An application of this theoretical formulation enables us to express the two parameters B_c, A_c defined by Moldauer[1] in terms of a single parameter p or q. They are given by

$$B_c=\left[\frac{|\langle\theta_{\mu c}{}^2\rangle_\mu|}{\langle|\theta_{\mu c}|^2\rangle_\mu}\right]^2, \tag{20}$$

$$A_c=\frac{\langle|\theta_{\mu c}|^4\rangle_\mu}{[\langle|\theta_{\mu c}|^2\rangle_\mu]^2}. \tag{21}$$

Using Eqs. (17a) and (17b), we get for case (a)

$$B_c=16[\exp(2q)-\exp(-2q)]^2 \times[\exp(4q)-\exp(-4q)+8q]^{-2}, \tag{22a}$$

$$A_c=\tfrac{4}{3}[\exp(2q)-\exp(-2q)]^2 \times[\exp(4q)-\exp(-4q)+8q]^{-2} \times[\exp(4q)+\exp(-4q)+16]. \tag{22b}$$

For case (b), using Eq. (16), we get

$$B_c=4\exp(2p^{-1})[\exp(4p^{-1})+1]^{-2}, \tag{23a}$$

$$A_c=\exp(2p^{-1})[\exp(4p^{-1})+1]^{-2}[\exp(8p^{-1})+5]. \tag{23b}$$

It is interesting to note from Eqs. (22b) and (23b) that if we carry out the limiting process indicated earlier, then $A_c=1.5$, which is in agreement with the value of A_c which results from the exact distribution of the width in two dimensions, given by Eq. (19), for the case of real-boundary condition.

As a further application, we consider the average value of the normalization constant[1]

$$N_\mu=\int_{\text{interior}}|X_\mu|^2d\tau. \tag{24}$$

With the help of Eqs. (2), (10), and (14) we see that $\langle N_\mu\rangle_\mu$ for case (a) is given by

$$\langle N_\mu\rangle_\mu=\tfrac{1}{4}[\exp(4q)-\exp(-4q)+8q] \times[\exp(2q)-\exp(-2q)]^{-1}. \tag{25}$$

For case (b), it is given by

$$\langle N_\mu\rangle_\mu=\tfrac{1}{2}[\exp(4p^{-1})+1]\exp(-p^{-1}). \tag{26}$$

Using Eqs. (25) and (26) and the limit $p\to\infty$, $q\to 0$, we get $\langle N_\mu\rangle_\mu=1$, which checks with the result of the real-boundary condition.

An extension of this formulation to N dimensions is presented in the following paper, and detailed application to the fluctuations of cross sections will be presented in a later article.

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Statistical Description of the Complex-Boundary-Value Problem. II. Distribution of the Parameters of the Collision Matrix

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A detailed statistical study is made of the parameters of the statistical collision matrix using the N-dimensional random complex orthogonal matrix. It is shown that, even without a complete knowledge of weight function which has to be introduced for the convergence of the normalization integral, certain relations between the average values of the parameters of the statistical collision matrix can be obtained and a statement can be made that the channel correlations of the parameters are always positive. A suitable form of the weight function is guessed, and the distributions of the parameters are also given. It is shown that under certain conditions the distribution of the parameters is close to the Porter-Thomas distribution except for small values. The resonance correlations of the parameters are also studied. Excellent agreement has been obtained between the values predicted by the present theoretical formulation and those obtained by a numerical calculation using the parameters of the real-boundary-value problem and a certain transformation matrix.

I. INTRODUCTION

In an earlier paper[1] we showed that the random-matrix hypothesis can be used to study the statistical properties of the statistical collision matrix introduced by Moldauer.[2] The earlier work[1] was intended to give the basic idea and its application to a simple two-dimensional case. In the applications of the statistical collision matrix to the study of the energy averages, fluctuations, and the correlations of the nuclear collision cross sections,[2] we need the N-dimensional generalization of the simple case discussed earlier. In this paper we shall study the distribution of the parameters of the statistical collision matrix. The results will be compared with the

[1] N. Ullah, preceding paper, Phys. Rev. **154**, 897 (1967).
[2] P. A. Moldauer, Phys. Rev. **135**, B642 (1964).

numerical calculations carried out by Moldauer.[3] In these calculations[3] the parameters of the real-boundary-value problem are used, and by numerically diagonalizing a complex symmetric level matrix the elements of a certain transformation matrix are obtained. The transformation matrix connects the parameters of the statistical collision matrix with those of the real-boundary-value problem. This connection is used to get the numerical values of the parameters of the statistical collision matrix. These numerical calculations can indicate certain trends in the behavior of the parameters but cannot be used to make definite statements. The advantage of using the random complex orthogonal matrix in studying the statistical properties of the parameters of the statistical collision matrix lies in the fact that certain relations between the parameters of the collision matrix can be obtained and definite statements about the nature of correlations can be made even without a complete knowledge of the weight functions which have to be introduced to make the normalization integral converge. In Sec. II we generalize the earlier results to N dimensions and study the distribution of the parameters of the collision matrix. We also try to guess a suitable form of the weight function and use it to calculate the average values of the parameters in Sec. III. Section IV is devoted to a discussion of both the channel correlations and the resonance correlations.

II. GENERAL FORMULATION

A straightforward procedure to generalize the results to N dimensions would have been to suitably parametrize the N-dimensional complex orthogonal matrix a, as was done earlier for the case of two dimensions, and to calculate the volume element and the ensemble averages. But it turns out that the actual calculation of the ensemble averages becomes too complicated. The same difficulty arises in the case of the ensembles of real orthogonal, unitary, and symplectic matrices if the ensemble averages are calculated, using a suitable parametrization.[4] A technique has been developed to overcome this difficulty for the calculation of ensemble averages.[5] We shall use this technique to calculate the ensemble averages for the complex orthogonal case.

To illustrate the method of calculating the ensemble averages we shall first consider a single column vector of the N-dimensional random complex orthogonal matrix. Since we are considering a single column vector, we shall suppress the column index β from the component $a_{\alpha\beta}$. Let us denote the real and imaginary parts of the component a_α by $a_\alpha{}^r$ and $a_\alpha{}^i$, respectively. Then the ensemble average of some quantity $Q(\{a_\alpha{}^r,a_\alpha{}^i\})$, which is a function of a_α's is expressed as[5]

$$\langle Q\rangle = K^{-1}\int Q(\{a_\alpha{}^r,a_\alpha{}^i\})\delta\Big(\sum_{\alpha=1}^{N}(a_\alpha{}^r)^2-\sum_{\alpha=1}^{N}(a_\alpha{}^i)^2-1\Big)\times\delta\Big(\sum_{\alpha=1}^{N}a_\alpha{}^r a_\alpha{}^i\Big)\prod_{\alpha=1}^{N}da_\alpha{}^r da_\alpha{}^i,\quad(1)$$

where $\langle\ \rangle$ denotes the ensemble average and K is the normalization integral, the same integral as in Eq. (1) but without the quantity Q in it.

As was the case in two dimensions, the normalization integral K diverges. To ensure its convergence we have to introduce a weight function $\rho_N(\{a_\alpha{}^r,a_\alpha{}^i\})$. We now make the simplifying assumption that the weight function ρ is of the form $\rho(\sum_{\alpha=1}{}^N(a_\alpha{}^i)^2)$. This assumption will be justified when we calculate the ensemble averages of the physical quantities and compare them with their numerically calculated values. Introducing this weight function in Eq. (1) and making a simple transformation, we can rewrite Eq. (1) as

$$\langle Q\rangle = K^{-1}\int Q(\{(1+\lambda)^{1/2}u_\alpha,\lambda^{1/2}v_\alpha\})[\lambda(1+\lambda)]^{\frac{1}{2}(N-3)}\times\rho_N(\lambda)\delta(\sum u_\alpha{}^2-1)\delta(\sum v_\alpha{}^2-1)\times\delta(\sum u_\alpha v_\alpha)d\lambda\prod_\alpha du_\alpha dv_\alpha.\quad(2)$$

It can be shown that the earlier results for the case of two dimensions can be obtained by a proper choice of the weight function $\rho_2(\lambda)$.

The theoretical formulation which we have described enables us to obtain relations between the parameters of the collision matrix without any knowledge of the weight function $\rho_N(\lambda)$. We recall that the complex amplitude $\theta_{\mu c}$ is given by[1]

$$\theta_{\mu c}=\sum_\nu a_{\nu\mu}J_{\nu c},\quad(3)$$

where $J_{\nu c}$ is an overlap integral defined in the earlier paper.[1] We introduce the quantity $g_{\mu c}$ given by[2]

$$g_{\mu c}=\Omega_c(2P_c)^{1/2}\theta_{\mu c},\quad(4)$$

where P_c is the penetrability and Ω_c is defined in Ref. 2. The normalization constant N_μ is given by[1]

$$N_\mu=\sum_\nu|a_{\nu\mu}|^2.\quad(5)$$

The parameters B_c, A_c are defined in terms of the moments of the complex amplitude $\theta_{\mu c}$ as[2]

$$B_c=\left[\frac{|\langle\theta_{\mu c}{}^2\rangle|}{\langle|\theta_{\mu c}|^2\rangle}\right]^2,\quad(6)$$

$$A_c=\frac{\langle|\theta_{\mu c}|^4\rangle}{[\langle|\theta_{\mu c}|^2\rangle]^2}.\quad(7)$$

[3] P. A. Moldauer, Phys. Rev. **136**, B947 (1964).
[4] C. E. Porter, *Statistical Theories of Spectra, Fluctuations* (Academic Press, Inc., New York, 1965), p. 64.
[5] N. Ullah, Nucl. Phys. **58**, 65 (1964).

TABLE I. Comparison of the values of B_c, $\langle\Gamma_{\mu c}\rangle$, and $S(|g_{\mu c}|^2)$ with their numerically calculated values, assuming the values of $\langle N_\mu\rangle$, $S(N_\mu)$, and $\langle|g_{\mu c}|^2\rangle$ to be given.

No. of channels	$\langle N_\mu\rangle$	$S(N_\mu)$	$\langle\|g_{\mu c}\|^2\rangle$	Numerical calculation B_c	$\langle\Gamma_{\mu c}\rangle$	$S(\|g_{\mu c}\|^2)$	Present calculation B_c	$\langle\Gamma_{\mu c}\rangle$	$S(\|g_{\mu c}\|^2)$
20	1.18	0.01	0.144	0.53	0.097	1.67	0.72	0.122	1.63
100	1.52	0.066	0.108	0.39	0.064	1.48	0.43	0.071	1.46
300	1.69	0.07	0.081	0.33	0.047	1.40	0.35	0.048	1.39

The quantities $\Gamma_{\mu c}$, $\Theta_{\mu c}$ are defined by[2]

$$\Gamma_{\mu c}=|g_{\mu c}|^2/N_\mu, \tag{8}$$

$$\Theta_{\mu c}=(2\pi/D)N_\mu|g_{\mu c}|^2, \tag{9}$$

where D is the mean spacing.[2] Using Eqs. (2)–(6), and (8) it can be easily shown that

$$B_c=[\langle N_\mu\rangle]^{-2}, \tag{10}$$

$$\langle\Gamma_{\mu c}\rangle=\langle|g_{\mu c}|^2\rangle/\langle N_\mu\rangle. \tag{11}$$

Assuming the values of $\langle N_\mu\rangle$ and $\langle|g_{\mu c}|^2\rangle$ to ge given, we calculate the values of B_c and $\langle\Gamma_{\mu c}\rangle$, using Eqs. (10) and (11), and compare them with Moldauer's[3] numerically calculated values in Table I. An inspection of Table I shows that the agreement is quite good. Other relations of this type are

$$A_c=N(N+2)^{-1}[1+2\langle N_\mu^2\rangle][\langle N_\mu\rangle]^{-2}, \tag{12}$$

$$\langle\Theta_{\mu c}\rangle=(2\pi/D)\langle\Gamma_{\mu c}\rangle\langle N_\mu^2\rangle. \tag{13}$$

