniform and equal to ρ_0 over the spheroidal volume contained inside $R=R_0(1+\beta Y_{20})$,

$$\nabla\rho_c(\mathbf{r})=-\rho_0\delta(r-R_0)\hat{\mathbf{r}}+\rho_0\{\delta(r-R_0)[\hat{\mathbf{r}}+i\hat{\mathbf{r}}\times\mathbf{l}]$$
$$+\rho_0R_0\delta'(r-R_0)\hat{\mathbf{r}}\}(\beta Y_{20})$$
$$+(\text{higher order in }\beta Y_{20}). \quad (33)$$

Then

$$H_{L-S}'=\left(\tfrac{1}{3}M\rho_0\int r^2\frac{V_{L-S}}{\hbar^2}d\tau\right)$$
$$\times\{2\mathbf{s}_P\cdot\boldsymbol{\omega}[\mathcal{S}(r_P-R_0)-\tfrac{1}{3}R_0\delta(r_P-R_0)]$$
$$+(\boldsymbol{\omega}\cdot\hat{\mathbf{r}}_P\mathbf{s}_P\cdot\hat{\mathbf{r}}_P-\tfrac{1}{3}\mathbf{s}_P\cdot\boldsymbol{\omega})R_0\delta(r_P-R_0)\}, \quad (34)$$

where $\boldsymbol{\omega}$ is defined in Eq. (16) as $\omega_\kappa=Q_\kappa/\mathcal{I}_\kappa$, and $\mathcal{S}(r_P-R_0)$ is the usual step-function. The numerical weighting factor,

$$\tfrac{1}{3}M\rho_0\int r^2\frac{V_{L-S}}{\hbar^2}d\tau\sim-0.8, \quad (35)$$

appearing in Eq. (34) is a fair measure of the strength of H_{L-S}' relative to the Coriolis interaction. The operators appearing in the brackets of Eq. (34) have more structure than the simple $\mathbf{j}_P\cdot\boldsymbol{\omega}$ of the Coriolis interaction, and so have different detailed selection rules. While only specific calculations can delineate the precise numerical values, it is also clear that, generally speaking, the matrix elements in (34) are of the same order as those contributed by the spin term of the Coriolis interaction.

For the irrotational case

$$H_{L-S}'=-\tfrac{1}{2}iM\beta\sum_{\kappa,m}(M_\kappa)_{0m}(Q_\kappa/\mathcal{I}_\kappa)$$
$$\mathbf{s}_P\cdot\int\rho_c(\mathbf{r}_c)(\mathbf{r}_c-\mathbf{r}_P)\times(\nabla r_c{}^2Y_{2m})\frac{V_{L-S}}{\hbar^2}d\tau_c. \quad (36)$$

Expanding the integrand, just as we did in handling the rigid-rotational case, we obtain for H_{L-S}'

$$-\tfrac{1}{6}iM\beta\left(\int r^2\frac{V_{L-S}}{\hbar^2}d\tau\right)$$
$$\times\sum_{\kappa,m}(M_\kappa)_{0m}(Q_\kappa/\mathcal{I}_\kappa)\mathbf{s}_P\cdot\nabla\rho_c(\mathbf{r}_P)\times\nabla r_P{}^2Y_{2m}. \quad (37)$$

It might be noted that only a $\nabla\rho_c$ term appears in (37). The term involving ρ_c itself vanishes because it involves the coupling of the spin-current to an irrota-

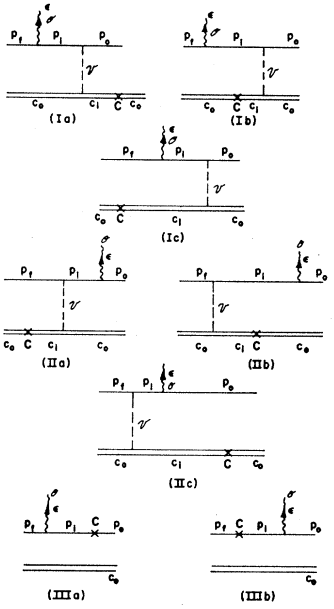

FIG. 1. Diagrams corresponding to the interaction of the particle-core potential $\mathcal{V}$, the Coriolis force C, and a transition operator $\mathcal{O}$; an energy ϵ is given up by the system. The core states are labeled c_0, c_i, c_f; the particle states are labeled p_0, p_i, p_f.

