# Raman Spectra of TiO<sub>2</sub>, MgF<sub>2</sub>, ZnF<sub>2</sub>, FeF<sub>2</sub>, and MnF<sub>2</sub>

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First-order Raman spectra have been measured at room temperature in five materials of the  $D_{4h}$  point group: TiO<sub>2</sub>, MgF<sub>2</sub>, ZnF<sub>2</sub>, FeF<sub>2</sub> and MnF<sub>2</sub>. The experiments utilized an argon laser light source and photoelectric detection. The four Raman-active phonon frequencies, predicted by group theory to have symmetries  $A_{1g}$ ,  $B_{1g}$ ,  $B_{2g}$ , and  $E_g$ , were observed and classified in each material. Each spectrum exhibited strong lines of  $A_{1g}$  and  $E_g$  symmetries, a weak high-frequency line of  $B_{2g}$  symmetry, and a very sharp  $B_{1g}$  line at quite low frequency (less than  $100 \text{ cm}^{-1}$  in all the materials except TiO<sub>2</sub>).

### INTRODUCTION

HE phonon frequencies and symmetries in the metal fluorides ZnF2, MgF2, MnF2, and FeF2 which have the rutile (TiO<sub>2</sub>) structure,  $D_{4h}$ , are of interest for several reasons. For example, "phononterminated" laser action has been reported recently in doped  $MgF_2$  and  $ZnF_2$ .<sup>1</sup> Also several transition metal fluorides, such as MnF<sub>2</sub> and FeF<sub>2</sub>, exhibit antiferromagnetism at low temperatures. In these cases a detailed knowledge of the phonon spectra is desired from the viewpoint of possible phonon-magnon interactions. The infrared-active phonons have been observed in TiO<sub>2</sub>,<sup>2</sup> MgF<sub>2</sub> and ZnF<sub>2</sub>,<sup>3</sup> and FeF<sub>2</sub>.<sup>4</sup> Raman spectra have been reported for TiO2 5-7 and MgF2.8,9 However in only one of the Raman experiments was a laser source used, so that polarization data were often lacking or unreliable. In the lone laser study<sup>9</sup> the crystal was not fully oriented so that complete phonon symmetry assignments could not be made.

In all the experiments described here a linearly polarized laser light source and completely oriented single-crystal samples have made possible unambiguous determination of the symmetries as well as the frequencies for the Raman-active phonons.

In the next section of this paper, we discuss briefly the experimental details, including the possibly misleading effects of improper crystal orientation. In the final section the results of our Raman experiments are presented and discussed in terms of both infrared and, where applicable, other Raman scattering results. Our conclusions concerning TiO<sub>2</sub> disagree with all but one of the previously reported Raman experiments, that of Krishnamurti. The earlier reports<sup>8,9</sup> of Raman spectra in MgF<sub>2</sub> not only disagree with each other, but with our results as well.

#### EXPERIMENTAL

To obtain the Raman spectra, we use  $\sim$ 75 mW of linearly polarized, 4880-Å light from an argon ion laser as the exciting source. Light scattered through 90° is passed through a double Czerny Turner spectrometer and detected photoelectrically. All samples used are oriented single crystals. All designations of the Raman tensor elements are referred to the crystal axes of the unit cell shown in Fig. 1. There are two molecules (six atoms) in the unit cell, implying a total of 15 vibrational modes. These modes have the irreducible representation<sup>10</sup>

 $1A_{1g} + 1A_{2g} + 1A_{2u} + 1B_{1g} + 1B_{2g} + 2B_{1u} + 1E_g + 3E_u$ .

Each E vibration is twofold degenerate. Group theory reveals four distinct infrared frequencies: one of symmetry  $A_{2u}$ , and three of symmetry  $E_u$ . In addition, as discussed by Barker,3 internal fields lift the degeneracies between the transverse and longitudinal infrared phonons. There are four Raman-active modes with symmetries  $A_{1g}$ ,  $B_{1g}$ ,  $B_{2g}$ , and  $E_g$ .