We can also calculate the normalized mean square deviations defined by

$$S(x_\mu)=(\langle x_\mu^2\rangle-\langle x_\mu\rangle^2)/\langle x_\mu\rangle^2. \tag{14}$$

A simple calculation will show that

$$S(|g_{\mu c}|^2)=(N+2)^{-1}[N-2+2NS(N_\mu)+N(\langle N_\mu\rangle)^{-2}]. \tag{15}$$

Again assuming the value of $S(N_\mu)$ to be given, we calculate the value of $S(|g_{\mu c}|^2)$. Table I shows that it is in excellent agreement with its numerically calculated value. The calculation of the other mean square deviations $S(\Gamma_{\mu c})$, $S(\theta_{\mu c})$ will involve a knowledge of the weight function and will be taken up later.

As a check on our calculation we note that for the real-boundary-value problem, Eq. (12) gives

$$A_c=3N/(N+2), \tag{16}$$

which for large values of N becomes 3, in agreement with the value obtained with the Porter-Thomas distribution of the partial width.[2]

We shall next consider the distribution of the parameters. We shall work out in detail the distribution of $|\theta_{\mu c}|^2$. The distribution of the other parameters can be worked out in similar fashion but will not be given here.

Using Eq. (3), we get

$$|\theta_{\mu c}|^2=(\sum_\alpha a_\alpha^r J_{\alpha c})^2+(\sum_\alpha a_\alpha^i J_{\alpha c})^2. \tag{17}$$

It will be convenient to define a quantity

$$x=|\theta_{\mu c}|^2/J_c, \tag{18}$$

where

$$J_c=\frac{1}{N}\sum_\alpha J_{\alpha c}^2,$$

and find its distribution.

Using Eqs. (1), (2), (17), and (18), we can write the distribution of the quantity x as

$$P(x)dx=L^{-1}\bigg\{\int\delta[(1+\lambda)(\sum_\alpha u_\alpha J_{\alpha c})^2+\lambda(\sum_\alpha v_\alpha J_{\alpha c})^2-xJ_c]\times\delta(\sum u_\alpha^2-1)\delta(\sum v_\alpha^2-1)\delta(\sum u_\alpha v_\alpha)[\lambda(1+\lambda)]^{\frac{1}{2}(N-3)}\times\rho_N(\lambda)d\lambda\prod_\alpha du_\alpha dv_\alpha\bigg\}J_c dx, \tag{19}$$

where L is the normalization integral. Let us make a real orthogonal transformation on the variables u_α, v_α:

$$u_\beta'=\sum_\alpha u_\alpha C_{\alpha\beta}, \tag{20a}$$

$$v_\beta'=\sum_\alpha v_\alpha C_{\alpha\beta}, \tag{20b}$$

and choose

$$C_{\alpha 1}=J_{\alpha c}/[\sum_\alpha J_{\alpha c}^2]^{1/2}; \tag{20c}$$

then since C is an orthogonal matrix,

$$\sum u'^2_\alpha=\sum u_\alpha^2,\quad \sum v'^2_\alpha=\sum v_\alpha^2,\quad \sum u_\alpha' v_\alpha'=\sum u_\alpha v_\alpha,$$
$$\prod_\alpha du'_\alpha dv_\alpha'=\prod_\alpha du_\alpha dv_\alpha.$$

Equation (19) now becomes

$$P_N(x)dx=(NL)^{-1}\bigg\{\int\delta[(1+\lambda)u_1'^2+\lambda v_1'^2-x/N]\delta(\sum u_\alpha'^2-1)\times(\sum v_\alpha'^2-1)\delta(\sum u_\alpha' v_\alpha')[\lambda(1+\lambda)]^{\frac{1}{2}(N-3)}\times\rho_N(\lambda)d\lambda\prod_\alpha du_\alpha' dv_\alpha'\bigg\}dx. \tag{21}$$

Integrating over all u_α''s and v_α''s except u_1' and v_1', and calling the latter u and v, we can write (21) as

$$P_N(x)dx=(NL)^{-1}\left\{\int \delta[(1+\lambda)u^2+\lambda v^2-x/N]\times[1-u^2-v^2]^{\frac{1}{2}(N-4)}[\lambda(1+\lambda)]^{\frac{1}{2}(N-3)}\times\rho_N(\lambda)d\lambda du d_v\right\}d_x, \quad (22a)$$

where now the normalization integral L is given by

$$L=\int (1-u^2-v^2)^{\frac{1}{2}(N-4)}[\lambda(1+\lambda)]^{\frac{1}{2}(N-3)}\times\rho_N(\lambda)d\lambda du dv. \quad (22b)$$

The u, v integration is inside the circle $(u^2+v^2)\leqslant 1$, and $N\geqslant 3$. In general, the u, v integration is difficult to carry out, but for large values of N, which is the situation in practice, we can approximately replace the factor $[1-u^2-v^2]^{\frac{1}{2}(N-4)}$ with $\exp[-\frac{1}{2}N(u^2+v^2)]$ and take the limits of integration on u, v from $-\infty$ to ∞. Using the Fourier transform of the δ function and carrying out the integration over the variables u and v, we get for large values of N

$$P_N(x)dx=\tfrac{1}{2}L^{-1}\left\{\int d\lambda\, \rho_N(\lambda)[\lambda(1+\lambda)]^{\frac{1}{2}(N-4)}\times\exp\left[-\left(\frac{x}{4}\frac{2\lambda+1}{\lambda(1+\lambda)}\right)\right]I_0\left(\frac{x}{4\lambda(1+\lambda)}\right)\right\}dx, \quad (23a)$$

where L is now the integral

$$L=\int d\lambda\, \rho_N(\lambda)[\lambda(1+\lambda)]^{\frac{1}{2}(N-3)}, \quad (23b)$$

and I_0 is the modified Bessel function of the first kind.[6]

III. SUITABLE FORM OF THE WEIGHT FUNCTION

The results obtained in Sec. II are all independent of the form of the weight function. To make further progress we need to know the weight function $\rho_N(\lambda)$. We have tried to guess its form by looking at the distribution of N_μ whick has been obtained numerically.[3] It is easy to show that the probability distribution $P(N_\mu)$ is given by

$$P(N_\mu)dN_\mu=q^{-1}(N_\mu{}^2-1)^{\frac{1}{2}(N-3)}\times\rho_N[\tfrac{1}{2}(N_\mu-1)]dN_\mu, \quad 1\leqslant N_\mu\leqslant\infty, \quad (24)$$

where q is the normalization integral

$$q=\int_1^\infty (N_\mu{}^2-1)^{\frac{1}{2}(N-3)}\rho_N[\tfrac{1}{2}(N_\mu-1)]dN_\mu.$$

[6] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (Cambridge University Press, New York, 1962), p. 372.

A suitable choice for $\rho_N(\lambda)$ turns out to be

$$\rho_N(\lambda)=[\lambda(1+\lambda)]^{-\frac{1}{2}(N-5)}\exp(-a\lambda), \quad (25)$$

where a is a constant to be fixed by comparing some known physical quantity with its calculated value. If we fix the value of a by assuming the value of $\langle N_\mu\rangle$ to be given, then we can calculate the normalized mean-square deviations $S(N_\mu)$, $S(\Gamma_{\mu c})$, $S(\Theta_{\mu c})$. These values are given in Table II and are found to be in good agreement with the values obtained using the numerical calculation.[3]

The distribution $P(N_\mu)$, using Eqs. (24) and (25), is given by

$$P(N_\mu)d_\mu=\tfrac{1}{8}a^3(a+2)^{-1}(N_\mu{}^2-1)\times\exp[-\tfrac{1}{2}a(N_\mu-1)]dN_\mu, \quad 1\leqslant N_\mu\leqslant\infty. \quad (26)$$

This fits nicely the curve which has been obtained by Moldauer.[3]

The distribution of the quantity x based on expressions (23) and (25) turns out to be

$$P_N(x)dx=\tfrac{1}{2}a^3(a+2)^{-1}\left\{\int_0^\infty d\lambda\,[\lambda(1+\lambda)]^{1/2}\times\exp\left[-\left(a\lambda+\frac{x}{4}\frac{2\lambda+1}{\lambda(1+\lambda)}\right)\right]I_0\left(\frac{x}{4\lambda(1+\lambda)}\right)\right\}dx. \quad (27)$$

The interesting result which follows from Eq. (27) is that for large values of a and for x not too small, we can expand I_0 for large argument[6] and approximately evaluate the integral. This gives a distribution for x which resembles closely the Porter-Thomas distribution,[7] except for small values of x. This distribution which we have indicated agrees with that obtained by actual numerical calculation.[3]

IV. CORRELATIONS

In this section we shall study the channel correlations and the resonance correlations of the parameters of the collision matrix. We shall show that definite statements about the nature of channel correlations can be made even without a complete knowledge of the weight function.

Let us consider the channel correlation of $|\theta_{uc}|^2$.

TABLE II. Comparison of the normalized mean-square deviations $S(N_\mu)$, $S(\Gamma_{\mu c})$, $S(\Theta_{\mu c})$, assuming $\langle N_\mu\rangle$ to be given.

No. of channels	$\langle N_\mu\rangle$	a	Numerical calculation $S(N_\mu)$	$S(\Gamma_{\mu c})$	$S(\Theta_{\mu c})$	Present calculation $S(N_\mu)$	$S(\Gamma_{\mu c})$	$S(\Theta_{\mu c})$
20	1.18	23.1	0.01	1.60	1.81	0.01	1.67	1.82
100	1.52	8.4	0.066	1.38	1.88	0.057	1.40	1.98
300	1.69	6.5	0.07	1.31	1.76	0.08	1.34	2.14

[7] C. E. Porter and R. G. Thomas, Phys. Rev. 104, 483 (1956).

The channel correlation coefficient $\rho_{|\theta|^2}{}^{(c,c')}$ is given by

$$\frac{\langle|\theta_{uc}|^2|\theta_{uc'}|^2\rangle-\langle|\theta_{uc}|^2\rangle\langle|\theta_{uc'}|^2\rangle}{[(\langle|\theta_{\mu c}|^4\rangle-\langle|\theta_{\mu c}|^2\rangle^2)(\langle|\theta_{\mu c'}|^4\rangle-\langle|\theta_{\mu c'}|^2\rangle^2)]^{1/2}}. \quad (28)$$

The calculation of the ensemble average $\langle|\theta_{\mu c}|^2|\theta_{\mu c'}|^2\rangle$ will be somewhat similar to the calculation of the distribution of the quantity x described in Sec. II, and therefore we shall only give the final result. The ensemble averages which are needed for the correlation coefficient given by Eq. (28) can be written for large values of N as

$$\langle|\theta_{\mu c}|^2|\theta_{\mu c'}|^2\rangle=J_cJ_{c'}[\langle(2\lambda+1)^2\rangle +2p^2\langle 2\lambda^2+2\lambda+1\rangle], \quad (29a)$$

$$\langle|\theta_{\mu c}|^4\rangle-\langle|\theta_{\mu c}|^2\rangle^2=2J_c^2[2\langle(\lambda-\langle\lambda\rangle)^2\rangle +\langle 2\lambda^2+2\lambda+1\rangle], \quad (29b)$$

where we have used the notation

$$\langle\lambda^n\rangle=\frac{\int\lambda^n[\lambda(1+\lambda)]^{\frac{1}{2}(N-3)}\rho_N(\lambda)d\lambda}{\int[\lambda(1+\lambda)]^{\frac{1}{2}(N-3)}\rho_N(\lambda)d\lambda}, \quad (30)$$

and

$$p=\frac{\sum_\alpha J_{\alpha c}J_{\alpha c'}}{[(\sum J_{\alpha c}^2)(\sum J_{\alpha c'}^2)]^{1/2}}. \quad (31)$$

Using Eqs. (28) and (29) we get the channel correlation coefficient

$$\rho_{|\theta|^2}{}^{(c,c')}=\frac{p^2+2\langle(\lambda-\langle\lambda\rangle)^2\rangle/[1+2\langle\lambda(1+\lambda)\rangle]}{1+2\langle\lambda-\langle\lambda\rangle)^2\rangle/[1+2\langle\lambda(1+\lambda)\rangle]}. \quad (32)$$

A similar calculation shows that the channel correlation coefficients of the partial width $\Gamma_{\mu c}$, namely $\rho_\Gamma{}^{(c,c')}$ and $\rho_\Theta{}^{(c,c')}$ are given by

$$\rho_\Gamma{}^{(c,c')}=p^2, \quad (33)$$

$$\rho_\Theta{}^{(c,c')}=\frac{\langle[(1+2\lambda)^2-\langle(1+2\lambda)^2\rangle]^2\rangle+2p^2\langle 1+6\lambda+14\lambda^2+16\lambda^3+8\lambda^4\rangle}{S(\Theta_{\mu c})\langle N_\mu{}^2\rangle^2}. \quad (34)$$

The expressions (32), (33), and (34) establish the important result that the channel correlation is always positive. This is also indicated by the numerical calculation.[3]

For the case of 100 black channels, Moldauer[3] has given the values of $\rho_g{}^{(c,c')}$, $\rho_\Gamma{}^{(c,c')}$, and $\rho_\Theta{}^{(c,c')}$. Using Eq. (25), which gives the form of the weight function, and assuming the value of $\rho_g{}^{(c,c')}$ to be given, we find with the help of Eqs. (32), (33), and (34) that $p^2=0.087$, which gives $\rho_\Theta{}^{(c,c')}=0.22$ and $\rho_\Gamma{}^{(c,c')}=0.09$. The numerically calculated values for these quantities are $\rho_\Theta{}^{(c,c')}=0.020$ and $\rho_\Gamma{}^{(c,c')}=0.02$. The value of $\rho_\Theta{}^{(c,c')}$ is in excellent agreement with our value, but the value of $\rho_\pi{}^{(c,c')}$ is not in such good agreement.

We now consider the resonance correlations. The resonance correlation of N_μ is defined by

$$\rho_N{}^{(n)}=\frac{\langle N_\mu N_{\mu+n}\rangle-\langle N_\mu\rangle^2}{\langle N_\mu{}^2\rangle-\langle N_\mu\rangle^2}. \quad (35)$$

The resonance correlations are more difficult to calculate, as they involve two different columns of the complex orthogonal matrix. Let us calculate $\rho_N{}^{(1)}$. For this calculation we need the ensemble average $\langle N_\mu N_{\mu+1}\rangle$. It can be shown after some calculation, the details of which are not given here, that the ensemble average $\langle N_\mu N_{\mu+1}\rangle$ for large dimensions of the complex orthogonal matrix can be expressed approximately as

$$\langle N_\mu N_{\mu+1}\rangle=k^{-1}\int(2\lambda_1+1)(2\lambda_2+1)\rho_N(\lambda_1)\rho_N(\lambda_2) \times[\lambda_1(1+\lambda_1)\lambda_2(1+\lambda_2)]^{\frac{1}{2}(N-3)}[(1+\lambda_1+\lambda_2+2\lambda_1\lambda_2) \times(\lambda_1+\lambda_2+2\lambda_1\lambda_2)]^{-1/2}d\lambda_1d\lambda_2, \quad (36)$$

where k is the same integral as in Eq. (36) but without the factor $(2\lambda_1+1)(2\lambda_2+1)$. Using the form of $\rho_N(\lambda)$ given by Eq. (25), we have estimated roughly the integrals in Eq. (36) and find $\rho_N{}^{(1)}$ to be 0.2. The value obtained by Moldauer[3] is 0.48. We feel that a better estimate of the integrals in Eq. (36) will improve our value and bring it closer to Moldauer's value.

The resonance correlations of the other parameters can be calculated in a similar way.

Our object in this paper has been the statistical study of the parameters of the statistical collision matrix using the random complex orthogonal matrix and comparison of their predicted values with those obtained by the numerical calculation.[3] We have succeeded in showing that the predictions of our theoretical formulation are in good agreement with the results obtained by the numerical calculation.

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