IV. CONCLUSIONS

The main features of the behavior of the $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ phase shifts and damping parameters for $C^{12}+p$ at $E_p \leqq 6.5$ MeV have been explained by a coupled-channels calculation. The $d_{5/2}$ resonance at $E_p=4.808$ MeV and the $d_{3/2}$ anomaly at $E_p \approx 5.3$ MeV have been shown to be caused by a low energy s-wave resonance in the $(C^{12*}+p)$ channel, closely corresponding to the 0.461-MeV $s_{1/2}$ resonance in the elastic channel. The diagonal parts of the potentials used have the following relation:

$$|V(_{5/2})| > |V(s_{1/2})| > |V(d_{3/2})| .$$

This relation is consistent with the existence of a spin-orbit interaction with the usual sign.

ACKNOWLEDGMENTS

The author is extremely grateful to Professor T. A. Tombrello for the encouragement, help and advice he has given on many occasions. Dr. W. P. Timlake gave substantial help on the mathematical aspects of this problem and Professor Ian Duck and Professor E. B. Paul contributed through several enlightening discussions.

PHYSICAL REVIEW VOLUME 155, NUMBER 4 20 MARCH 1967

Theory of Pion Absorption Applied to the Reaction $Li^6(\pi^-,2n)He^4$†

D. S. KOLTUN
Department of Physics and Astronomy, University of Rochester, Rochester, New York

AND

A. REITAN*
Department of Physics and Astronomy, University of Rochester, Rochester, New York
and
Norges Tekniske Høgskole, Trondheim, Norway

(Received 3 October 1966)

The absorption of pions from a low-energy S state, by a nucleus with the emission of two fast nucleons, is treated in an impulse approximation in which the pion interacts only with the pair of ejected nucleons. We consider the particular reaction $Li^6(\pi^-,2n)He^4$. The two-nucleon absorption process is given by a theory, previously applied to the pion absorption by a deuteron, which is developed from low-energy pion-nucleon scattering theory. The relative motion of the absorbing nucleon pair is treated in detail, including effects of short-range repulsion and tensor potentials. Distributions of the emitted nucleons in momentum and angle are calculated in this theory, and are compared with other results for some theories and with experimental data.

I. INTRODUCTION

THE reaction in which a nucleus absorbs a slow pion and emits two fast nucleons has been attractive for theoretical consideration since kinematical conditions seem favorable to some form of impulse approximation. The notion is that two colliding nucleons in a nucleus may convert the rest energy of the pion to kinetic energy with little transfer of momentum to the residual nucleus. If the probability is high that these two nucleons will escape with little further interaction with the residual nucleus, it is inviting to try to treat the problem in two parts: the absorption of a pion by two nucleons, and the motion of the two nucleons in a nucleus. This approach was first introduced in the work of Brueckner, Serber, and Watson[1] who attempted to relate the rate for pion absorption in nuclei to the absorption of a pion by a free deuteron.

More recently a model along these lines has been applied to the absorption of S-wave pions by various light nuclei, by Eckstein[2] for He^4, by Divakaran[3] for He^3, by Sakamoto[4] for Li^6, and by M. Ericson[5] for several cases. These calculations have in common a model interaction which includes both the pion interaction with a nucleon pair, and the short-range correlation of the nucleon pair in its initial state. The strength parameters of the assumed interaction are fixed to give the experimental cross sections for thresh-

† Work supported in part by the U. S. Atomic Energy Commission.

* Permanent address: Norges Tekniske Høgskole, Trondheim, Norway.

[1] K. Brueckner, R. Serber, and K. Watson, Phys. Rev. **84**, 258 (1951).
[2] S. G. Eckstein, Phys. Rev. **129**, B387 (1963).
[3] P. P. Divakaran, Phys. Rev. **139**, B387 (1965).
[4] Y. Sakamoto, Nuovo Cimento **37**, 775 (1965).
[5] M. Ericson, Compt. Rend. **258**, 1471 (1964).

old production of pions in nucleon-nucleon collisions,

$$N+N \rightarrow N+N+\pi \quad (1)$$

which is the time-reverse of pion absorption. However, this determination has the kinematics appropriate to the nuclear absorption only for the limiting case in which the full rest energy of the pion goes into relative motion of the pair of nucleons; the dependence of the absorption amplitude on the center-of-mass energy given to the pair cannot be determined empirically, so it is assumed to have some simple form in these papers.[2–5] A second limitation of this approach is that it is assumed that only pairs in relative S states in the target contribute to the absorption, and that the short-range correlation for all such pairs is the same. These assumptions, while possibly reasonable, cannot be checked within the model.