Corresponding to each Raman-active mode, there is a scattering tensor  $\alpha$  having a distinctive symmetry. For the four allowed Raman transitions in materials of the  $D_{4h}$  point group these tensors have the form<sup>11</sup>

$$\alpha(A_{1g}) = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix},$$
  
$$\alpha(E_g) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & d & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & d \\ 0 & 0 & 0 \\ d & 0 & 0 \end{pmatrix}$$
  
$$\alpha(B_{1g}) = \begin{pmatrix} c & 0 & 0 \\ 0 & -c & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \alpha(B_{2g}) = \begin{pmatrix} 0 & e & 0 \\ e & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

To examine experimentally a given component  $\alpha_{ij}$ one merely arranges the experimental geometry such that the incident light is polarized in the "i" direction while only that scattered light of "j" polarization is

<sup>&</sup>lt;sup>1</sup> L. F. Johnson, R. E. Dietz, and H. J. Guggenheim, Phys. Rev. Letters 11, 318 (1963).

 <sup>&</sup>lt;sup>2</sup> D. M. Eagels, J. Phys. Chem. Solids 25, 1243 (1964).
 <sup>3</sup> A. S. Barker, Jr., Phys. Rev. 136, A1290 (1964).
 <sup>4</sup> M. Balkunski, P. Moch, and G. Parisot, J. Chem. Phys. 44, 1440 (1964). 940 (1966).

 <sup>&</sup>lt;sup>6</sup> B. Dayal, Proc. Indian Acad. Sci. 32A, 304 (1950).
 <sup>6</sup> P. S. Narayanan, Proc. Indian Acad. Sci. 37A, 411 (1953).
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 <sup>8</sup> D. S. Krishnumart, M. Katimarti, Proc. Indian Acad. Sci. 55A, 290 (1962). <sup>8</sup> R. S. Krishnan and R. S. Katiyar, J. Phys. Radium 26, 627 (1965).

R. S. Krishnan and J. P. Russell, British J. Appl. Phys. 17, 501 (1966).

<sup>&</sup>lt;sup>10</sup> P. S. Narayanan, Proc. Indian Acad. Sci. 32A, 279 (1950). <sup>11</sup> R. Loudon, Proc. Phys. Soc. (London) 82, 393 (1963).



FIG. 1. Tetragonal unit cell for  $D_{4h}$  materials. Shaded and open circles represent positive and negative ions, respectively.  $a_1=a_2=a$ ;  $a_3=c\neq a$ .

observed. For further details on experimental geometries see Ref. 12.

As the unit cell of Fig. 1 suggests, the crystals studied here are uniaxial. Thus, determination of the c axis (z direction) may be conveniently done optically as well as by the use of x rays. However, proper determination of the a axes (x-y plane orientation) is sometimes difficult. Improper sample orientation in the x-yplane complicates interpretation of Raman data. For a sample whose laboratory axes x'y' are oriented at an angle  $\theta$  with respect to the crystal *x*-*y* axes, the strengths of the  $\alpha_{xx}$  and  $\alpha_{xy}$  scatterings are multiplied by factors  $(2\sin\theta\cos\theta)^2$  and  $(\cos^2\theta - \sin^2\theta)^2$ , respectively. We have verified this orientation dependence in a  $TiO_2$  sample cut in the form of a cylinder about its c axis. As the sample was rotated the intensities of the  $B_{1q}$  and  $B_{2q}$ lines varied according to the above trigonometric expressions. Thus, for a  $D_{4h}$  crystal an error in x-y orientation can result in incorrect assignments of the  $B_{1g}$  and  $B_{2g}$  phonons. An additional consequence of improper orientation is the inability to discriminate against two-phonon processes on the basis of scattering tensor symmetries.

## RESULTS AND DISCUSSION

The frequencies and symmetry assignments for the Raman-active phonons in the five materials studied here are summarized in Table I.

TABLE I. Raman-active phonons.

	$\operatorname{TiO}_{2}$ (cm <sup>-1</sup> )	$\mathrm{MgF}_{2}$ (cm <sup>-1</sup> )	$ZnF_2$ (cm <sup>-1</sup> )	FeF <sub>2</sub> (cm <sup>-1</sup> )	MnF <sub>2</sub> (cm <sup>-1</sup> )
$B_{1g}$	143	92	70	73	61
$E_{q}$	447	295	253	257	247
$A_{1a}$	612	410	350	340	341
$B_{2q}$	826	515	522	496	476

<sup>12</sup> T. C. Damen, S. P. S. Porto, and B. Tell, Phys. Rev. 142, 570 (1966).

