effects is given by[8]

$$\alpha(F)=\frac{P\theta_f^2}{\omega}\int_\beta^\infty (t-\beta)^{1/2}\left\{\left|\frac{dA_i(t)}{dt}\right|^2-t|A_i(t)|^2\right\}dt,$$

where $P$ includes matrix elements and fundamental constants;

$\beta$ in this case equals $(E_g-\hbar\omega\pm h\nu)/\hbar\theta_f$;

$h\nu$ is the phonon energy with signs indicating absorption or emission of a phonon.

The corresponding value of $d\alpha(F)/dF$ is given by

$$\frac{d\alpha(F)}{dF}=\frac{4}{3}\frac{P\theta_f^2}{F}\left[\int_\beta^\infty dt(t-\beta)^{1/2}\left(1+\frac{\beta}{4(t-\beta)}\right)\times\left\{\left|\frac{dA_i(t)}{dt}\right|^2-t|A_i(t)|_2\right\}\right].$$

It may be noted that at $\beta=0$, the magnitude of the extrema vary as $F^{4/3}$.

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# Pressure Coefficients of Phonon-Assisted Tunneling Current in Germanium*

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The interband tunneling current of germanium $p$-$n$ junctions has been measured to stresses, $S=5\times10^9$ dyn cm$^{-2}$, along the [001] direction. The pressure dependence of the phonon-assisted tunneling current $\pi=d\ln I/dP$, has been obtained by measuring the stress dependence of the second derivative of the current, $I''$, with respect to voltage at the phonon threshold voltage. The $\pi$ values for the transverse acoustic (TA), longitudinal acoustic (LA), longitudinal optic (LO), and transverse optic (TO) phonon branches are given. The $\pi$ values for the LO and TO branches have not been previously reported. The results indicate that the $\pi$ for each phonon branch is symmetric with respect to forward and reverse bias voltage. Previous experiments gave $\pi$'s that had an asymmetry of $\approx19\%$ for the LA branch. Furthermore, the relative magnitudes of $I''$ at the phonon threshold voltage, here reported, differ from previously accepted values. The experiment is compared with theory and good qualitative agreement is found. However, the relative difference in magnitude of the tunneling currents between branches and the relative difference in $\pi$ between branches is still not fully explained.

THE study of phonon-assisted electron tunneling through semiconductor $p$-$n$ junctions has yielded a wealth of information about the lattice dynamics of semiconductors.[1–6] Until recently the theories developed to explain this effect have not been too successful.[7–13] The works of Tiemann and Fritzsche[14] and Kleinman[15] have improved agreement between theory and experiment considerably by investigating not only the

TABLE I. Pressure coefficients of the phonon-assisted tunneling current, $\pi=d\ln I/dP$ (in units of $10^{-10}$ cm$^2$ dyn$^{-1}$).

| Phonon branch | $-\pi$ This experiment | $-\pi$ Hydrostatic pressure (see Ref. 13) | $-\pi$ Theory TF (see Ref. 14) | $-\pi$ Theory K (see Ref. 15) | $\alpha$ |
|---|---|---|---|---|---|
| | | Forward bias | | | |
| TA | 1.61±0.05 | 1.47±0.03 | 2 | 2.38 | 0.03 |
| LA | 1.75±0.05 | 2.07±0.03 | 2 | 2.62 | 0.05 |
| LO | 1.44±0.05 | ··· | | | 0.02 |
| TO | 1.16±0.05 | ··· | | | 0.05 |
| | | Reverse bias | | | |
| TA | 1.60±0.05 | 1.56±0.03 | 2 | 2.47 | 0.04 |
| LA | 1.79±0.05 | 2.46±0.03 | 2 | 3.13 | 0.03 |
| LO | 1.3 ±0.1 | ··· | | | 0.09 |
| TO | 1.0 ±0.2 | ··· | | | 0.04 |

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TABLE II. Relative phonon-peak amplitudes 1.15°K and Sb concentration of $\approx 5\times 10^{18}$ carriers $cm^{-3}$.

| Phonon branch | Crystallographic tunneling direction (this experiment) [100] | [110] | [111] | Theory TF (see Ref. 14) | Theory K (see Ref. 15) |
|---|---|---|---|---|---|
| | | Forward bias | | | |
| TA | 1.00 | 1.00 | 1.00 | 1.0 | 1.0 |
| LA | 1.6 ±0.05 | 1.5 ±0.1 | 1.5 ±0.1 | 1.7 | 3.8 |
| LO | 0.48±0.05 | 0.46±0.05 | 0.46±0.05 | ··· | ··· |
| TO | 0.30±0.05 | 0.35±0.05 | 0.36±0.05 | ··· | ··· |
| | | Reverse bias | | | |
| TA | −0.86±0.05 | −0.88±0.05 | −0.87±0.05 | −0.65 | −0.74 |
| LA | −1.6 ±0.05 | −1.5 ±0.1 | −1.5 +0.1 | −0.37 | −3.0 |
| LO | −0.31±0.05 | −0.29±0.05 | −0.30±0.05 | ··· | ··· |
| TO | −0.12±0.05 | −0.16±0.05 | −0.14±0.05 | ··· | ··· |

phonon-assisted tunneling current $I$ but its pressure dependence $\pi = d\ln I/dP$ as well. In this way the questions about junction thickness and junction carrier concentrations can be separated out.

This paper reports the measurement of the tunneling pressure coefficients $\pi$ by a different technique $\pi^* = d\ln I''/dP$, where the differentiation of $I$ is with respect to voltage. See Table I. The values for $\pi$ for each branch and the relative phonon-assisted tunneling amplitudes $I''$, also reported in Table II, disagree, quantitatively, with the latest theoretical calculations of Kleinman.[15]

The experiment has been explained in detail in Ref. 5. The samples used where antimony-doped germanium $p$-$n$ junctions similar to the sample $D1B$ of Ref. 5.

The quantity $\pi^* = d\ln I''/dP$ can be related to $\pi$ if the equation for the tunneling current can be separated into a product of a pressure-dependent term $G(P)$ and a voltage-dependent term $Y(V)$. Then

$$\pi^* = \frac{d\ln(GY)''}{dP} = \frac{d\ln}{dP}[GY'' + 2G'Y' + \cdots].$$

The differentiation is with respect to voltage. For the case of $G$ independent of $V$ and $Y$ independent of $P$ then

$$\pi^* = \frac{d\ln G}{dP} + \frac{d\ln}{dP}\left(\frac{dY^2}{dV^2}\right) = \pi.$$

In fact, if only $Y$ is independent of $P$ and if $G$ is a slowly varying function of $V$ in the region of the threshold voltage $V_t$ for phonon emission, then $\pi = \pi^*$ to the order of $\alpha = 2G'Y'/GY''$. To obtain this end, the tunneling equation 1 of Ref. 5 can be used, $I = \int YGdE$, where $G$ is the tunneling current per unit energy interval $dE$ and $Y$ is the Fermi function product $f(E)[1 - f(E - eV \pm E_b)]$. The plus (minus) defines forward (reverse) bias. Then $Y$ is assured of being independent of pressure[16] and the tunneling data will reveal the ratio $\alpha$. The second derivative of $Y$ effectively introduces a delta function in $I''$ at the threshold voltages for phonon emission $V_t = \pm E_b/e$ at absolute zero temperature. The quantity $E_b$ is just the phonon energy of the branch $b$. This delta function makes it unnecessary to consider the effects of integration in the tunneling equation for low temperatures.

Although the experiment utilized stress $S$ not pressure $P$, the two can be related by $P = S/3$ as long as the stress direction is chosen not to split the conduction- and valence-band extrema. For Germanium the conduction-band minima are not split by stress along the [001] direction. However, the valence-band degeneracy between light and heavy hole bands is split by this stress. It is estimated that this splitting is small in comparison to the Fermi level used in the experiment[17] and should have equal effect on tunneling for all the phonon branches.

The results for $\pi$ obtained from stress measurements in the [001] direction are shown in Table I, column 1. Column 2 shows the low hydrostatic pressure $\pi$'s.[13] Columns 3 and 4 give theoretical values.[14,15] One cannot compare the absolute values of the $\pi$'s to better than 10% because of stress calibration limitations. However, the relative values are to the accuracy indicated in the table.

It can be seen from Table I, column 1, that the present experiment shows no forward to reverse bias symmetry in $\pi$ to within the accuracy of the experiment. However, the hydrostatic pressure experiment (column 2) has an asymmetry of 19% for the LA branch. This difference between the two experiments is not fully understood but may be due to the analysis of the hydrostatic $\pi$'s. If more than one branch is considered, then

$$\pi = d\ln I/dP = (\pi_{TA}I_{TA} + \pi_{LA}I_{LA} + \pi_{LO}I_{LO} + \pi_{TO}I_{TO})/I,$$

where $I_b$ is the partial current due to that branch $b$

[16] The Fermi functions have little pressure dependence (to the order of the change in phonon energies with pressure) because the energy $E$ is measured relative to the Fermi level. Further, only the peak amplitude is measured irrespective of its position.

[17] The valence band-splitting deformation potential is roughly 0.5 eV. Under the maximum stress of $5\times 10^9$ dyn $cm^{-2}$, the splitting of the light and heavy hole bands is $\approx 5$ meV. This is only 3.3% of the Fermi level of the valence band $\mathcal{L} = 150$ meV for this experiment See S. H. Koenig and J. J. Hall, Phys. Rev. Letters **5**, 550 (1960).

and $\pi_b$ is the equivalent pressure coefficient.[18] The analysis required first the finding of $\pi_{TA}$ and graphically separating $I_{TA}$ and $I_{LA}$ to obtain the $\pi_{LA}$.[17] Since the experiment was performed at 4.2°K, the LO and TO modes were not separated from the LA. The correction to $\pi_{LA}$ is then $\pi_{LA}$ (measured)$=\pi_{LA}+(\pi_{LO}I_{LO})/I$. Experimentally $I_{LO}/I$ for reverse bias increases much more rapidly in absolute voltage than $I_{LO}/I$ from forward bias. As a consequence, the $\pi_{LA}$ (measured) coefficients are larger than $\pi_{LA}$ and have an asymmetry with respect to forward and reverse bias.[13]

The present experiment alleviated the analysis problem in three ways. First, it was performed at lower temperatures (1.15°K) so that the effects of the LO and TO branches are separated out. They are also listed in Table I. Second, $I''$ has a sharp peak at the threshold voltage $V_t$ for each phonon branch. This peaking effect selects out only that branch for measurement and makes it unnecessary to graphically analyze the data. Finally, the error introduced by the assumptions made to obtain the relation between $\pi$ and $\pi^*$ can be obtained by measuring $\alpha$ for Sb-doped Ge (Table I, column 5).[19] It can be seen from column 5 that $\alpha$ is the same order of magnitude as the error in column 1. Consequently, its pressure dependence is neglected.[20]

The differences between the two experiments can also be explained by arguing that since the tunneling directions are different ([110] for stress and [100] for hydrostatic pressure) the $\pi$ coefficients should be different. This experiment, however, shows that the relative $I''$ phonon-peak amplitudes between branches for phonon-assisted tunneling in Sb-doped Ge is independent of crystallographic direction to within experimental error.

Table II gives the phonon peak amplitudes for the three major crystallographic directions. Each column is normalized relative to the TA phonon branch.[19] This result indicates that either the current direction is poorly defined by the junction manufacturing process, allowing the tunneling current in one direction to dominate over all the others; or the tunneling mechanism is the *same* for all phonon branches making the relative peak amplitudes independent of direction. In either case, for Sb-doped Ge the $\pi$ coefficients should be independent of the tunneling direction.

It should also be noted that Table II gives values for the relative phonon peak amplitudes that are different from the estimates obtained graphically from several authors.[1,12,15] These graphical values were roughly 2 to 3 for the LA phonon relative to the TA phonon. It is of interest that the relevant experiments[1,12] were performed at 4.2°K and again did not take into account the effects of the LO and TO phonons.

In order to compare this experiment with the theoretical results, it is necessary to note that for transitions involving only the indirect band gap the LA branch is forbidden in Ge junctions. In order to explain the presence of the LA branch in tunneling, Tiemann and Fritzsche (TF)[14] proposed that the tunneling went via intermediate electronic states and calculated the allowed transitions for the various phonon branches. Their calculation utilizing the $\mathbf{K}\cdot\mathbf{p}$ technique to mix states in the different bands and the phonon coupling to cause the transition, resulted in the $\pi$ coefficients listed in column 3 of Table I and the relative phonon peak amplitudes found in column 4 of Table II. Their results give good qualitative agreement with experment except that the average $\pi_{LA}-\pi_{TA}$ is comparable with the bulk compressibility while both experiments yield significantly larger values. The present experiment gives a value for $\pi_{LA}-\pi_{TA}$ which is 13 times the bulk compressibility. Also significant is the large asymmetry of the theoretical relative phonon peak amplitudes between forward and reverse bias. See Table II, column 5.

Kleinman (K) extended the theory of (TF) by calculating the first-order direct-tunneling transition with subsequent emission of a phonon to mix states in the same band. His results are very approximate since the integration over the junction barrier field was not explicitly performed. However, his $\pi$ coefficients, listed in column 4 of Table I, have an average $\pi_{LA}-\pi_{TA}$ value comparable with experiment and a tunneling current roughly two orders of magnitude larger than the (TF) theory. In other respects the calculation is inadequate. The $\pi$ coefficients are larger than experiment, a definite $\pi$ asymmetry is present for the LA branch, and the relative phonon-peak amplitudes are significantly different from experiment. It would appear that the theory of Kleinman is approximately correct but that the assumptions are not quite valid. It might be profitable to repeat the calculation by explicitly integrating over the junction barrier field, possibly using a different junction potential, and finding $\pi$- and phonon-peak amplitudes for the LO and TO branches.

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[18] For antimony-doped Ge, the current $I$ is dominately due to indirect tunneling. In other materials the effects of direct tunneling must be included.

[19] The $I''$ peak amplitude was also measured relative to the pre-threshold value in order to remove effects due to other branches noted in Ref. 5.

[20] For direct-tunneling semiconductors, it would be easy to measure the $\pi^*$ for nonthreshold voltages to obtain the errors.