tional flow, or, more succinctly, because $\rho_c\nabla\times\mathbf{p}_c=0$. Equation (37) reduces, on again expanding ρ_c and keeping only the lowest order in β, to the simple form

$$-(\tfrac{3}{2})^{1/2}\beta\left(\frac{M}{3}\rho_0\int r^2\frac{V_{L-S}}{\hbar^2}d\tau\right)\times\sum_\mu\left\{\delta(r-R_0)\mathbf{s}_P\cdot(\mathbf{l}Y_{2-\mu})\frac{(Q_1+i\mu Q_2)}{2\mathcal{I}}\right\}. \quad (38)$$

Thus, the K-breaking part of H_{LS}' is

$$\sim 0.5\delta(r-R_0)\sum_\mu \mathbf{s}_P\cdot(\mathbf{l}Y_{2-\mu})\frac{(I_1+i\mu I_2)}{2\mathcal{I}}. \quad (39)$$

A direct comparison with the Coriolis interaction is clearly not possible for a number of reasons. First, the particle dependence of the interaction (39) is that of a tensor of second rank as opposed to the first rank tensor $\mathbf{j}_\mu$, appearing in the Coriolis interaction. Once again, we are trying to compare an $E2$ with an $M1$ operator. Secondly, the surface radial dependence must be compared with the volume weighting of the Coriolis interaction. Parenthetically, we note that this latter kind of radial weighting also appears in the rigid rotation form of H_{L-S}', Eq. (34). Different assumptions for the radial behavior of particle wave functions indicates that the surface weighting results in a radial integral 1.5 to 4 times larger than the volume weighting. On the other hand, the angular dependence in (39) seems to favor the Coriolis term by a factor of ~ 1.5. In summary, it is reasonable to expect that (39) is of the order of the spin-dependent term in the Coriolis interaction.

This is as far as we can credibly go with such a simple evaluation and model. In Sec. IV, we present a microscopic description using the cranking model[6] as an intermediary.

IV. PARTICLE DESCRIPTION OF THE CORE

Using the cranking model, the core is now to be described as particles moving in the orbits determined by an average potential and perturbed by the core-Coriolis interaction:

$$\sum_\kappa\sum_{c=\text{core particles}} j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa. \quad (40)$$

The Q_κ are operators on the angles describing the orientation of the core. It is to be noted that the Coriolis interaction in (40) involves the core particles only and is to be distinguished from the K-breaking Coriolis interaction of the particle outside the core. In fact, the core-Coriolis interaction (40) serves to introduce the angular velocity dependence that we had in our semiclassical treatments. We will first calculate the core orbits in first-order perturbation theory and then consider, in a typical transition matrix element, the K-breaking effects of both velocity-dependent and velocity-independent interactions between the extra particle and the core particles.

The wave function ψ consists of a factor $\mathfrak{D}_{MK}{}^I(\Theta)$, describing the orientation of the core, a factor $\chi^{\epsilon P}$, describing the orbit of the outside particle, and a factor ψ_c describing the core particles.

$$\psi=\psi_c\mathfrak{D}_{MK}{}^I(\Theta)\chi^{\epsilon P}(\mathbf{r}_P). \quad (41)$$

First we consider the part ψ_c. To first order in perturbation theory, it is

$$|\psi_c\rangle=|c_0\rangle+\sum_i\frac{|c_i\rangle\langle c_i|-\sum_{c,\kappa}j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa|c_0\rangle}{E_{c0}-E_{ci}}. \quad (42)$$