TABLE II. Infrared-active, transverse optical phonons.

	${ m TiO_{2^{a}}}$ (cm <sup>-1</sup> )	$\mathrm{MgF_{2^{b}}}$ (cm <sup>-1</sup> )	ZnF <sub>2</sub> <sup>b</sup> (cm <sup>-1</sup> )	FeF <sub>2</sub> ° (cm <sup>-1</sup> )	$MnF_2$
$E_u$	183	247	173	200	• • •
$E_{u}$	388	410	244	320	
$E_u$	500	450	380	480	
$A_{2u}$	167	399	294	440	••••

<sup>b</sup> Reference 3.
 <sup>c</sup> Reference 4.

TABLE III. Longitudinal optical phonons.

	$\operatorname{TiO}_{2^{a}}$ (cm <sup>-1</sup> )	$\mathrm{MgF_{2^{b}}}_{\mathrm{(cm^{-1})}}$	$ m ZnF_{2^{b}}$ (cm <sup>-1</sup> )	FeF2	$MnF_2$
$E_{u}$	373	303	227		
$E_{u}$	458	415	264	•••	• • •
$E_{u}$	806	617	498	• • •	•••
$A_{2u}$	811	625	488	•••	•••

<sup>a</sup> Reference 2. <sup>b</sup> Reference 3.

For completeness, Tables II and III summarize the previously reported results of infrared experiments, where available, by other workers. We shall discuss



FIG. 2. Raman spectrum of TiO<sub>2</sub>; instrumental width ~1.5 cm<sup>-1</sup>. (a)  $\alpha_{zz}$  component showing strong combination bands. The  $\alpha_{zz}$  component of the  $A_{1q}$  phonon, 612 cm<sup>-1</sup>, is obscured by two-phonon processes at room temperature, but dominates at 77°K. (b)  $\alpha_{zz}$  component taken at liquid-nitrogen temperature, showing clearly the emergence of the 612-cm<sup>-1</sup> line as a fundamental above the neighboring combination bands. All other spectra exhibited in this paper were obtained at room temperature. (c)  $\alpha_{zz}$  component showing phonon of  $E_q$  symmetry at 447 cm<sup>-1</sup>. (d)  $\alpha_{xx}$  component showing  $B_{1q}$  and  $A_{1q}$  phonons at 143 and 612 cm<sup>-1</sup>, respectively, as well as very strong two-phonon bands. (e)  $\alpha_{yx}$  component showing the  $B_{2q}$  phonon at 826 cm<sup>-1</sup> and two-phonon bands.



FIG. 3. Raman spectrum of MgF<sub>2</sub>; instrumental width  $\sim 3 \text{ cm}^{-1}$ . (a)  $\alpha_{zz}$  component showing  $A_{1g}$  phonon at 410 cm<sup>-1</sup>. (b)  $\alpha_{zx}$  component showing  $B_{g}$  phonon at 295 cm<sup>-1</sup>. (c)  $\alpha_{xx}$  component showing  $B_{1g}$  and  $A_{1g}$  phonons at 92 and 410 cm<sup>-1</sup>, respectively. Gain increase by a factor of 10 shows clearly the  $B_{1g}$  phonon. (d)  $\alpha_{yy}$  and  $\alpha_{xy}$  components together showing  $B_{1g}$  and  $A_{1g}$  again. Notice the small peak at 515 cm<sup>-1</sup> which is due to the  $B_{2g}$  phonon.

each material in turn in the light of the observed Raman spectra (see Figs. 2-6).

(a)  $TiO_2$ . Though several authors<sup>5-7</sup> have reported observation of the Raman effect in TiO<sub>2</sub>, there has not been agreement among them as to phonon frequencies and symmetries. In large measure, these disagreements arise from the relatively strong two-phonon processes in TiO<sub>2</sub>. Without careful polarization studies, the clear-cut symmetries associated with fundamental (one-phonon) processes cannot be exploited. For example, Fig. 2 shows a strong band at  $\sim 235$  cm<sup>-1</sup> in TiO<sub>2</sub>, which under high resolution exhibits considerable structure. Largely because of its strength, several workers have assigned this band to a fundamental process.<sup>5,9</sup> Detailed examination of the scattering tensor associated with the 235cm<sup>-1</sup> peak, however, does not reveal a simple symmetry corresponding to any of the one-phonon Raman tensors displayed in Sec. II. This peak exhibits nonzero tensor elements  $\alpha_{xx}$ ,  $\alpha_{yy}$ ,  $\alpha_{zz}$ , and  $\alpha_{xz}$ ; and is therefore a combination line. Further, from the room-temperature spectrum in Fig. 2(a), the 612-cm<sup>-1</sup> phonon is nearly obscured by combination bands. However, Fig. 2(b), taken at liquid-nitrogen temperature, shows clearly the 612-cm<sup>-1</sup> line as a fundamental process. This observation is complemented by the room-temperature  $\alpha_{xx}$ 

spectrum, Fig. 2(d), which completes identification of the 612-cm<sup>-1</sup> line as  $A_{1g}$ . The sharp  $B_{1g}$  phonon at 143 cm<sup>-1</sup> and the weak  $B_{2g}$  phonon at 826 cm<sup>-1</sup> are evident in the  $\alpha_{xx}$  and  $\alpha_{yx}$  spectra, respectively. The  $E_g$  phonon is clearly identified in Fig. 2(c) at 447 cm<sup>-1</sup>.

(b)  $MgF_2$ . The recent reports of Raman studies in  $MgF_2$  by Krishnan and Katiyar<sup>8</sup> and by Krishnan and Russell<sup>9</sup> are in disagreement with each other and with our results. First, the line at 72 cm<sup>-1</sup> reported in Refs. 8 and 9 and identified as  $B_{1g}$  and  $B_{g}$ , respectively, is not observed by us. This is clearly shown in Fig. 3. (The notation  $B_g$  is employed by Krishnan and Russell to indicate the inability to distinguish between  $B_{1g}$  and



FIG. 4. Raman spectrum of ZnF<sub>2</sub>; instrumental width  $\sim 3 \text{ cm}^{-1}$ . (a)  $\alpha_{zz}$  component showing the  $A_{1g}$  phonon at 350 cm<sup>-1</sup> and twophonon bands. (b)  $\alpha_{zz}$  component showing the  $E_g$  phonon at 253 cm<sup>-1</sup>. (c)  $\alpha_{xz}$  component showing the  $B_{1g}$  phonons at 70 and 350 cm<sup>-1</sup>, respectively. A factor of 3 increase in gain shows clearly the  $B_{1g}$  phonon. (d)  $\alpha_{yz}$  component showing the  $B_{2g}$  phonon at 522 cm<sup>-1</sup> superimposed on the tail of a weak fluorescence band. A small amount of leakage of the  $A_{1g}$  and  $E_g$  phonon lines is evident.

 $B_{2g}$  in their experiment.) Instead, we identify as the  $B_{1g}$  phonon the 92-cm<sup>-1</sup> line clearly visible in the figure. This line was also observed by Krishnan and Katiyar but was ignored in their analysis. Another line reported at ~155 cm<sup>-1</sup> and identified as  $E_g$  and  $B_g$  in Refs. 8 and 9, respectively, was not observed by us. Figure 4 shows clearly that the line whose symmetry is  $E_g$  has a frequency of 295 cm<sup>-1</sup>, while the  $A_{1g}$  phonon is at 410 cm<sup>-1</sup>. The  $B_{2g}$  line we observe at 515 cm<sup>-1</sup> is extremely weak, as are the  $B_{2g}$  lines for all of the  $D_{4h}$  crystals studied. However, the symmetry of the scattering tensor is that expected for the  $B_{2g}$  vibration.

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(c)  $ZnF_2$ . As indicated in the tables above,  $ZnF_2$  has been thoroughly studied in the infrared by Barker.<sup>3</sup> We find that the frequencies and symmetry assignments for the Raman-active phonons follow the same general pattern as those discussed above in  $TiO_2$  and  $MgF_2$ . The Raman spectra are displayed in Fig. 4 where it is clear that the lines at 70, 253, 350, and  $522 \text{ cm}^{-1}$  are due



FIG. 5. Raman spectrum of FeF<sub>2</sub>; instrumental width  $\sim$ 3 cm<sup>-1</sup>. The sample was misoriented in the x-y plane by 19°. (a)  $\alpha_{zz}$  component showing the  $A_{1g}$  phonon at 340 cm<sup>-1</sup>. (b)  $\alpha_{zz}$  compo component showing the  $A_{1g}$  phonon at 340 cm<sup>-1</sup>. (b)  $\alpha_{zz}$  component with a small leakage of  $\alpha_{yx}$  due to 19° misorientation. Thus the  $B_{1g}$ and  $A_{1g}$  phonons at 73 and 340 cm<sup>-1</sup>, respectively, appear together with the weak  $B_{2g}$  phonon at 496 cm<sup>-1</sup>. (d)  $\alpha_{yz}$  component with small amount of  $\alpha_{zz}$  component. The  $B_{1g}$  and  $B_{2g}$  phonons at 73 and 496 cm<sup>-1</sup> are evident.



FIG. 6. Raman spectrum of  $MnF_2$ ; instrumental width  $\sim 3$  cm<sup>-1</sup>. (a)  $\alpha_{zz}$  component showing the  $A_{1g}$  phonon at 341 cm<sup>-1</sup> and a very weak two-phonon band. (b)  $\alpha_{zx}$  component showing the  $E_g$  phonon weak two-photon ball. (b)  $\alpha_{xx}$  component showing the  $A_{1g}$  and  $B_{1g}$  phonons at 247 cm<sup>-1</sup>. (c)  $\alpha_{xx}$  component showing the  $A_{1g}$  and  $B_{1g}$  phonons at 341 and 61 cm<sup>-1</sup>, respectively. The gain is increased by a factor of 10 to show the  $B_{1g}$  phonon clearly. (d)  $\alpha_{yx}$  component showing the  $B_{2g}$  phonon at 476 cm<sup>-1</sup> and a leakage of the much stronger  $E_g$  phonon.

to phonons with symmetries  $B_{1g}$ ,  $E_g$ ,  $A_{1g}$ , and  $B_{2g}$ , respectively. In this case the  $B_{2g}$  phonon is clearly visible above the noise.

(d)  $FeF_2$  and  $MnF_2$ . Figures 5 and 6 reveal the Raman spectra of FeF<sub>2</sub> and MnF<sub>2</sub> to be quite similar to those already discussed. Additional interest arises in these cases, however, because these materials become antiferromagnetic at low temperatures ( $T_N = 67.7$  °K for MnF<sub>2</sub>; 78.5°K for FeF<sub>2</sub>). The magnons in the magnetic state have been studied in both optical<sup>13</sup> and infrared absorption<sup>14</sup> experiments, as well as by light scattering.<sup>15</sup> In FeF<sub>2</sub><sup>15</sup> the magnon frequency increases from 53 cm<sup>-1</sup> at the center of the Brillouin zone to  $\sim 77 \text{ cm}^{-1}$  at the

<sup>&</sup>lt;sup>18</sup> R. L. Green, D. D. Sell, W. M. Yen, A. L. Schawlow, and R. M. White, Phys. Rev. Letters **15**, 656 (1965) and references cited therein.

 <sup>&</sup>lt;sup>14</sup> S. J. Allen, R. Loudon, and P. L. Richards, Phys. Rev. Letters 16, 463 (1966), and references cited therein.
 <sup>15</sup> P. A. Fleury, S. P. S. Porto, L. E. Cheesman, and H. J. Guggenheim, Phys. Rev. Letters 17, 84 (1966).

zone edge. The low-frequency  $B_{1g}$  phonon we have observed at 73 cm<sup>-1</sup> in FeF<sub>2</sub> may possibly interact with the magnon away from the center of the zone. In MnF2<sup>15</sup> a similar situation exists: The magnon frequency increases from  $\sim 8.5 \text{ cm}^{-1}$  to  $\sim 56 \text{ cm}^{-1}$  from zone center to zone edge; and the  $B_{1g}$  phonon frequency is 61 cm<sup>-1</sup>. Of course, the phonon frequencies given here are room-temperature values, while the magnon frequencies were obtained at low  $(<20^{\circ}K)$  temperatures. Some frequency shift with temperature will occur in the phonons, and some dispersion in phonon frequency will occur in going from the zone center to the zone boundary. Nevertheless, neither of these effects should be large enough to eliminate completely the possibility of phonon-magnon interaction in these materials. In fact, measurements of the zone-center  $B_{1g}$  phonon frequencies at  $\sim 20^{\circ}$ K reveal  $\sim 71$  cm<sup>-1</sup> and  $\sim 57$  cm<sup>-1</sup> for FeF<sub>2</sub> and MnF<sub>2</sub>, respectively.

Finally, some comments on the general features of the phonons in the  $D_{4h}$  compounds are in order. Matossi<sup>16</sup> has proposed a model for the vibrational modes in the rutile structure based on seven force constants and the masses and bond angles of the atoms involved. We present a few of his results in order to assess the validity of his model. Two results of Matossi are expressed in Eqs. (1) and (2):

$$\omega_2^2 = \omega_4^2 + \omega_3^2 - \omega_1^2, \qquad (1)$$

$$\omega_2^2 = \omega_3^2 \left( \frac{\cos \psi + \sin \psi}{\cos \psi - \sin \psi} \right)^2. \tag{2}$$

Here  $\omega_1$  is the  $A_{1g}$  phonon frequency;  $\omega_2$  is  $A_{2g}$ ;  $\omega_3$  is  $B_{1g}$ ; and  $\omega_4$  is  $B_{2g}$ . All but the  $A_{2g}$  frequency are measured in the Raman effect. The  $A_{2g}$  phonon is neither Raman nor infrared active. The angle  $\psi$  is defined by Matossi<sup>16</sup> and depends only on the lattice parameters of the solid. Equations (1) and (2) together

with our measurements of  $\omega_1, \omega_3$ , and  $\omega_4$  provide a test of Matossi's model. Our Raman data and Eq. (1) yield  $A_{2g}$  frequencies  $\omega_2$  of 653, 326, 394, 368, and 338 cm<sup>-1</sup> for TiO<sub>2</sub>, MgF<sub>2</sub>, ZnF<sub>2</sub>, FeF<sub>2</sub>, and MnF<sub>2</sub>, respectively. Using values of  $\psi$  computed from measured lattice parameters,<sup>17</sup> together with Eq. (2), results in values for  $\omega_2$  of 686, 434, 335, 361, and 282 cm<sup>-1</sup>, respectively. With regard to this simple test of internal consistency, Matossi's model is moderately successful-the maximum discrepancy being  $\sim 30\%$  in the case of MgF<sub>2</sub>. However, agreement between calculations and experiment is much poorer in the case of some infrared phonons, particularly A 2u. Such discrepancies have discouraged us from including any values for the  $B_{2u}$ phonon frequencies calculated from Matossi's model. Since both infrared and Raman data are now available on several of the  $D_{4h}$  materials, a re-examination of their lattice dynamics seems in order.

#### SUMMARY

The first-order Raman effect has been measured at room temperature in the crystals of  $D_{4h}$  symmetry: TiO<sub>2</sub>, MgF<sub>2</sub>, ZnF<sub>2</sub>, FeF<sub>2</sub>, and MnF<sub>2</sub>. The four Ramanactive phonons predicted by group theory were found for each crystal. Each spectrum exhibits strong lines of  $A_{1g}$  and  $E_g$  symmetries, a sharp low-frequency line of  $B_{1g}$  symmetry, and a weak high-frequency line of  $B_{2g}$ symmetry. The somewhat richer structure of the TiO<sub>2</sub> spectra is due to large second-order Raman effect, not evident in the fluorides studied.

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<sup>&</sup>lt;sup>16</sup> F. Matossi, J. Chem. Phys. **19**, 1543 (1951).

<sup>&</sup>lt;sup>17</sup> R. W. G. Wycoff, *Crystal Structures* (Interscience Publishers, Inc., New York, 1960), Vol. 1, p. 251.