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Quasi-Elastic Scattering near Zero Recoil Momentum

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Finite resolution in the detectors in a coplanar symmetric $(p,2p)$ experiment leads to out-of-plane scattering which is dominant over in-plane scattering when the cross section has a minimum. The amount of filling in that this causes is calculated and compared with experiment.

1. INTRODUCTION

It is well known that the differential cross section in the high-energy symmetric coplanar $(p,2p)$ reaction for the case in which the target proton has angular momentum $l\neq0$ has a minimum at the scattering angle corresponding to zero linear momentum $\mathbf{P}$ of the target proton in the laboratory system. The momentum of the recoiling core, which is the more commonly used quantity, is of course $\mathbf{Q}=-\mathbf{P}$. Theoretical predictions of this minimum,[1,2] using the distorted-wave impulse approximation, have invariably made this minimum too deep, compared with experiment, unless distorting potentials of quite unreasonable size were employed. An attempt to obtain the filling in of the minimum by folding in a Gaussian factor to simulate finite resolution in angle and energy was also unsuccessful.[2] All analyses so far, however, have been made on the assumption that in a symmetric coplanar experiment the vector $\mathbf{Q}$ must lie along the line of the momentum of the incident proton. This is, of course, strictly correct for perfect angular and energy resolution, and it is approximately correct even at finite experimental resolution, provided that Q is not small. The situation is quite different for small Q, when the direction of $\mathbf{Q}$ may depart substantially from this line until, for very small Q (depending on the experimental resolution), all directions of $\mathbf{Q}$ will contribute to the reaction. This effect enhances the differential cross section near the minimum, while it clearly has little influence elsewhere.[3]

2. FORMALISM

We shall perform the analysis in terms of the motion of the center of mass of the two out going protons, and it will therefore be convenient to use the "di-proton" formalism of the previous paper.[4] In the center-of-mass system of incident particle and target nucleus, let the momenta of the incident proton, the target proton, the two scattered protons, and the di-proton (i.e., the center of mass of the two scattered protons) be $\mathbf{k}_0$, $\mathbf{K}$, $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_B$, respectively. Let the corresponding momenta in the laboratory system be $\mathbf{p}_0$, $\mathbf{P}$, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_B$. Then[1]

$$\mathbf{k}_0=(1-c)\mathbf{p}_0,\quad \mathbf{k}_1=\mathbf{p}_1-c\mathbf{p}_0,\quad \mathbf{k}_2=\mathbf{p}_2-c\mathbf{p}_0,\quad \mathbf{k}_B=\mathbf{p}_B-2c\mathbf{p}_0,\quad \mathbf{K}=\mathbf{P}-c\mathbf{p}_0,\tag{1}$$

where $c=1/(A+1)$. The scattering angles are defined by

$$\theta_1=\cos^{-1}(\hat{k}_0\cdot\hat{k}_1),\quad \theta_2=\cos^{-1}(\hat{k}_0\cdot\hat{k}_2),\tag{2}$$

while the direction of $\mathbf{k}_B$ is given by the polar angles

$$\omega=\cos^{-1}(\hat{k}_0\cdot\hat{k}_B),\quad \varphi=\text{azimuthal angle about }\hat{k}_0.\tag{3}$$

We take the z axis along $\hat{k}_0$ and define the y-z plane as that in which the coplanar symmetric experiment takes place, i.e., for which $\theta_1=\theta_2=\theta$, say, $\omega=0$.

In all the experiments,[2,5–7] the energy resolution has been substantially better than the angle resolution, and in this analysis we shall assume it to be exact, i.e., we shall take the values of $\mathbf{p}_0$, $\mathbf{p}_1$, and $\mathbf{p}_2$ to be fixed. The finite angular resolutions will, however, lead to variations in the magnitude of $|\mathbf{p}_1+\mathbf{p}_2|$, i.e., of $\mathbf{p}_B$. Departures from the mean are as likely to be positive as negative, and, since we shall eventually perform an average, the effect of the finite angular resolution on the variation in magnitude of p_B only enters in second order. It will therefore be neglected, so that p_B may also be assumed fixed. Hence we shall be concerned entirely with contributions to the cross section at fixed energies from angles $\omega\neq0$. For the symmetric noncoplanar experiment, the maximum value of this angle is given by half the vertical opening angle of the detectors, while for nonsymmetric scattering, owing to the finite horizontal opening angle of the detectors, the vector $\mathbf{p}_B$ is confined within an ellipsoidal cone, with axis along $\mathbf{p}_0$ defined by the detector opening angles. We shall replace this by a circular cone of half-angle $\bar{\omega}$, which corresponds to the same solid angle as the quoted values of the solid angle Ω subtended by the detectors:

$$2\pi(1-\cos\bar{\omega})=\Omega,\quad \text{i.e.,}\quad \bar{\omega}\simeq(\Omega/\pi)^{1/2}.\tag{4}$$

