Electromagnetic Properties of Hadrons in the Quark Model

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A general review of the electromagnetic properties of the hadrons when interpreted as composite states of quarks is given. Electromagnetic mass formulas are derived using two-body forces. The one-particle contributions to the magnetic moments of the baryons and to the electromagnetic decays of the vector mesons are calculated without any assumptions about the U-spin transformation properties of the photon. The electromagnetic decays of the spin-$\frac{3}{2}$ baryons are calculated in a way analogous to the Becchi-Morpurgo calculation of the rate for $\omega \rightarrow \pi^0\gamma$. A quark model of photoproduction is presented, and its predictions are accompanied by the explicit kinematic correction factors necessary for a comparison with experiment.

I. INTRODUCTION

THE possible existence of three elementary particles of third integral charge was proposed by Gell-Mann and Zweig in 1964.[1,2] Zweig performed some calculations of the properties of hadrons in this model under assumptions that included $SU(3)$ invariance. The discovery of $SU(6)$ and its striking successes in certain applications seemed to indicate that the results obtained from the quark model were only manifestations of this symmetry of strong interactions and that the quarks were merely a useful tool for deriving results following from $SU(6)$, rather than physical substructures of the hadrons. On the other hand $SU(6)$ soon ran into well known difficulties; it could not be made consistent with relativity and quantum theory.

Lately, several groups have therefore continued the investigation of the quark model.[3,4] In particular its applications to elastic- and inelastic-scattering processes, to strong and electromagnetic mass differences, and to baryon and meson decays[5–7] have shown that the assumption of a quark structure for hadrons and some very simple dynamical assumptions concerning quarks are sufficient to derive a number of relations which agree remarkably well with experiment. In most cases, no higher symmetry, like $SU(3)$ or $SU(6)_W$, is assumed to govern the quark dynamics. The resulting predictions are generally consistent with (but not identical to) relations obtained from assuming that the interactions of the particles are invariant under $SU(3)$ or $SU(6)_W$, and ignoring quark structure; some predictions are stronger and some are weaker.

Among the stronger predictions which seem particularly interesting are the $SU(3)$-independent relations for total meson-baryon and baryon-baryon scattering cross sections,[3,4] the electromagnetic transition moments for vector-meson decays and the mass relations for the baryons.[6,7] In view of the success of this very simplified "independent quark model" in a variety of problems in hadron physics, it is tempting to take more seriously the assumption that the hadrons are really bound states of quarks and antiquarks. This would lead us to the opposite standpoint from the one mentioned at the beginning: The apparent invariance of strong interactions under $SU(3)$ or $SU(6)_W$ (and also the well-known "symmetry-breaking" effects) would now be a manifestation of the dynamical structure of mesons and baryons as quark-antiquark and three-quark bound states respectively. "Symmetry-breaking" effects, for instance, would then reflect the fact that the nonstrange and the strange quarks have different interactions.

In this paper we are concerned with the electromagnetic properties of mesons and baryons. It is our aim to discuss electromagnetic mass differences, magnetic moments, magnetic dipole transitions and photoproduction cross sections, without assuming that the photon has any definite transformation properties under the strong interactions. In particular, we will not insist on the U-spin invariance of the electromagnetic interactions. The use of U spin is an elegant way to discuss electromagnetic interactions,[8] provided one assumes that the strong interactions are invariant under $SU(3)$ or some still higher symmetry, and this is just the assumption that we are interested in testing. We obtain

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[1] M. Gell-Mann, Phys. Letters 8, 214 (1964).

[2] G. Zweig, CERN reports No. TH. 401 and TH. 412, 1964 (unpublished).

[3] E. M. Levin and L. L. Frankfurt, JETP Pis'ma v Redaktsiyu 2, 105 (1965) [English transl.: JETP Letters 2, 65 (1965)]. V. V. Anisovich, *ibid.* 2, 439 (1965) [English transl.: *ibid.* 2, 272 (1965)].

[4] H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).

[5] H. J. Lipkin, F. Scheck, and H. Stern, Phys. Rev. (to be published), and further references in this paper. Also G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters 17, 412 (1966).

[6] P. Federmann, H. R. Rubinstein, and I. Talmi, