a sufficiently accurate description of the low-energy collective spectrum—at least as far as the 0^+ and 2^+ states are concerned, which alone can participate in the dipole states. Then one may hope to achieve quite a good description when adding the dipole excitation since, as already mentioned, the two modes contain to a large extent different single-particle states.

The most conspicious discrepancy between theory and experiment is, however, the structure at the low-energy side of the resonance. No consistent explanation of this discrepancy has as yet been given. It seems very likely that the excess cross section should be associated with some of those states which in the schematic model[26] have been swept clear of any transition strength.[27]

In the language of the collective model the giant resonance is an isospin wave. It can be coupled to the spin wave. This would result in a splitting of the giant resonance, as observed in the calculations concerning O^{16} where two states carry appreciable dipole strength, the upper being the spin-flip state. It is possible that the same coupling would lead to a structure on the low-energy side of the giant resonance. However, it is very unlikely that this structure would be as complicated as that in praseodymium, Fig. 11. This point thus merits a quantitative exploration.

[26] G. E. Brown and M. Bolsterli, Phys. Rev. Letters **3**, 472 (1959); G. E. Brown, *Modified Theory of Nuclear Models* (North-Holland Publishing Company, Amsterdam, 1964).

[27] M. Danos and E. G. Fuller, Ann. Rev. Nucl. Sci. **15**, 29 (1965).

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Properties of the Projected Spectra for Finite Nuclei

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The general features of the expressions useful for numerical calculations of the projected deformed Hartree-Fock (HF) spectra for finite nuclei are investigated. It is proved that the projected deformed HF wave-function gives the possible nuclear spins as $I=0, 2, 4, \ldots, I_{\max}$ for a $K=0$ band and $I=K, K+1, \ldots, I_{\max}$ for a $K\neq 0$ band. It is further proved that if the energy $E_{I=K}$ of the projected $I=K$ state is greater (less) than the HF energy $E_K^{\rm HF}$, then E_I is greater (less) than E_I' for $I<I'$. A plausible reason why one should use the deformed HF state rather than any other deformed state is also pointed out.

1. INTRODUCTION

RECENTLY there has been considerable interest in Hartree-Fock (HF) calculations for finite nuclei. In nuclear HF calculations there is a special difficulty due to the nucleon-nucleon interaction inside the finite nucleus. Various different approaches are suggested in the literature to cope with this difficulty.[1] Here we will not be concerned with this aspect of the problem. We simply assume some effective inter-nucleon potential inside the nucleus. In the literature, two types of HF calculations are reported: (1) radial HF calculations for nearly closed-shell nuclei,[2] and (2) deformed HF calculations for nonspherical nuclei. Following the finding that one can obtain nearly the same physical results for a nucleus by doing the intermediate coupling calculations or by doing the deformed HF calculations and then projecting good angular momentum states from it, deformed HF calculations gained popularity.[3] By deformed HF calculations, we mean those in which the radial parts of the single-particle orbitals are taken as harmonic-oscillator radial wave functions while the angular momentum parts are determined from the HF variational calculation. Here we will be dealing with deformed HF calculations only.

We investigate the general broad features of the low-lying excited states of nuclei as obtained by projecting the good angular momentum states from the deformed HF wave function, and we derive the properties of the projected spectrum. We also give a justification of why one should project from the HF state

[1] K. A. Brueckner, J. L. Gammel, and H. Weitzner, Phys. Rev. **110**, 431 (1958); S. A. Moszkowski and B. L. Scott, Ann. Phys. (N. Y.) **11**, 657 (1960); R. K. Bhaduri and E. L. Tomnsiak, Proc. Phys. Soc. (London) **86**, 451 (1965); C. Shakin and Y. R. Waghmare, Phys. Rev. Letters **16**, 403 (1966).

[2] Nazakat Ullah and R. K. Nesbet, Nucl. Phys. **39**, 239 (1962); **46**, 254 (1963); Phys. Rev. **134**, B308 (1964); R. Muthukrishnan and M. Baranger, Phys. Letters **18**, 160 (1965); A. K. Kerman, J. P. Svenne, and F. M. H. Villars, Phys. Rev. **147**, 710 (1966).

[3] M. Redlich, Phys. Rev. **110**, 468 (1958); D. Kurath and L. Picman, Nucl. Phys. **10**, 313 (1959); W. H. Bassichis, B. Giraud, and G. Ripka, Phys. Rev. Letters **15**, 980 (1965).

rather than any other deformed state to obtain the low-lying states of nuclei.