Using (1) and (3), it is easily shown that, to first order in c, the out-of-plane angle in the laboratory system ω_L

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[1] D. F. Jackson and T. Berggren, Nucl. Phys. **62**, 353 (1965).
[2] H. Tyrén *et al.*, Nucl. Phys. **79**, 321 (1966).
[3] The existence of this effect was pointed out to the authors by Professor B. Gottschalk.
[4] D. F. Jackson, preceding paper, Phys. Rev. **155**, 1065 (1967). This paper is referred to as I.
[5] J. P. Garron *et al.*, Nucl. Phys. **37**, 126 (1962).
[6] G. Tibell *et al.*, Arkiv Fysik **25**, 433 (1963).
[7] J. C. Roynette, thesis, University of Paris, 1966 (unpublished).

is given by

$$\cos\omega=\cos\omega_L-2c(p_0/p_B)(1-\cos\omega_L)\,, \tag{5}$$

so that ω differs from ω_L only in second-order terms, which are negligible.

From conservation of momentum, we have

$$\mathbf{p}_0+\mathbf{P}=\mathbf{p}_B\,,$$

so that in terms of $\mathbf{Q}=-\mathbf{P}$, we have, for the symmetric case,

$$Q^2=p_0^2+p_B^2-2p_0p_B\cos\omega\,,$$
$$Q_x=-p_B\sin\omega\,,\quad Q_z=p_B\cos\omega-p_0\,.$$

To first order in ω, we then have

$$Q=|p_0-p_B|\,,\quad Q_x=-p_B\omega\,,\quad Q_z=p_B-p_0\,.$$

To examine the effect of the Q_x component on the differential cross section [I. Eq. (1)],

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE}\propto\sum_m |g_l{}^m|^2\,, \tag{6}$$

we note that in plane-wave approximation with oscillator wave functions (I, Sec. 2 A) we have

$$g_1{}^0\propto b^{5/2}Q_z e^{-\frac{1}{2}Q^2b^2}\,,\quad g_1{}^{\pm1}\propto b^{5/2}Q_x e^{-\frac{1}{2}Q^2b^2}\,, \tag{7}$$

where b is the oscillator-length parameter. We therefore expect $g_1{}^{\pm1}$ to increase parabolically with ω, while $g_1{}^0$ remains constant, for small ω. To relate the out-of-plane effect to the distorted-wave effect, which also leads to contributions from the transverse momentum component Q_x, we write $g_l{}^m$ in the distorted-wave formalism (I Sec. 2 B),

$$g_l{}^m=\int\chi_B{}^{-*}(\mathbf{k}_B,\mathbf{r})\psi_l{}^m(\mathbf{r})\chi_p{}^+(\mathbf{k}_0,a\mathbf{r})d\mathbf{r}\,, \tag{8}$$

where $a=(A-1)/A$, and we use the WKB-approximation expressions for the distorted waves[8,9]:

$$\chi_B{}^{-*}(\mathbf{k}_B,\mathbf{r})=e^{-i\mathbf{k}_B\cdot\mathbf{r}}D_B(\mathbf{k}_B,\mathbf{r})\,, \tag{9}$$

$$\chi_p{}^+(\mathbf{k}_0,a\mathbf{r})=e^{i\mathbf{k}_0\cdot a\mathbf{r}}D_p(\mathbf{k}_0,a\mathbf{r})\,, \tag{10}$$

$$D_B(\mathbf{r})=\exp\left\{-\frac{i}{\hbar v_B}\int_0^\infty V_B(\mathbf{r}-\hat{k}_Bs)ds\right\}\,, \tag{11}$$

$$D_p(a\mathbf{r})=\exp\left\{-\frac{i}{\hbar v_p}\int_0^\infty V_p[a(\mathbf{r}+\hat{k}_0s)]ds\right\}\,. \tag{12}$$

Here V_p and V_B are the complex optical potentials for the incident proton and the scattered diproton, and v_p and v_B are the corresponding velocities. For forward scattering, which we have here, we can integrate along the z axis, i.e., replace $\hat{k}_B$ by $\hat{k}_0$ in $D_B(\mathbf{k}_B,\mathbf{r})$, so that the product of the two distorted waves then leads to the sum of two integrals

$$S(x,y,z)=-\frac{1}{\hbar v_p}\int_{-\infty}^{z}V_p(a\rho,az')dz' -\frac{1}{\hbar v_B}\int_{z}^{\infty}V_B(\rho,z')dz'\,, \tag{13}$$

where $\rho^2=x^2+y^2$. For an oscillator wave function $\psi_l{}^m(\mathbf{r})$ this yields

$$g_1{}^0\propto\int z e^{i(Q_xx+Q_zz)}e^{iS(x,y,z)}e^{-Q^2/2b^2}d\mathbf{r}\,, \tag{14}$$

$$g_1{}^{\pm1}\propto\int(x+iy)e^{i(Q_xx+Q_zz)}e^{iS(x,y,z)}e^{-Q^2/2b^2}d\mathbf{r}\,. \tag{15}$$

This demonstrates clearly that the transverse momentum Q_x has an effect similar to that of a distortion of the incident and scattered waves. It may be noted that the further approximation,[10] by which V_p and V_B are replaced by an average potential and v_p and v_B by an average velocity so that the z integration in (13) can be taken from $-\infty$ to $+\infty$, leads to a phase S which is independent of z. In this approximation $g_1{}^0=0$ for $Q_z=0$, in contrast to either the more accurate WKB calculation or the exact partial-wave analysis performed in the next section. Conclusions based on this further approximation, e.g., those relating to localization, are therefore of doubtful validity.

3. NUMERICAL RESULTS

The evaluation of $g_1{}^m$ near $Q=0$ was actually performed by means of the partial-wave expansion [I Eq. (7)], and results were obtained for Li^7 and C^{12} at 150 MeV. The oscillator parameters, $b=1.72$ F for Li^7 and $b=1.64$ F for C^{12}, have been obtained from

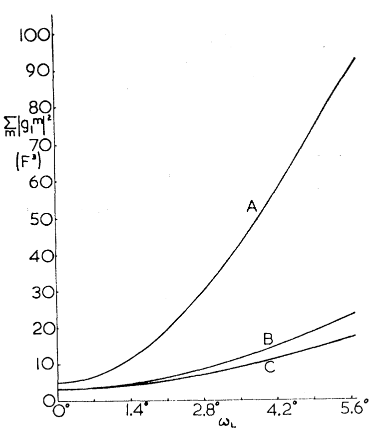

FIG. 1. Out-of-plane scattering in the $(p,2p)$ reaction at 150 MeV. The curves correspond to A: Li^7, $b=2.24$ F; B: Li^7, $b=1.72$ F; C: C^{12}, $b=1.64$ F. The laboratory scattering angle θ_L has been chosen so that $Q=0$, and is 42.6° for Li^7, 41.5° for C^{12}.

[8] D. F. Jackson, Nucl. Phys. **35**, 194 (1962).

[9] T. Berggren and G. Jacob, Nucl. Phys. **47**, 481 (1963).

[10] P. A. Benioff, Phys. Rev. **129**, 1355 (1963).

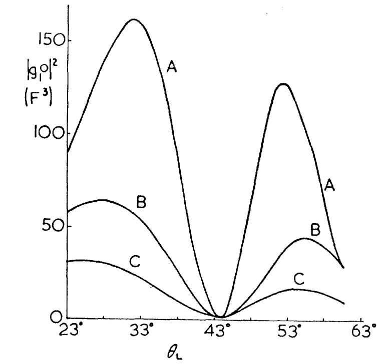

FIG. 2. Coplanar symmetric scattering in the $(p,2p)$ reaction at 150 MeV. For A, B, and C see the caption of Fig. 1.

elastic electron scattering,[11,12] but for Li^7 we have also used $b=2.24$ F, which gives a better fit to inelastic proton scattering.[13] Absolute cross sections can be obtained by multiplying the curves by the appropriate kinematical factors and the free cross section [I Eq. (1)], and by taking the spectroscopic factor to