Here, $|c_0\rangle$ describes the core particles occupying the orbits that would exist if the determining potential did not rotate. The remaining terms of (42) correct for the fact that the potential is, in fact, rotating with angular velocity given by $\omega_\kappa=Q_\kappa/\mathcal{I}_\kappa$. The correction is stated in terms of the various eigenstates, $|c_i\rangle$, appropriate to the nonrotating potential. It is instructive, at this point, to evaluate a component of the core angular momentum:

$$\langle\psi_c|\sum_c j_{c\lambda}|\psi_c\rangle=\langle c_0|\sum_c j_{c\lambda}|c_0\rangle+\sum_i\frac{\langle c_0|\sum_c j_{c\lambda}|c_i\rangle\langle c_i|-\sum_{c,\kappa}j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa|c_0\rangle}{E_{c0}-E_{ci}}+\sum_i\frac{\langle c_0|-\sum_{c,\kappa}j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa|c_i\rangle\langle c_i|\sum_c j_{c\lambda}|c_0\rangle}{E_{c0}-E_{ci}}. \quad (43)$$

[6] D. Inglis, Phys. Rev. **96**, 1059 (1954); **103**, 1786 (1956); Nucl. Phys. **8**, 125 (1958).

The first term is zero because all angular momenta are paired off in the state $|c_0\rangle$. Since the second two terms are equal to each other, (43) can be more simply written as

$$2\sum_i \frac{\langle c_0|\sum_c j_{c\lambda}|c_i\rangle\langle c_i|\sum_c j_{c\lambda}|c_0\rangle}{E_{c_i}-E_{c_0}} Q_\lambda/\mathcal{I}_\lambda. \quad (43a)$$

On recognizing the cranking-model formula for $\mathcal{I}_\lambda$, one obtains finally the usual result

$$\langle\psi_c|\sum_c j_{c\lambda}|\psi_c\rangle = Q_\lambda. \quad (44)$$

This well-known result has been repeated here to emphasize that the rotating properties of the core enter only because of the presence of the second, core-Coriolis induced term of the wave function, (42).

To fix on the relevant aspects of K breaking, we consider a transition matrix element of an operator with definite K-selection rules:

$$\mathcal{O} = \sum_\mu \theta_\mu^L(\mathbf{r}_P)\mathfrak{D}_{M\mu}^L(\Theta).$$

The transition consists in the change of the outside-particle orbit only, an energy ϵ being carried off. While the full form of the wave function is as given in Eq. (41), here we omit explicit reference to the Θ-dependent factor, it being understood that the Θ operations are carried out later.

We now want to write out the perturbative effects on the transition matrix element of the residual interaction $\mathcal{V}$ between the extra particle and the core particles. This will be done to first order in the core-Coriolis interaction, the residual interaction, and, of course, in the operator $\mathcal{O}$. The third-order perturbation theory is most easily visualized in pictorial form. In Fig. 1 we have displayed the relevant terms of ordinary Schrödinger perturbation theory. The terms corresponding to Figs. 1, diagrams (Ia), (Ib), (Ic), are

$$\text{(Ia)}\quad \sum_i \frac{\langle p_f|\theta_\mu^L|p_i\rangle\langle p_ic_0|\mathcal{V}|p_0c_i\rangle\langle c_i|-\sum_{c,\kappa} j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa|c_0\rangle}{[\epsilon_{p_0}-\epsilon_{p_i}][E_{c_0}-E_{c_i}]}$$

$$\text{(Ib)}\quad \sum_i \frac{\langle p_f|\theta_\mu^L|p_i\rangle\langle c_0|-\sum_{c,\kappa} j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa|c_i\rangle\langle p_ic_i|\mathcal{V}|p_0c_0\rangle}{[\epsilon_{p_0}-\epsilon_{p_i}][(\epsilon_{p_0}-\epsilon_{p_i})+(E_{c_0}-E_{c_i})]} \quad (45)$$

$$\text{(Ic)}\quad \sum_i \frac{\langle c_0|-\sum_{c,\kappa} j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa|c_i\rangle\langle p_f|\theta_\mu^L|p_i\rangle\langle p_ic_i|\mathcal{V}|p_0c_0\rangle}{[(\epsilon_{p_0}-\epsilon_{p_f}-\epsilon)+(E_{c_0}-E_{c_i})][(\epsilon_{p_0}-\epsilon_{p_i})+(E_{c_0}-E_{c_i})]}.$$