2. GENERAL EXPRESSIONS AND THEIR PROPERTIES

Let

$$H=\sum_i T_i+\tfrac{1}{2}\sum_{i\neq j} v_{ij}$$

be the Hamiltonian of the nucleus under consideration. Since H is invariant under time inversion, its eigenfunctions should have definite symmetry under that operation. If, however, the actual eigenfunctions are approximated by the HF wave functions, one should project the good angular momentum wave functions from the latter in order to obtain the same symmetry. Let ϕ_K be the axially-symmetric deformed HF state with the azimuthal quantum number K and HF energy E_K^{HF}. For $K=0$, ϕ_K itself has definite symmetry under time inversion (T), namely $T\phi_0=\phi_0$. For $K\neq 0$, however, we employ the states $\psi_K=(\phi_K+T\phi_K)/\sqrt{2}$, which satisfy $T\psi_K=\psi_K$.[3a] The operators which project the good angular momentum (IM)states from ϕ_K are[4]

$$P_M{}^I=\frac{2I+1}{8\pi^2}\int_0^{2\pi}d\alpha\int_0^{2\pi}d\gamma\int_0^{\pi}\sin\beta d\beta \times D_{MK}{}^{I*}(\alpha\beta\gamma)e^{-i\alpha J_z}\,e^{-i\beta J_y}\,e^{-i\gamma J_z}. \quad (1)$$

In order to project good IM states from ψ_K, the proper operators are

$$P'_M{}^I=\frac{2I+1}{8\pi^2}\int_0^{2\pi}d\alpha\int_0^{2\pi}d\gamma\int_0^{\pi}\sin\beta d\beta \times[D_{MK}{}^{I*}(\alpha\beta\gamma)+(-)^{I-K}D_{M-K}{}^{I*}(\alpha\beta\gamma)] e^{-\alpha J_z}\,e^{-i\beta J_y}\,e^{-i\gamma J_z}. \quad (2)$$

The required good IM states are $P_M{}^I\phi_0$ and $P'_M{}^I\psi_K$. Since the energies of these states do not depend on M, we consider the $M=K$ projection only. The low-lying excited states of the nucleus under consideration can then be approximated by

$$E_0{}^I=\langle P_0{}^I\phi_0|H|P_0{}^I\phi_0\rangle/\langle P_0{}^I\phi_0|P_0{}^I\phi_0\rangle,$$

and

$$E_K{}^I=\langle P'_K{}^I\psi_K|H|P'_K{}^I\psi_K\rangle/\langle P'_K{}^I\psi_K|P'_K{}^I\psi_K\rangle. \quad (3)$$

Similarly the expectation value in the projected state of any other operator (such as quadrupole moment, magnetic moment, etc.) can be found. Using $(P_K{}^I)^2=P_K{}^I$, $[H,P_K{}^I]=0$ and similar relations for $P'_K{}^I$, Eq. (3) can be written in the following form valid for all K:

$$E_K{}^I=\int_0^{\pi} d_{KK}{}^I(\theta)\langle\phi_K|H(\exp-i\theta J_y)|\phi_K\rangle\sin\theta d\theta \Big/ \int_0^{\pi} d_{KK}{}^I(\theta)\langle\phi_K|(\exp-i\theta J_y)|\phi_K\rangle\sin\theta d\theta$$
$$=h_K{}^I/p_K{}^I. \quad (4)$$

From Eq. (4) it is clear that one needs to know

$$\langle\phi_K|H\exp(-i\theta J_y)|\phi_K\rangle \quad\text{and}\quad \langle\phi_K|\exp(-iJ\theta_y)|\phi_K\rangle$$

for the numerical computation of $E_K{}^I$. We evaluate these expressions below for the axially symmetric deformed single-particle HF orbitals

$$\varphi_{m_i\tau_i}{}^{(i)}=\sum_{j_i} c_{m_i\tau_i}{}^{j_i}|j_im_i\tau_i\rangle.$$

After some straightforward but lengthy algebra one obtains

$$\langle\phi_K|\exp(-i\theta J_y)|\phi_K\rangle=D^N(a_{\alpha\beta})D^P(a_{\gamma\delta}) \quad (5)$$

and

$$\langle\phi_K|H\exp(-i\theta J_y)|\phi_K\rangle=T^PD^N(a_{\alpha\beta})+T^ND^P(a_{\alpha\beta}) +V^{PP}D^N(a_{\alpha\beta})+V^{NN}D^P(a_{\alpha\beta})+V^{PN}, \quad (6)$$

where

$$T^N=\sum_{i=1}^N\sum_{j_ij'_i}(j_im_i|T|j'_im_i)\sum_{k=1}^N(-)^{i+k}c_{m_i}{}^{j_i}c_{m_k}{}^{j'_i}d_{m_im_k}{}^{j'_i}(\theta)D_{i,k}{}^{N-1}(a_{\alpha\beta}),$$

$$V^{NN}=\sum_{i_1<i_2}^N\sum_{j_{i_1}j_{i_2}}\sum_{j'_{i_1}j'_{i_2}m'_{i_1}m'_{i_2}}(j_{i_1}m_{i_1},j_{i_2}m_{i_2}|v^{NN}|j'_{i_1}m'_{i_1},j'_{i_2}m'_{i_2})$$
$$\times\sum_{k_1<k_2}^N(-)^{i_1+i_2+k_1+k_2}c_{m_{i1}}{}^{j_{i1}}c_{m_{k1}}{}^{j'_{i1}}d_{m'_{i1}m_{k1}}{}^{j'_{i1}}(\theta)c_{m_{i2}}{}^{j_{i2}}c_{m_{k2}}{}^{j'_{i2}}d_{m'_{i2}m_{k2}}{}^{j'_{i2}}(\theta)D_{i_1i_2,k_1k_2}{}^{N-2}(a_{\alpha\beta}). \quad (7)$$

T and v^{NN} are the single-particle Hamiltonian and the neutron-neutron interaction, respectively; T^P and V^{PP} in Eq. (6) are given by similar expressions to T^N and V^{NN} with the replacement of N by P. The neutron-proton

[3a] This holds for integral values of K; for half integral values of K,

$$\psi_K=(\phi_K+iT\phi_K)/\sqrt{2}.$$

[4] E. P. Wigner, *Group Theory* (Academic Press Inc., New York, 1959).

interaction term V^{PN} is given by

$$V^{PN}=\sum_{p=1}^{P}\sum_{n=1}^{N}\sum_{j_pj_n}\sum_{j'_pj'_nm'_pm'_n}(j_pm_p,j_nm_n|v^{PN}|j'_pm'_p,j'_nm'_n)$$

$$\times\sum_{p'=1}^{P}\sum_{n'=1}^{N}(-)^{p+n+p'+n'}c_{m_p}{}^{j_p}c_{m_{p'}}{}^{j'_p}d_{m'_pm_{p'}}{}^{j'_p}(\theta)c_{m_n}{}^{j_n}c_{m_{n'}}{}^{j'_n}d_{m'_nm_{n'}}{}^{j'_n}(\theta)D_{p,p'}{}^{P-1}(a_{\alpha\beta})D_{n,n'}{}^{N-1}(a_{\gamma\delta}). \quad (8)$$

One can convince oneself that the expectation value of the quadrupole moment in the projected state with angular momentum I is given by

$$Q_K{}^I(P)=(I+\tfrac{1}{2})\int_0^\pi\int_0^\pi d_{KK}{}^I(\theta)d_{KK}{}^I(\theta')\sum_{M=-2}^{2}d_{0M}{}^2(\theta')Q_{2M}{}^P(\theta-\theta')\sin\theta d\theta\sin\theta' d\theta'$$

$$\Big/\int_0^\pi d_{KK}{}^I(\theta)\langle\phi_K|\exp(-i\theta J_y)|\phi_K\rangle\sin\theta d\theta.$$

It can also be written as

$$Q_M{}^I(P)=(I2M0|IM)\sum_\mu(I2K-\mu\,\mu|IK)\int_0^\pi d_{K-\mu,K}{}^I(\theta)Q_{2\mu}{}^P(\theta)\sin\theta d\theta/p_K{}^I,$$

where

$$Q_{2\mu}{}^P(\theta)=\sum_{i=1}^{P}\sum_{j_ij'_i}(j_im_i|(16\pi/5)^{1/2}e_pr_i{}^2y_{2\mu}(i)|j_i'm_i-\mu)\sum_{k=1}^{P}(-)^{i+k}c_{m_i}{}^{j_i}c_{m_k}{}^{j'_i}d_{m_i-\mu,m_k}{}^{j'_i}(\theta)D_{i,k}{}^{P-1}(a_{\alpha\beta}), \quad (9)$$

and $(J_1J_2M_1M_2|JM)$ stands for the Clebsch-Gordan coefficient. The definition of the quantities appearing in Eqs. (5)–(9) is as follows:

$$a_{\alpha\beta}=\sum_{j_\alpha}c_{m_\alpha}{}^{j_\alpha}c_{m_\beta}{}^{j_\alpha}d_{m_\alpha m_\beta}{}^{j_\alpha}(\theta).$$

$D_{i_1i_2\cdots i_p,k_1k_2\cdots k_p}{}^{N-p}(a_{\alpha\beta})$ denotes the determinant of rank $N-p$ whose elements are $a_{\alpha\beta}$, where α takes all the values from 1 to N excluding i_1, i_2, $\cdots i_p$, and β takes all the values from 1 to N excluding $k_1,k_2,\cdots k_p$.

Let

$$h_K(\theta)=\langle\psi_K|H\exp(-i\theta J_y)|\psi_K\rangle$$

and

$$P_K(\theta)=\langle\psi_K|\exp(-i\theta J_y)|\psi_K\rangle.$$

For $K=0$ band, ψ_K should be replaced by ϕ_0 in the above equations. We then have

$$h_K(\pi-\theta)=\langle\psi_K|H\exp(-i\theta J_y)|\exp(-i\pi J_y\psi_K\rangle$$
$$=\langle\psi_K|H\exp(-i\theta J_y)|T\psi_K\rangle=h_K(-\theta). \quad (10)$$

In arriving at Eq. (10), we have used $T\psi_K=\psi_K$. Similarly one can see that $P_K(\pi-\theta)=P_K(-\theta)$. Expanding the exponential in $h_K(\theta)$, we see that

$$h_K(\theta)=\sum_{n=0}^{\infty}\langle\psi_K|H(-iJ_y)^n|\psi_K\rangle\theta^n/n!$$
$$=\sum_n[1+(-)^n]/2$$
$$\times\langle\psi_K|H(-iJ_y)^n|\psi_K\rangle\theta^n/n!=h_K(-\theta). \quad (11)$$

In deriving Eq. (11), we have used the facts that H is diagonal in azimuthal quantum number K and that $\langle\phi_K|H(-iJ_y)^n|T\phi_K\rangle$ is real. The latter statements holds for the Condon-Shortley[5] phase convention. Combining Eqs. (10) and (11) we have proved that

$$h_K(\pi-\theta)=h_K(-\theta)=h_K(\theta), \quad (12)$$

and similarly

$$P_K(\pi-\theta)=P_K(-\theta)=P_K(\theta). \quad (13)$$

It is clear from Eqs. (12) and (13) that $dh_K/d\theta$ and $dP_K/d\theta$ are zero at $\theta=\frac{1}{2}\pi$. Furthermore,

$$[dh_K/d\theta]_{\theta=0}=\langle\psi_K|H(-iJ_y)|\psi_K\rangle=0,$$
$$[dP_K/d\theta)_{\theta=0}=\langle\psi_K|(-iJ_y)|\psi_K\rangle=0, \quad (14)$$

and

$$[dh_K/d\theta]_{\theta=\pi}=\langle\psi_K|H(-iJ_y)|T\psi_K\rangle$$
$$=\langle\psi_K|H(-iJ_y)|\psi_K\rangle=0,$$
$$[dP_K/d\theta]_{\theta=\pi}=\langle\psi_K|(-iJ_y)|T\psi_K\rangle$$
$$=\langle\psi_K|(-iJ_y)|\psi_K\rangle=0. \quad (15)$$

In obtaining Eqs. (14) and (15), we have used the same arguments as those used in deriving Eq. (11). Collecting together the results on the slopes of h_K and P_K and the fact that for $\theta\neq 0$, $\pi/2$, and π these will not vanish in general, we have proved that $dh_K/d\theta$ and $dP_K/d\theta$ will vanish only at $\theta=0$, $\frac{1}{2}\pi$, and π, i.e.,

$$[dh_K/d\theta]_{\theta=0,\,\pi/2,\,\pi}=0\,;\quad[dP_K/d\theta]_{\theta=0,\,\pi/2,\,\pi}=0. \quad (16)$$

In order to study the behavior of $h_K(\theta)$ and $P_K(\theta)$, let us expand these in the neighborhood of $\theta=0$. From arguments similar to those used in deriving Eq. (11), it follows that the linear term in θ vanishes. Thus for

[5] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1935).

$\theta \approx 0$ we have,

$$h_K(\theta) \approx E_K{}^{\mathrm{HF}} - \langle iJ_y\psi_K | H | iJ_y\psi_K\rangle \tfrac{1}{2}\theta^2,$$
$$P_K(\theta) \approx 1 - \langle iJ_y\psi_K | iJ_y\psi_K\rangle \tfrac{1}{2}\theta^2. \quad (17)$$

It follows from Eq. (15) that $P_K(\theta)$ is a decreasing function of θ while $h_K(\theta)$ is an increasing function of θ in the region $(0, \frac{1}{2}\pi)$. In arriving at this conclusion, we have used the fact that the expectation value of H is negative and that the slopes of $h_K(\theta)$ and $P_K(\theta)$, in general, cannot vanish except at $\theta = 0$, $\frac{1}{2}\pi$, and π [proved in Eq. (16)]. A sketch of $h_K(\theta)$ and $P_K(\theta)$ as given by Eqs. (12), (13), (16), and (17) is shown in Fig. 1.

The number of zeros of $d_{KK}{}^I(\theta)$ increases as I increases, and hence $h_K{}^I \langle h_K{}^{I'}$ and $p_K{}^I \rangle p_K{}^{I'}$ for $I < I'$ [see Eq. (4)]. However, this does not help us to say anything about $E_K{}^I$.

From Eq. (4) we have

$$E_K{}^I = h_K{}^I / p_K{}^I = \int_0^\pi d_{KK}{}^I(\theta) h_K(\theta) \sin\theta d\theta \Big/ \int_0^\pi d_{KK}{}^I(\theta) P_K(\theta) \sin\theta d\theta, \quad (18)$$

where $h_K(\theta)$ and $P_K(\theta)$ are the same as defined earlier with ψ_K replaced by ϕ_K. Changing the range of integration from $(0,\pi)$ to $(0,\frac{1}{2}\pi)$ and using Eq. (12) we have

$$h_K{}^I = \int_0^{\pi/2} [d_{KK}{}^I(\theta) + d_{KK}{}^I(\pi-\theta)] h_K(\theta) \sin\theta d\theta. \quad (19)$$

Using the relation

$$d_{KK}{}^I(\pi-\theta) = (-)^{I-K} d_{K,-K}{}^I(\theta),$$

one sees that $h_K{}^I = 0$ for odd I in the case of a $K=0$ band and $h_K{}^I \neq 0$ for $K \neq 0$. A similar result holds for $p_K{}^I$. This shows that the $E_K{}^I$ are not defined for odd I in the case of a $K=0$ band, so that these members can be treated as nonexistent. Thus, one has the projected nuclear states with $I = 0, 2, 4, \cdots I_{\max}$ for a $K=0$ band, and $I = K, K+1, \cdots I_{\max}$ for a $K \neq 0$ band.

3. IF $E_K{}^{I=K} < E_K{}^{\mathrm{HF}}$ $(E_K{}^{I=K} > E_K{}^{\mathrm{HF}})$ THEN $E_K{}^I < E_K{}^{I'}$ $(E_K{}^I > E_K{}^{I'})$ FOR $I < I'$

An alternative definition of the projection operator P_{I_1} which projects out the state with total angular momentum $I = I_1$ is

$$P_{I_1} = \prod_{i \neq 1} [\mathbf{J}^2 - I_i(I_i+1)] \Big/ \prod_{i \neq 1} [I_1(I_1+1) - I_i(I_i+1)], \quad (20)$$

where $\mathbf{J}$ is the total angular momentum operator and I_i are all possible eigenvalues of $\mathbf{J}$ in the state to be projected. With this definition one can easily obtain

$$E_K{}^{I_1} - E_K{}^{I_2} = [I_1(I_1+1) - I_2(I_2+1)] N_K / D_K, \quad (21)$$

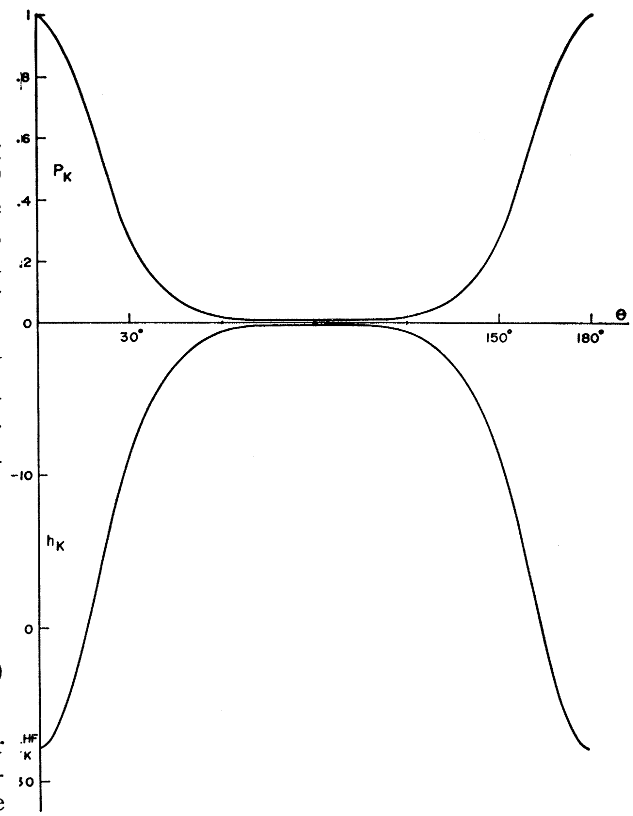

FIG. 1. Sketch of $h_K(\theta)$ and $P_K(\theta)$ as a function of θ.

where

$$N_K = \langle H\alpha\rangle\langle J^2\alpha\rangle - \langle HJ^2\alpha\rangle\langle\alpha\rangle,$$
$$D_K = [\langle J^2\alpha\rangle - I_1(I_1+1)\langle\alpha\rangle][\langle J^2\alpha\rangle - I_2(I_2+1)\langle\alpha\rangle],$$
$$\alpha = \prod_{i \neq 1,2} [\mathbf{J}^2 - I_i(I_i+1)],$$

and $\langle\ \rangle$ denotes the expectation value in the HF state $\phi_K{}^{\mathrm{HF}}$. One can express N_K in the form

$$N_K = \sum_i{}' [\langle H\alpha\rangle\langle\phi_K{}^{\mathrm{HF}}|J^2|\phi_K{}^i\rangle\langle\phi_K{}^i|\alpha|\phi_K{}^{\mathrm{HF}}\rangle - \langle\phi_K{}^{\mathrm{HF}}|H\alpha|\phi_K{}^i\rangle\langle\phi_K{}^i|J^2|\phi_K{}^{\mathrm{HF}}\rangle\langle\alpha\rangle]$$
$$= \sum_i{}' [\langle H\alpha\rangle\langle\phi_K{}^i|\alpha|\phi_K{}^{\mathrm{HF}}\rangle - \langle\phi_K{}^{\mathrm{HF}}|H\alpha|\phi_K{}^i\rangle\langle\alpha\rangle]\langle\phi_K{}^{\mathrm{HF}}|J^2|\phi_K{}^i\rangle. \quad (22)$$

The prime on the summation in Eq. (22) denotes that in the complete set of HF states $\phi_K{}^i$, the ground state $\phi_K{}^{\mathrm{HF}}$ is to be excluded. We first observe that not all the off-diagonal elements $\langle\phi_K{}^i|\alpha|\phi_K{}^{\mathrm{HF}}\rangle$, $\langle\phi_K{}^{\mathrm{HF}}|H\alpha|\phi_K{}^i\rangle$, and $\langle\phi_K{}^{\mathrm{HF}}|J^2|\phi_K{}^i\rangle$ can be zero since, if they were zero, $\phi_K{}^{\mathrm{HF}}$ would be a good angular momentum state and an eigenstate of H. Moreover, not all $\langle H\alpha\rangle\langle\phi_K{}^i|\alpha|\phi_K{}^{\mathrm{HF}}\rangle$ will be equal to $\langle\alpha\rangle\langle\phi_K{}^{\mathrm{HF}}|H\alpha|\phi_K{}^i\rangle$. It is thus clear that N_K in Eq. (21) will never vanish, and therefore, that $E_{I_1} = E_{I_2}$ will not vanish for $I_1 \neq I_2$; thus $E_K{}^I$ is either an increasing or a decreasing function of I. It is worth pointing out that if the expression N_K/D_K remains nearly constant and is positive, one will obtain a spectrum similar to the rotational spectrum with the moment of inertia $\mathcal{J}=D_K/2N_K$.

Using the property

$$\sum_I p_K{}^I=1\,,$$

one can easily obtain the following relation from Eq. (4):

$$\sum_I (E_K{}^I-E_K{}^{\rm HF})p_K{}^I=0\,, \tag{23}$$

where $p_K{}^I=\langle\phi_K{}^{\rm HF}|P_K{}^I|\phi_K{}^{\rm HF}\rangle$. From Eq. (23) we have,

$$\sum_{I\neq K} (E_K{}^I-E_K{}^{\rm HF})p_K{}^I=(E_K{}^{\rm HF}-E_K{}^K)p_K{}^K.$$

If $E_K{}^K<E_K{}^{\rm HF}(E_K{}^K>E_K{}^{\rm HF})$ one obtains

$$\sum_{I\neq K} (E_K{}^I-E_K{}^{\rm HF})p_K{}^I>0 \quad \Big(\sum_{I\neq K} (E_K{}^I-E_K{}^{\rm HF})p_K{}^I<0\Big)\,, \tag{24}$$

since from the definition of $P_K{}^I$, one has $P_K{}^I>0$. From Eq. (24) it follows that there must exist at least one $I\neq K$ state such that

$$E_K{}^I>E_K{}^{\rm HF} \quad (E_K{}^I<E_K{}^{\rm HF}).$$

However, we have proved that $E_K{}^I$ is either an increasing or a decreasing function of I. Combining these facts, one obtains the result that if $E_K{}^K<E_K{}^{\rm HF}$ $(E_K{}^K>E_K{}^{\rm HF})$ then $E_K{}^I<E_K{}^{I'}$ $(E_K{}^I>E_K{}^{I'})$ for $I<I'$.

Let us now prove the converse of the above result, namely, if $E_K{}^I<E_K{}^{I'}$ $(E_K{}^I>E_K{}^{I'})$ for $I<I'$, then

$$E_K{}^K<E_K{}^{\rm HF}\ (E_K{}^K>E_K{}^{\rm HF}).$$

Because $E_K{}^I<E_K{}^{I'}$ $(E_K{}^I>E_K{}^{I'})$ for $I<I'$, we have from Eq. (23)

$$0=\sum_I (E_K{}^I-E_K{}^{\rm HF})p_K{}^I >\sum_I (E_K{}^K-E_K{}^{\rm HF})p_K{}^I>E_K{}^K-E_K{}^{\rm HF}\,,$$

$$0=\sum_I (E_K{}^I-E_K{}^{\rm HF})p_K{}^I <\sum_I (E_K{}^K-E_K{}^{\rm HF})p_K{}^I<(E_K{}^K-E_K{}^{\rm HF}). \tag{25}$$

Q.E.D.

4. JUSTIFICATION FOR THE USE OF THE HF WAVE FUNCTION

What we have proved so far could also be proved for any deformed state with good K. Here we explain why one should take the deformed HF state for projection. Let ϕ_K be the HF state and ϕ'_K be any other determinantal state having the same symmetries and class of variation (the single-particle basis) as ϕ_K has. Then from the variational principle, the corresponding energies satisfy

$$\langle\phi_K|H|\phi_K\rangle=E_K\langle E'_K=\langle\phi'_K|H|\phi'_K\rangle. \tag{26}$$

Let $E_K{}^K$ and $E'_K{}^K$ be the corresponding energies of the lowest $I=K$ states projected from ϕ_K and ϕ'_K, respectively. From Eq. (4),

$$E_K{}^K=h_K{}^K/p_K{}^K \quad\text{and}\quad E'_K{}^K=h'_K{}^K/p'_K{}^K.$$

Using arguments similar to those used in deriving Eq. (11), we have

$$h_K(\theta)-h'_K(\theta)=\langle\phi_K+\phi'_K|He^{-i\theta J_y}|\phi_K-\phi'_K\rangle.$$

The matrix element on the right-hand side, in general, will not vanish, and hence $h_K(\theta)-h'_K(\theta)\neq 0$. Combining this with the fact that

$$h_K(0)=E_K<E'_K=h'_K(0)\,,$$

we obtain the result $h_K(\theta)<h'_K(\theta)$. Integrating both sides over the region $(0,\pi)$, we get $h_K{}^K<h'_K{}^K$. Then, since $P_K(0)=P'_K(0)=1$ in both the states and most of the contributions to $p_K{}^K$ and $p'_K{}^K$ come from the region near $\theta=0$, we have

$$p_K{}^K-p'_K{}^K=2\int_0^{\pi/2} d_{KK}{}^K(\theta) \times\langle\phi_K+\phi'_K|\exp-i\theta J_y|\phi_K-\phi'_K\rangle\sin\theta d\theta\approx 0\,.$$

If we take $p_K{}^K\approx p'_K{}^K$ approximately, then $E_K{}^K<E'_K{}^K$.