An alternative approach which avoids some of these objections, and which is still based on the notation of a two-nucleon absorption process, has been advanced by several authors.[6] The pion absorption interaction is assumed to be the nonrelativistic pseudovector pion-nucleon interaction

$$\mathcal{H}(i) \propto \boldsymbol{\sigma}_i \cdot (\mathbf{v}_\pi - \mathbf{v}_i), \quad (2)$$

where $\mathbf{v}_\pi$ and $\mathbf{v}_i$ are the velocities of the pion, and the ith nucleon, respectively. The strength of this interaction is fixed by the effective-range formula for p-wave pion-nucleon scattering.[7] With such an interaction, the two-nucleon absorption can be computed for all kinematical situations, and for different assumptions about pair correlations in complex nuclei. Of course this demands confidence in the pseudovector interaction as given.

The problem with this second approach is that it does not seem to be consistent with the known cross sections for pion production at threshold (1), particularly for the reaction

$$p+p \rightarrow d+\pi^+, \quad (3)$$

which is equivalent to the absorption of π^- at rest by the deuteron. The situation is reviewed by Woodruff,[8] who proposed that a process which competes with that given by absorption through the one-nucleon interaction (2), is the S-wave scattering of the pion from one nucleon before absorption by the second. The present authors showed[9] that the combination of these processes does yield the correct cross section (3), when the initial and final nucleon states are solutions of a phenomenological potential, like that of Hamada and Johnston.[10]

In the present paper we apply the theory developed in I (Ref. 9) to a particular example of two-nucleon absorption:

$$\pi^-+\mathrm{Li}^6 \rightarrow \mathrm{He}^4+2n. \quad (4)$$

As in other treatments, we use an impulse model so that the reaction amplitude can be factored into a part for the absorption of the pion by the "valence" nucleons of Li^6, and a factor for the recoil of the He^4. This treatment differs from those discussed above in the following particulars:

(1) The interaction between the pion and the two absorbing nucleons is calculated from more elementary pion-nucleon interactions, but is more complicated than the pseudovector interaction (2). The derived interaction is a two-nucleon operator, and is consistent with the threshold cross section (3).

(2) Two-nucleon correlations are included explicitly for the absorbing pair in the initial and final states. In the final state, the pair is considered free of the recoiling He nucleus, so the relative motion of the pair is given by the Schrödinger equation for a two-nucleon system at the relative energy appropriate to the kinematics. We integrate the equation using the potential of Hamada and Johnston.[10]

In the initial state the absorbing pair is treated by the method of Dawson, Talmi, and Walecka,[11] in which the relative motion of the pair is modified from that given by the harmonic-oscillator shell model, by integrating a Schrödinger equation for each state of relative motion with a combination of harmonic oscillator and phenomenological two-nucleon potential. This induces not only a short-range correlation into the pair motion, but also orbital angular-momentum admixtures through the tensor potential, like the $^3D_1$ component of the deuteron. Both types of correlation were found important in the process[9] (3).

The details of the theory are given in Sec. II. The results, in the form of distributions of neutrons in momentum and angle, are given in Sec. III, and comparison with experiment and with other theories is discussed in Sec. IV.

II. THEORY

We are interested in the amplitude $M(\mathbf{k}_1,\mathbf{k}_2)$ for the reaction $\mathrm{Li}^6(\pi^-,2n)\mathrm{He}^4$, where the pion is initially in an S state, with negligible momentum with respect to the target, and $\mathbf{k}_1$, $\mathbf{k}_2$ are the momenta of the two emitted neutrons. We shall express this amplitude in terms of a transition operator T which describes the absorption interaction in terms of nucleon variables. Assuming the

[6] P. Huguenin, Z. Physik **167**, 416 (1962); T. Ericson, Phys. Letters **2**, 278 (1962); M. Jean, Nuovo Cimento Suppl. **2**, 400 (1964); G. M. Shklyarevskii, Zh. Eksperim. i Teor. Fiz. **45**, 698 (1963) [English transl.: Soviet Phys.—JETP **18**, 480 (1964)]; R. I. Jibuti and T. I. Kipaleishvilli, Nucl. Phys. **55**, 337 (1964); R. M. Spector, Phys. Rev. **134**, B101 (1964); T. Kohmura, Progr. Theoret. Phys. (Kyoto) **34**, 234 (1965).

[7] See, e.g., J. D. Jackson, *The Physics of Elementary Particles* (Princeton University Press, Princeton, New Jersey, 1958), p. 24.

[8] A. E. Woodruff, Phys. Rev. **117**, 1113 (1960).

[9] D. S. Koltun and A. Reitan, Phys. Rev. **141**, 1413 (1966), which will be referred to in this paper as I.

[10] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).

[11] J. F. Dawson, I. Talmi, and J. D. Walecka, Ann. Phys. (N. Y.) **18**, 339 (1962); **22**, 133 (1963).

interaction is weak, we have, to lowest order,

$$M(\mathbf{k}_1,\mathbf{k}_2)=\langle \mathrm{He}^4,\mathbf{k}_1,\mathbf{k}_2|\,T\,|\mathrm{Li}^6\rangle \tag{5}$$

which is to be understood as a *nuclear* matrix element: The matrix element for the pion is implicit in our definition of T. Following the theory of I, T includes contributions from absorption of the pion by a single nucleon, and from the scattering of the pion from one nucleon and absorption by a second. Thus T is a sum of one- and two-nucleon operators [see I, Eq. (4)]. Our impulse approximation consists of factoring (5)

$$M\simeq\langle \mathbf{k}_1,\mathbf{k}_2|\,T(1,2)\,|\alpha(1,2)\rangle\Gamma\,, \tag{6}$$

where Γ gives the overlap of the Li^6 ground state with the recoiling He^4, plus a proton-neutron pair in the state $\alpha(1,2)$. Γ is a function of the recoil momentum of the He^4 nucleus. In approximation (6) we have considered only the factoring of the Li^6-state function into He^4 core plus valence particles in the shell-model sense. Correct treatment of antisymmetry introduces additional amplitudes in which the recoil He^4 is "made" of 2 core $+2$ valence or 3 core $+1$ valence particles. However, amplitude (6) is largest from symmetry considerations, and is the only term we have retained.

In making approximation (6), we are also neglecting certain three-body processes in which one of the emitted particles was not directly involved in the absorption.

It is convenient to express both initial and final two-body states in (6) in terms of relative and center-of-mass coordinates and momenta:

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2\,,\qquad \mathbf{R}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)\,,$$
$$\mathbf{k}=\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)\,,\quad \mathbf{K}=\mathbf{k}_1+\mathbf{k}_2\,.$$

The final state is taken as a product

$$\psi(\mathbf{k},\mathbf{K};\mathbf{r},\mathbf{R})=\exp(i\mathbf{K}\cdot\mathbf{R})\phi_k(\mathbf{r})\,, \tag{7}$$

where $\phi_k(\mathbf{r})$ is a solution of the free two-nucleon scattering equation for relative energy $E=k^2/m$ (nucleon mass$=m$). Spin is implicit in the relative wave function.