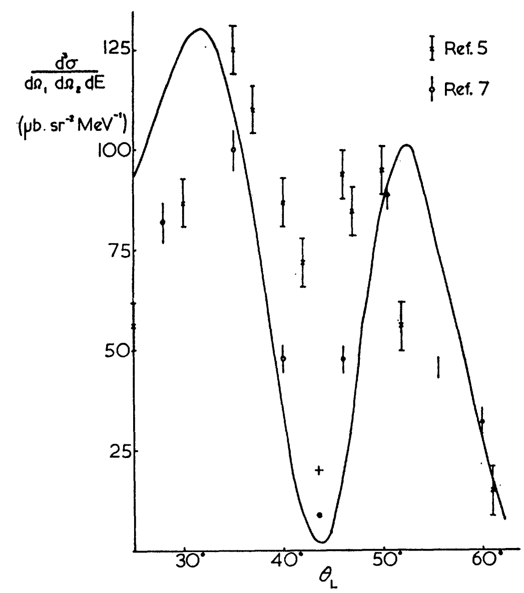

FIG. 3. The $Li^7(p,2p)$ reaction at 150 MeV. The full curve is for the coplanar symmetric case, i.e., for $g_1^{\pm 1}=0$, while the two points at 43° marked + and ● indicate the filling is due to out-of-plane scattering for the experiments of Refs. 5 and 7, respectively.

TABLE I. Peak-to-valley ratio (for larger peak) for $Li^7(p,2p)$ at 150 MeV. ($\bar{\omega}=0$ corresponds to no out-of-plane scattering.)

	Ref. 5	Ref. 7	
Ω (msr)	10.0	2.1	
$\bar{\omega}$	3.2°	1.5°	0°
Peak/valley: expt.	$\sim$2	$\sim$3	...
$b=1.72$ F	9	16	47
$b=2.24$ F	8	17	80

be the extreme jj-coupling value which introduces a factor of $N_l/(2l+1)$, where N_l is the number of protons in the shell.[9] The product of these quantities varies rather slowly with angle so that the cross section in μb sr^{-2} MeV^{-1} can be estimated from our figures by multiplying the Li^7 curves by 0.8 and the C^{12} curves by 3.2. Results are shown in Fig. 1. It will be noted that $g_1^{\pm 1}$ rises much more rapidly for larger b, i.e., for wave functions with a long range. This is in line with the plane-wave result (7), according to which $g_1^{\pm 1}$ has a maximum for $bQ_x=2.5$. Clearly, the larger b is, the smaller Q_x is and hence the value of ω for which this maximum is reached. It is also clear that the rise is approximately parabolic, as predicted by the plane-wave formula.

Away from $Q=0$, the $g_1^{\pm 1}$ terms do not contribute significantly to the cross section. (I, Fig. 3.) The cross sections are then proportional to $|g_1^0|^2$, which has been plotted for the coplanar case as a function of θ in Fig. 2. The filling in of the minima in these curves is obtained by adding the $|g_1^{\pm 1}|^2$ contributions near $Q=0$, averaged over ω up to $\bar{\omega}$, i.e.,

$$\text{av}|g_1^{\pm 1}|^2 = \int_0^{\bar{\omega}} |g_1^{\pm 1}|^2 \sin\omega d\omega \bigg/ \int_0^{\bar{\omega}} \sin\omega d\omega .$$

Results for Li^7 at 150 MeV are shown in Table I and Fig. 3 for $\bar{\omega}=1.5°$ and 3.2°, corresponding to the stated accuracies of the experiments.[5,7] (The later experiment actually separates out reactions leading to the ground and first excited states of He^6, but in order to maintain a comparison with the earlier experiment we have summed the two cross sections.) Although the filling in of the minimum is still inadequate it is significant and its dependence on $\bar{\omega}$ is what is found experimentally. The peak-to-valley ratio is remarkably independent of the size of the oscillator parameter when the effect of finite resolution is taken into account, but the distance between the cross-section maxima depends strongly on b and indicates a value even larger than 2.24 F.

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[11] M. Bernheim, thesis, University of Paris, 1964 (unpublished).
[12] H. F. Ehrenberg *et al.*, Phys. Rev. **113**, 666 (1959).
[13] J. Mahalanabis, Ph.D. thesis, University of London, 1964 (unpublished).