Using the conservation of energy, $\epsilon_{p_0}=\epsilon_{p_f}+\epsilon$, terms (Ia), (Ib), and (Ic) can be combined simply into

$$\sum_{p_i}\frac{\langle p_f|\theta_\mu^L|p_i\rangle}{[\epsilon_{p_0}-\epsilon_{p_i}]}\Big\{\sum_{c_i}\frac{1}{[E_{c_0}-E_{c_i}]}$$
$$[\langle p_ic_0|\mathcal{V}|p_0c_i\rangle\langle c_i|-\sum_{c,\kappa} j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa|c_0\rangle+$$
$$\langle c_0|-\sum j_{c\kappa}Q_\kappa/\mathcal{I}_\kappa|c_i\rangle\langle p_ic_i|\mathcal{V}|p_0c_0\rangle]\Big\}. \quad (46)$$

This is to be compared with the effect of the usual particle-Coriolis interaction on the transition matrix element (IIIa) of Fig. 1,

$$\sum_{p_i}\frac{\langle p_f|\theta_\mu^L|p_i\rangle}{[\epsilon_{p_0}-\epsilon_{p_i}]}\{\langle p_i|-\sum_\kappa I_\kappa j_\kappa/\mathcal{I}|p_0\rangle\}. \quad (47)$$

From this we can immediately see that the factor in curly brackets in (46) is to be compared with the factor in curly brackets in (47). If we neglect exchange, we are, then, to compare

$$H'(\mathbf{r}_p,\mathbf{p}_p) = -\sum_{c_i,\kappa}\frac{1}{[E_{c_0}-E_{c_i}]}\{\langle c_0|\sum_c \mathcal{V}(\mathbf{r}_P\mathbf{p}_P;\mathbf{r}_c\mathbf{p}_c|c_i\rangle$$
$$\times\langle c_i|\sum_c j_{c\kappa}|c_0\rangle+\langle c_0|\sum_c j_{c\kappa}|c_i\rangle$$
$$\times\langle c_i|\mathcal{V}(\mathbf{r}_P\mathbf{p}_P;\mathbf{r}_c\mathbf{p}_c)|c_0\rangle\}I_\kappa/\mathcal{I}_\kappa, \quad (48)$$

with

$$H_c'(\mathbf{r}_P,\mathbf{p}_P) = -\sum_\kappa j_{P\kappa}I_\kappa/\mathcal{I}_\kappa.$$

The similar set of terms from (IIa), (IIb), and (IIc) lead to similar results, and let us derive the Hermitian conjugate of $H'(\mathbf{r}_P,\mathbf{p}_P)$.

There is a very simple but useful conclusion that can be drawn from the earlier form, Eq. (46). Thus, $\mathcal{V}$ appears in (46) only in matrix elements that involve at least 2-particle transitions, $p_0\to p_i$, $c_i\to c_0$. This at once assures us that it is only the residual interaction that contributes and, also, that none of the effect contained in (46) is included elsewhere.

We can also see explicitly that an interaction depending only on the position coordinates contributes

zero to (46), or (48), provided that we omit exchange terms. To do this we have only to expand the interaction according to the tensorial dependence on $\mathbf{r}_P$, $\mathbf{r}_c$. We have then, a sum of terms of the form (radial function $(-1)^\mu Y_{l,\mu}(\mathbf{r}_P)Y_{l,-\mu}(\hat{\mathbf{r}}_c)$, and, excluding exchange we find for the lth term that the core factors in (46) involve

$$\sum_i \frac{1}{E_{c0}-E_{ci}}\{\langle c_0|\sum_c Y_{l-\mu}(\hat{\mathbf{r}}_c)|c_i\rangle\langle c_i|-\sum_c j_{c\mu}|c_0\rangle + \langle c_0|-\sum_c j_{c\mu}|c_i\rangle\langle c_i|\sum_c Y_{l-\mu}(\hat{\mathbf{r}}_c)|c_0\rangle\}. \quad (49)$$

Now, the unperturbed core state c_0 is described by the occupation of pairs of orbits, each member of the pair the time reverse of the other. It then follows immediately that the odd l are knocked out by parity and the remaining even l terms are knocked out by time reversal. Explicitly, there is a cancelation in (48) between the first and second terms. Labeling the occupied core orbits by their 3-axis component of angular momentum, the cancelation can be seen to be between the transitions $m \to m\pm1 \to m$, and the time-reversed transitions $-m \to -m\mp1 \to -m$. If the interaction is velocity dependent, the time-reversal properties of the terms in the interaction expansion are clearly different. As an example, consider

$$\sum_{\mu,\nu}(\mathbf{r}_c\times\mathbf{p}_c)_\mu Y_{2\nu}(12\mu\nu|12LM) \quad (50a)$$

as a replacement for Y_{LM}. Such velocity-dependent factors give a nonzero result, and we have in Sec. III approximated the results of Eq. (48) by using simple models.

It might be noted in passing that a factor such as

$$\sum_{\mu,\nu}(\sigma_c)_\mu Y_{2\nu}(12\mu\nu|12LM) \quad (50b)$$

would also have the parity and time-reversal properties necessary for a nonzero result in (47). Such terms would come from a tensor force. There are also the core-spin parts of v_{L-S}, Eq. (12), that would contribute. In the semiclassical models these terms are zero, and the spin contributions more obviously require a microscopic particle model. We note, also, that the inclusion of exchange terms will result in a nonzero contribution even for velocity- and spin-independent interactions.

We have, so far, omitted exchange. Had we included such terms, in the zero-range approximation and with the forces, v_L, v_{L-S}, we have used, the numerical strengths in H_L', H_{L-S}' would have been somewhat altered, but no new forms introduced. While we could examine these explicitly, the semiclassical model itself is not so easily understood in the presence of exchange. We leave the exchange terms of v_L, v_{L-S} together with the exchange terms of finite range Wigner forces to be dealt with later in a microscopic, particle description.

We also omit here any description of the rotational motion better than that given by the cranking model. This description, together with a unified treatment of the related higher-order corrections to Hartree-Fock, will be left for a later paper.

V. PARITY MIXING AND K BREAKING

We have so far considered the K-breaking parts of the strong interparticle forces. Similar considerations apply to the weak, parity-breaking parts of the nuclear force. Michel,[7] from the current-current hypothesis, deduces a parity-breaking two-particle interaction

$$W=-\sum_{i>j}\frac{Gh}{(8)^{1/2}M}(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\{(\mathbf{p}_i-\mathbf{p}_j)\delta(\mathbf{r}_i-\mathbf{r}_j)+\delta(\mathbf{r}_i-\mathbf{r}_j)(\mathbf{p}_i-\mathbf{p}_j)\}T_{ij} -\sum_{i>j}(\mu_p-\mu_n+1)\frac{Gh}{(8)^{1/2}M}(i\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot[(\mathbf{p}_i-\mathbf{p}_j)\delta(\mathbf{r}_i-\mathbf{r}_j)-\delta(\mathbf{r}_i-\mathbf{r}_j)(\mathbf{p}_i-\mathbf{p}_j)]T_{ij}, \quad (51)$$

where T_{ij} is the charge-exchange operator, and G is the weak-coupling constant.

The presence of the charge-exchange operator requires an explicit particle description. It can be noted parenthetically that it is only charge exchange that appears so that the semiclassical models are still applicable after some manipulation. The matrix element of the effective single-particle interaction, H_P', that results from integrating W over the core-particle[7] variables is

$$\int d\tau_p\chi_{Pf}^*(\mathbf{r}_P)H_P'(\mathbf{r}_P,\boldsymbol{\sigma}_P,\mathbf{p}_P)\chi_{P0}(\mathbf{r}_P)=(\mu_p-\mu_n+1)\frac{G}{(8)^{1/2}M} \times\sum_{ci}\int d\tau_p\int d\tau_c\chi_{Pf}^*(\mathbf{r}_p)\omega_{ci}^*(\mathbf{r}_c)i\boldsymbol{\sigma}_P\times\boldsymbol{\sigma}_c \cdot[(\mathbf{p}_P-\mathbf{p}_c)\delta(\mathbf{r}_P-\mathbf{r}_c)-\delta(\mathbf{r}_P-\mathbf{r}_c)(\mathbf{p}_P-\mathbf{p}_c)] \times\omega_{ci}(\mathbf{r}_P)\chi_{P0}(\mathbf{r}_c). \quad (52)$$

The core-particle orbit wave functions ω_{ci} are understood to be perturbed by the core-Coriolis interaction. The sum over c_i is over the core orbits occupied by the neutrons (protons) if the extra particle is a proton (neutron). Here, we have omitted the contributions from the first term of W, Eq. (51), because, as before, the semiclassical model to be used below has the core spins coupled to zero. (In a microscopic theory, such as that outlined in Sec. III, this is not so.) Straight-

[7] F. Curtis Michel, Phys. Rev. **133**, B329 (1964).

forward calculation from (52) yields

$$H_P'=\frac{G}{(8)^{1/2}M}(\mu_P-\mu_n+1)\{2\boldsymbol{\sigma}_P\cdot\mathbf{p}_P\sum_{ci}[\omega_{ci}{}^*\omega_{ci}]_{rP}$$
$$-2\boldsymbol{\sigma}_P\cdot\sum[\tfrac{1}{2}\omega_{ci}{}^*\mathbf{p}\omega_{ci}+\tfrac{1}{2}(\mathbf{p}\omega_{ci})^*\omega_{ci}]_{rP}\}. \quad (53)$$

The first term is that of Michel.[7] The second term becomes, on recognizing

$$\sum_{ci}[\tfrac{1}{2}\omega_{ci}{}^*\mathbf{p}\omega_{ci}+\tfrac{1}{2}(\mathbf{p}\omega_{ci})^*\omega_{ci}]=\mathbf{p}_{\rm core}\rho_c', \quad (54)$$

$$H_{P,2}'=-\frac{G}{(2)^{1/2}M}(\mu_p-\mu_n+1)\boldsymbol{\sigma}_P\cdot\mathbf{p}_{\rm core}\rho_c', \quad (55)$$

where ρ_c' is $(N/A)\rho_c$ if we have an odd proton, $(Z/A)\rho_c$ if we have an odd neutron. In the rigid-rotation case, the K-breaking part of $H_{P,2}'$ is

$$H_{P,2}'=\frac{G}{(2)^{1/2}}(\mu_P-\mu_N+1)\rho_c'\sum_\kappa(\boldsymbol{\sigma}_P\times\mathbf{r}_P)_\kappa I_\kappa/\mathcal{I}_\kappa. \quad (56)$$

This is seen to be simultaneously a parity-mixing and K-breaking interaction.

To see the effect of this double mixing, we consider the simplest case of an $M1$ transition that is one-unit K-forbidden. The admixtures produced by $H_{P,2}'$ will permit a K-allowed electric-dipole $\tilde{E}1$. Since this $\tilde{E}1$ transition matrix element will not vanish in general, we can study it by taking the simplest shell-model Hamiltonian

$$H_P=p_P{}^2/2M+U(\mathbf{r}_P), \quad (57)$$

to which must be added the usual parity-mixing interaction,[7] the particle-Coriolis interaction, H_c' of Eq. (9), and $H_{P,2}'$ from Eq. (56).

$$H=\frac{p_P{}^2}{2M}+U(\mathbf{r}_P)+\frac{G\hbar}{(2)^{1/2}M}(\mu_P-\mu_N+1)\rho_c'\boldsymbol{\sigma}_c'\cdot\mathbf{p}_P+H_c'$$
$$+\frac{G\hbar}{(2)^{1/2}}(\mu_P-\mu_N+1)\rho_c'\sum_{\kappa=1,2}(\boldsymbol{\sigma}_P\times\mathbf{r}_p)_\kappa I_\kappa/\mathcal{I}_\kappa. \quad (58)$$