The initial two-nucleon state is based on the harmonic-oscillator shell model, modified by the method of Dawson, Talmi, and Walecka.[11] That is, we assign the absorbing pair to the first p shell, coupled to $L=0$, $S=1$ $[(0p)^2\,{}^3S_1]$ and write the wave function in relative and cm coordinates.[12]

$$\psi_\alpha(\mathbf{r},\mathbf{R})=(\sqrt{\tfrac{1}{2}})[\tilde{\phi}_{0s}(\mathbf{r})\phi_{1s}(\mathbf{R})-\tilde{\phi}_{1s}(\mathbf{r})\phi_{0s}(\mathbf{R})]. \tag{8}$$

In the absence of a two-body interaction, $\phi_{ns}(\mathbf{R})$ and $\tilde{\phi}_{ns}(\mathbf{r})$ are S-state solutions of the harmonic-oscillator wave equations in c.m. and relative coordinates, respectively. But in the presence of a two-body potential, the $\tilde{\phi}_{ns}(\mathbf{r})$ are taken (see Ref. 11) as solutions of the wave equation for the harmonic oscillator plus two-body potential, in relative coordinates. The wave function (8) may be thought of as an improvement, in the variational sense, over the shell-model function, in the presence of the two-body potential. Thus for a phenomenological potential applicable to nucleon-nucleon scattering, $\tilde{\phi}_{ns}$ will have S- and D-state components like the deuteron:

$$\tilde{\phi}_{ns}(r)=r^{-1}[\tilde{u}_n|\,{}^3S_1\rangle+\tilde{w}_n|\,{}^3D_1\rangle], \tag{9}$$

and it will exhibit short-distance structure due to short-range repulsion.

We have used radial wave functions of this type, which were calculated by Kim,[13] for the Hamada-Johnston potential. We have taken the single-particle harmonic-oscillator length $\beta=(\hbar/m\omega)^{1/2}$ as 1.6 F to make use of Kim's calculations, which are appropriate only for that value.

The operator $T(1,2)$ is given by the theory of I, with appropriate modifications due to the c.m. motion of the absorbing nucleon pair in the target nucleus. We obtain T from I, Eqs. (6), (7′), and (8′) by taking the nuclear isospin matrix element for the transition $T=0\rightarrow T=1$,

$$T(1,2)=\alpha\phi_\pi(0)\mathbf{S}\cdot[\boldsymbol{\nabla}+\gamma\mathbf{g}(r)]+\text{c.m. modifications}\,, \tag{10}$$

with

$$\alpha=-2i(4\pi)^{1/2}m^{-1}(2\mu)^{-1/2}f\,,$$
$$\gamma=(2\lambda_1+3\lambda_2)m/\mu\,,$$
$$\mathbf{g}(r)=\boldsymbol{\nabla}(\mu r)^{-1}\exp(-\nu r)\,,$$

and

$$\nu^2=\tfrac{3}{4}\mu^2\,,$$

where μ and m are the pion and nucleon masses, respectively, $\mathbf{S}$ is the total spin of the two nucleons, and $\boldsymbol{\nabla}$ is the gradient in nucleon relative coordinates. Elimination of the pion field in (5) introduces the pion initial-state wave function ϕ_π at the positions of the nucleons. For low-pion momentum on a Li target the wave function can be approximated by a constant, equal to the value at the origin, $\phi_\pi(0)$. As in I, we have taken the phenomenological pion-nucleon scattering constants $f^2=0.088$, $\lambda_1=0.005$, $\lambda_2=0.045$. We have dropped in (10) a term which was kept in I, which is a recoil correction in the rescattering of order $\mu/4m\sim0.04$.

The c.m. modifications in (10) have been estimated to be small, and are therefore omitted. One term is linear in the c.m. momentum $\mathbf{K}$, and gives a small correction for $K<k$. Also the form of $g(r)$ should vary with c.m. motion, but the calculation proved insensitive to such variation.

The result is that $T(1,2)$, under these approximations, reduces to an operator in relative coordinates only. With our assumption that the initial state is ${}^3S_1+{}^3D_1$, [Eqs. (8) and (9)], the final relative motion

[12] See, e.g., T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets* (Monografias del Instituto de Fisica, Mexico, 1960).