In order to compare the theoretical spectrum with the low-lying excited states of a nucleus, one must use that state for projection which gives the lowest energy for the ground state with $I=K$. Hence, in view of the result $E_K{}^K<E'_K{}^K$, one should use the HF state for projection rather than any other state having the same symmetry and class of variation.

5. CONCLUSIONS

It is found that the low-lying excited states of nuclei calculated by projecting the good angular momentum states from the deformed HF wave function have the possible spins $I=0, 2, 4, \cdots, I_{\max}$ for a $K=0$ band and $I=K,K+1,\cdots,I_{\max}$ for a $K\neq 0$ band. It is clear from the expressions in Sec. 2 that, in the evaluation of the projected $I=K$ state energy (relative to the HF energy $E_K{}^{\rm HF}$), the contribution of the single-particle part of the Hamiltonian is positive, while that of the two-body interaction part is negative. The sum of the two may be positive or negative, depending on the single-particle input spectrum and the strength of the two-body interaction. Depending on whether $E_K{}^K<E_K{}^{\rm HF}$ or $E_K{}^K>E_K{}^{\rm HF}$, we proved that the projected spectrum would manifest $E_K{}^I<E_K{}^{I'}$ or $E_K{}^I>E_K{}^{I'}$, respectively, for $I<I'$. From this, one observes that in order to obtain the realistic spectrum $E_K{}^I<E_K{}^{I'}$ for $I<I'$, one must have the single-particle input spectrum and the interaction such that $E_K{}^K<E_K{}^{\rm HF}$.

The spectrum whose ground state is lower than that of any other spectrum would be closer to the physical situation. This suggests that if the HF state gives the experimental spectrum quite correctly then $E_K{}^K < E'_K{}^K$, where $E'_K{}^K$ is the energy of the $I=K$ state projected from any other state ϕ'_K having the same symmetries and the single-particle basis as the HF state ϕ_K. In the last section we have shown, in a certain approximation, that $E_K{}^K < E'_K{}^K$, explaining thereby the importance of the HF state in obtaining the low-lying excited states of nuclei by the projection technique.

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Electric Quadrupole Transitions near $A=16$: the Lifetimes of the N^{16} 0.120-, F^{18} 1.125-, F^{19} 0.197-, and Ne^{19} 0.241-MeV Levels*

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Continuing a program to study $E2$ lifetimes in the neighborhood of $A=16$, we have remeasured the mean lifetimes of the following states: the 0.197-MeV level of F^{19}; the 0.241-MeV level of Ne^{19}; the 0.120-MeV level of N^{16}; and the 1.125-MeV level of F^{18}. We find $(129.9\pm2.3)\times10^{-9}$sec, $(26.6\pm1.2)\times10^{-9}$ sec, $(7.58\pm0.09)\times10^{-6}$ sec, and $(221\pm21)\times10^{-9}$ sec, respectively.

INTRODUCTION

As has been pointed out on numerous occasions, the independent particle model (IPM) characterized by the calculations of Kurath and others[1,2] describes quite well level schemes in the $1p$ and $(2s, 1d)$ shells; and new calculations[3,4] indicate even more success. However, the success in describing the level schemes has not extended itself to the description of electromagnetic transitions between these states, most particularly $E2$ transitions, although the most recent calculations[3,4] seem to indicate a more substantive agreement[5,6] with theory. Perhaps the IPM wave functions may be repaired to include collective effects of admixtures of higher states which will explain these strong $E2$ transitions.

The region near $A=16$ represents a fertile testing ground as the number of additional particles or holes past the filled $1p$ shell is small. Particularly, we have remeasured[7] the lifetimes of the first excited states of O^{17} and F^{17} since, representing as they do $2s\rightarrow1d$ jumps, they are most relevant to the problem of $E2$ enhancement. We have undertaken further remeasurements[8] on $E2$ transitions near $A=16$: the lifetimes of the first excited states of Ne^{19} (0.241-MeV) and N^{16} (0.120-MeV), the second excited state of F^{19} (0.197-MeV), and also the 1.125-MeV level of F^{18}.

Previous measurements of these mean lifetimes are

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