Following Michel,[7] the canonical transformation, $H\rightarrow e^{-iS}H_0e^{+iS}$,

$$S=+[G/(2)^{1/2}](\mu_P-\mu_n+1)\rho_c'\boldsymbol{\sigma}_P\cdot\mathbf{r}_P, \quad (59)$$

leaves us with

$$H_0=\frac{p_P{}^2}{2M}+U(\mathbf{r}_P)+\frac{G}{(2)^{1/2}}\hbar(\mu_P-\mu_N+1)\rho_c'$$
$$\times\sum_{\kappa=1,2}(\boldsymbol{\sigma}_P\times\mathbf{r}_P)_\kappa I_\kappa/\mathcal{I}_\kappa-H_c'+\mathcal{O}(G^2). \quad (60)$$

This transformation also transforms the complete convection current

$$[\mathbf{p}_P/M+(G\hbar/(2)^{1/2}M)(\mu_P-\mu_n+1)\rho_c'\boldsymbol{\sigma}_p]\rightarrow\mathbf{p}_P/M, \quad (61)$$

which is the basis of the proof[7] that electric-multipole admixtures $\tilde{E}L$ vanish in the absence of spin-orbit dependence in the particle Hamiltonian, H_P, and when $H_{P,2}'$ is ignored. For the K-forbidden $M1$ transition we are considering here, the $\tilde{E}1$ transition matrix element is

$$\frac{1}{\epsilon_f-\epsilon_0}\frac{\hbar}{M}\sum_i\left\{\frac{\langle f|\mathbf{p}\cdot\hat{e}|i\rangle\langle i|H_{P,2}'|0\rangle}{\epsilon_0-\epsilon_i}\right.$$
$$\left.+\frac{\langle f|H_{P,2}'|i\rangle\langle i|\mathbf{p}\cdot\hat{e}|0\rangle}{\epsilon_f-\epsilon_i}\right\}$$
$$=\sum_i\left\{\frac{\langle f|\mathbf{r}\cdot\hat{e}|i\rangle\langle i|H_{P,2}'|0\rangle}{\epsilon_0-\epsilon_i}\right.$$
$$\left.+\frac{\langle f|H_{P,2}'|i\rangle\langle i|\mathbf{r}\cdot\hat{e}|0\rangle}{\epsilon_f-\epsilon_i}\right\}. \quad (62)$$

We can compare this consequence of $H_{P,2}'$, with the results obtained in its absence.

Michel[7] obtains a nonzero result for an $\tilde{E}1$ admixture by considering the spin-orbit term in the particle Hamiltonian. The operator so obtained is $\mathfrak{M}(\tilde{E}1)$,

$$\mathfrak{M}(\tilde{E}1)=\gamma e(\boldsymbol{\sigma}\cdot\mathbf{r})r[(G/(8)^{1/2})\rho_c'(\mu_P-\mu_n+1)], \quad (63)$$

where γ is proportional to the strength of the spin-orbit interaction and has been taken[7] as

$$\gamma\simeq 1.3A^{-1/3}.$$

The $\tilde{E}1$ transition matrix element corresponding to Eq. (62) is

$$\sum_i\left\{\frac{\langle f|\mathfrak{M}(\tilde{E}1)|i\rangle\langle i|H_c'|0\rangle}{\epsilon_0-\epsilon_i}\right.$$
$$\left.+\frac{\langle f|H_c'|i\rangle\langle i|\mathfrak{M}(\tilde{E}1)|0\rangle}{\epsilon_f-\epsilon_i}\right\}. \quad (64)$$

An order of magnitude comparison between Eqs. (64) and (62) is simply

$$\sim\tfrac{1}{2}\gamma j_P\sim\tfrac{1}{8}j_P,$$

slightly favoring Eq. (62) based on $H_{P,2}'$. We leave detailed calculations to later work.

The analysis outlined in this section is also applicable to any two-particle operator, such as exchange charges and currents.

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