[13] Y. E. Kim, Phys. Letters **19**, 583 (1965), and (private communication).

state will be $^3P_1$, as in the deuteron absorption:

$$\phi_k(r)=\mp(\sqrt{2}/pr)iu_{1,1}(r) \times e^{i\delta_{1,1}}(2\pi\times 3)^{1/2}|^3P_1\rangle, \quad (M_J=\pm 1). \quad (11)$$

The calculation of the amplitude (6) now involves an integral over relative pair coordinates, which is done as in I, but for a range of relative momenta in the final state. The integrals over pair c.m., and He^4 c.m. coordinates are simply Fourier transforms (in K) of the respective harmonic-oscillator functions in configuration space, for the initial state, because of the plane-wave assumption (7). Thus for the He^4 recoil,

$$\Gamma(K)=\text{const}\times\exp(-\beta^2K^2/8),$$

while the $0s$ and $1s$ states of c.m. motion (8) have transforms, up to an over-all constant,

$$\phi_{0s}(R)\rightarrow\phi_{0s}(K_{ij})=\exp(-K^2\beta^2/4),$$
$$\phi_{1s}(R)\rightarrow-\phi_{1s}(K)=-(\sqrt{\tfrac{3}{2}})(1-\tfrac{1}{3}K^2\beta^2) \times\exp(-K^2\beta^2/4). \quad (12)$$

Substituting (7), (8), and (10) into (6), we may express the amplitude squared, summed over spin projections:

$$\sum|M|^2=\alpha^2\Gamma^2(K)\phi_\pi^2(0) \times|I_{0s}(k)\phi_{1s}(K)+I_{1s}(k)\phi_{0s}(K)|^2, \quad (13)$$

where

$$I_{ns}(k)=\sum_{i=1}^{4}K_i(n),$$

$$K_1(n)=\frac{\mu}{k}\int_0^\infty dr\,r^2\frac{u_{1,1}(r)}{r}\frac{d}{dr}\frac{\tilde{u}_n(r)}{r},$$

$$K_2(n)=(2)^{-1/2}\frac{\mu}{k}\int_0^\infty dr\,r^2\frac{u_{1,1}(r)}{r}\left(\frac{d}{dr}-\frac{3}{r}\right)\frac{\tilde{w}_n(r)}{r},$$

$$K_3(n)=\gamma\frac{\mu}{k}\int_0^\infty dr\,u_{1,1}(r)g'(r)\tilde{u}_n(r), \quad (14)$$

$$K_4(n)=(2)^{1/2}\gamma\frac{\mu}{k}\int_0^\infty dr\,u_{1,1}(r)g'(r)\tilde{w}_n(r),$$

$$g'(r)=\frac{d}{dr}g(r).$$

We can also calculate the squared amplitude for absorption, in which we do not include the possibility of scattering of pions between nucleons, by omitting K_3 and K_4 from I_{ns} in (13). This corresponds to the $\boldsymbol{\sigma}\cdot\mathbf{v}$ interaction alone; we shall designate the squared amplitude by $\sum|M'|^2$.

III. KINEMATICS AND RESULTS

The value of the integrals K_i for several values of relative energy $E=k^2/m$ are given in Table I. For comparison, we have included the values of K_i for the deuteron calculated in I (but here normalized $N^2=1$). We have also listed the sums I_{ns}, and for comparison, the sums $I_{ns}'=K_1(n)+K_2(n)$ in which pion scattering is omitted.

TABLE I. The partial relative integrals $K_i(n)$ and their sum I_{ns}, defined in Eqs. (13) and (14), with initial-state wave functions of form (9) as calculated by Kim (Ref. 13) and final-state wave functions (11) calculated for several values of relative energy $E=k^2/m$, with the Hamada-Johnston potential (Ref. 10). I_{ns}' are the integrals with the meson-scattering terms omitted. We include for comparison the integrals for the deuteron initial state at $E=\mu c^2\simeq 140$ MeV.

E (MeV)	ns	K_1	K_2	K_3	K_4	I_{ns}	I_{ns}'
140	$0s$	−0.034	0.121	−0.147	−0.026	−0.094	0.087
	$1s$	−0.082	0.085	−0.098	−0.020	−0.115	0.003
Deuteron		−0.084	0.088	−0.128	−0.036	−0.161	0.003
120	$0s$	−0.058	0.121	−0.155	−0.026	−0.118	0.063
	$1s$	−0.138	0.093	−0.099	−0.022	−0.166	−0.045
100	$0s$	−0.115	0.132	−0.164	−0.028	−0.175	0.017
	$1s$	−0.270	0.114	−0.098	−0.023	−0.277	−0.156
70	$0s$	−0.270	0.113	−0.177	−0.031	−0.365	−0.157
	$1s$	−0.625	0.113	−0.088	−0.027	−0.627	−0.512

The relative and c.m. momenta of the two neutrons are related through the energy equation (including recoil)

$$k^2+\tfrac{3}{8}K^2=m\Delta E, \quad (15)$$

where

$$\Delta E=\mu+M(\text{Li}^6)-M(\text{He}^4+2n).$$

In general, the summed squared amplitude (13) is a function of two kinematic variables: if we choose K as one, then either the c.m. angle θ, or the lab angle ψ, will serve as the second (see Fig. 1). The angular-momentum selection rules of the impulse approximation can be shown to lead to simple dependence of the squared amplitude on $z=\cos\theta$. For example, in the present case of Li^6, the assumption of pure S-state c.m. motion of the absorbing pair [Eq. (8)], leads to independence of expressions (13) on the variable z, that is, to isotropy of the intensity on the sphere (in the c.m. system). A more general study of such angular correlations will be presented in another paper.[14]

In Fig. 2 we show the calculated distribution in K for intensity $\sum|M|^2$ [Eq. (13)], as well as for $\sum|M'|^2$, the distribution for no meson scattering. We plot the intensities on a logarithmic scale, with arbitrary over-all normalization, against $X=K\beta/\sqrt{2}=K/(170\text{ MeV}/c)$.

The three-body phase space is particularly simple in

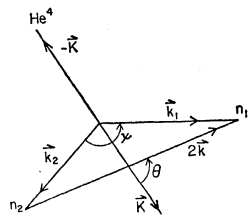

FIG. 1. Kinematics of the three-body final state for the reaction $Li^6(\pi^-,2n)He^4$.

[14] D. S. Koltun (to be published).

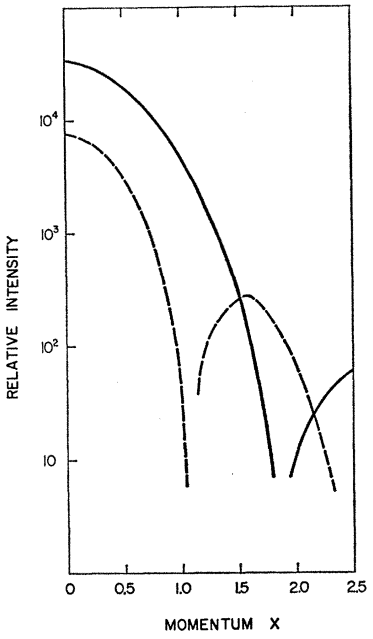

FIG. 2. The intensities (squared amplitudes) $\sum |M|^2$ for our theory (solid line), and $\sum |M'|^2$ for no meson scattering (dashed line), as functions of momentum $X=K\beta/\sqrt{2}=K/(170\ \mathrm{MeV}/c)$. The over-all normalization is arbitrary.

terms of K and $z(\cos\theta)$, so that the distribution of neutrons may be expressed

$$\frac{\partial^2 W(K,z)}{\partial K\partial z}=\text{const}\times\sum_{\text{spin}}|M(K,z)|^2kK^2. \quad (16)$$

The spectrum in momentum K of the emitted neutron pair, or alternatively of the recoiling α particles is obtained from (16)

$$\frac{\partial W(K)}{\partial K}\propto\sum|M|^2kK^2. \quad (17)$$

This quantity is plotted, with arbitrary normalization, in Fig. 3, both for our theory, $\partial W/\partial K$, and for the no-scattering theory, $\partial W'/\partial K$.

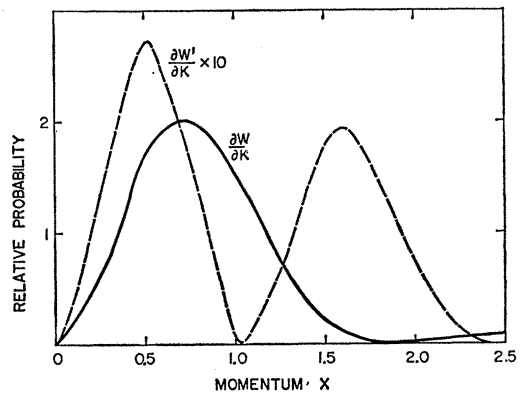

FIG. 3. The spectrum in momentum K of the neutron pair, $\partial W/\partial K$, for our theory (solid line), and $10\times\partial W'/\partial K$ for no meson scattering (dashed line). Again $X=K/(170\ \mathrm{MeV}/c)$, and over-all normalization is arbitrary.

Frequently experimental results are given in terms of the laboratory angle ψ. The distribution in K, $y=\cos\psi$ is given by

$$\frac{\partial^2 W(K,y)}{\partial K\partial y}\propto\frac{K(k^2-\frac{1}{4}K^2)^2\sum|M|^2}{y^2[(k^2+\frac{1}{4}K^2)^2y^2-(k^2-\frac{1}{4}K^2)^2]^{1/2}}. \quad (18)$$

This distribution has been measured for $\mathrm{Li}^6(\pi^-,2n)\mathrm{He}^4$ by a Liverpool group,[15] but only for $y=-1$ ($\psi=\pi$), for which (18) becomes

$$\frac{\partial^2 W(K,-1)}{\partial K\partial y}\propto\frac{(k^2-\frac{1}{4}K^2)^2}{k}\sum|M(K)|^2. \quad (19)$$

Our theoretical result is compared to the experimental distribution in Fig. 4.

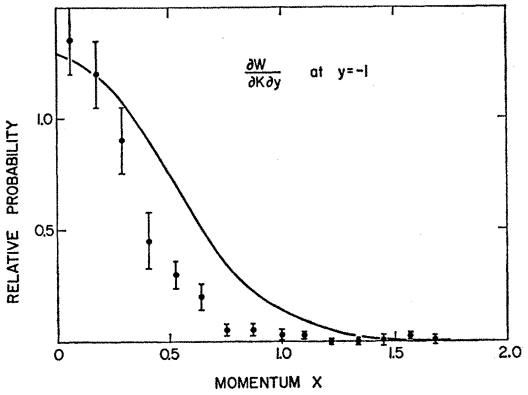

FIG. 4. Comparison of the momentum and angle distribution as a function of K at $y=\cos\psi=-1$, for our theory (solid line) and experiment of Ref. 15 (bars).

IV. CONCLUSIONS

In discussing the results of our calculations, we must distinguish those characteristics which are largely a feature of the impulse model, and those which depend on the detailed mechanism we have assumed. The dominant behavior of the distribution $\sum|M(K,\theta)|^2$ in our model is exhibited in Eqs. (12) and (13): The distribution is independent of θ and drops rapidly with increasing K. This general behavior results only from the assumptions that the center-of-mass motion of the absorbing nucleon pair is described in the initial state as an S state in a harmonic-oscillator potential, and in the final state by a plane wave. Thus similar behavior can be expected in other impulse theories in which similar assumptions about Li^6 are made.

The θ dependence for the reaction $\mathrm{Li}^6(\pi^-,2n)\mathrm{He}^4$ has not been measured, but the K dependence measured in the Liverpool experiment[15] is compared with our calculation in Fig. 4. The fact that the calculated distribution is wider than the experimental, is probably attributable to our use of Kim's calculated wave func-

[15] H. Davies, H. Muirhead, and J. N. Woulds, Nucl. Phys. 78, 663 (1966).

tions, which restrict us to his choice of the harmonic-oscillator length parameter $\beta=1.6$ F, which parameter is too small for the p-shell nulceons in Li^6.[16] We could, however, vary this parameter separately for the c.m. functions in (8), and fit the experimental width, if we chose to interpret Li^6 in a cluster-model sense.

Some features of the results of our theory differ from those of other impulse models, because of our explicit treatment of two-nucleon correlations, and the absorption mechanism (10) developed in I. For example, we have compared the effect of including meson scattering before absorption, to its neglect, in Table I, Figs. 2 and 3. From the table we see that the latter assumption leads to a smaller rate than does our theory, which was also true for the deuteron absorption. In the figures we see that neglect of scattering shifts the minimum and considerably increases the relative strength of the second maximum in the distributions. Thus it would be possible in principle to distinguish between our theory and those of Ref. 6, if it could also be shown that other corrections to both types of approach do not overwhelm the differences. Of course much greater experimental accuracy would also be required.

We also may compare our results with those based on the absorption mechanism given by Eckstein.[2-4] The main difference is in the dependence of the relative motion integrals $I_{ns}(k)$ on the relative momentum k [see Eq. (13)]. Eckstein assumes $I \propto k$, while our values of I_{ns} *decrease* with k, as is shown in Table I. As a result, our prediction of the absolute rate and momentum spectrum (17) is higher than for such theories, assuming they agreed at the threshold energy, $E=140$ MeV.

Several other consequences of our assumptions should be noted. As in the deuteron absorption, the contribution of the $^3D_1$ state is very important. The wave functions we have used in these calculations have $\sim 6\%$ D-state probability, based on Kim's calculations. A different treatment of the tensor force in the Li^6 wave function would alter our results.

Aside from tensor-induced contributions, the inclusion of two-nucleon scattering potentials in the initial and final states enhances the rate over that given by an ordinary shell-model initial state and plane-wave final state.

A more fruitful approach to testing the various reaction models is to calculate branchings to different final target states, as has been done by Kopaleishvili *et al.*[17] It would be more useful to have these branchings, both theoretical and experimental, as a function of K, particularly for small K, where impulse models are expected to work best.

ACKNOWLEDGMENTS

The authors wish to thank Dr. Y. E. Kim for supplying the results of his wave-function calculations (Ref. 13). They are also grateful to R. Abel for assistance in computing.

[16] See, e.g., L. R. B. Elton, *Nuclear Sizes* (Oxford University Press, Oxford, England, 1961), p. 25.

[17] T. I. Kopaleishvili, I. Z. Machabeli, G. Sh. Goksadge, and N. B. Krupennikova, Phys. Letters **22**, 181 (1966).

PHYSICAL REVIEW VOLUME 155, NUMBER 4 20 MARCH 1967

Two- and Four-Quasiparticle States in Spherical Vibrational Nuclei*

M. K. PAL,† Y. K. GAMBHIR, AND RAM RAJ
Saha Institute of Nuclear Physics, Calcutta, India
(Received 21 October 1966)

Expressions are derived in the modified Tamm-Dancoff (TD) approximation which take into account the mixing of zero-, two-, and four-quasiparticle states for the description of the collective vibrational states of spherical nuclei. The proposed method is free from two very important defects inherent in the higher random-phase approximation (HRPA), namely, nonorthonormality and redunancy of the four-quasiparticle basic states. The relative merits of the present method over the existing one (HRPA) are discussed.

1. INTRODUCTION

IN recent years a unified theory of nuclear structure has emerged. One now starts from a basic independent-particle model and then derives the collective properties of nuclei by suitably treating the residual two-body interactions. A straightforward diagonalization of the residual forces becomes unmanageable whenever the nucleus is away from a closed shell; this is because of the numerous near-degenerate configurations that are possible for a few nucleons moving in an unfilled major shell. The task has been greatly facilitated by the development of the quasiparticle theory[1] of the nucleus, which neatly takes into account the strong "pairing interaction" between nucleons by the Bogoliubov-Valatin transformation resulting in independent quasiparti-

* Preliminary reports on this work were published in the Proceedings of the Low-Energy Nuclear Physics Symposium (1964 and 1965) held by the Department of Atomic Energy, Government of India.

† Present address: Institute of Theoretical Science and the Department of Physics, University of Oregon, Eugene, Oregon.

[1] M. Baranger, Phys. Rev. **120**, 957 (1960); this work lists all other earlier publications.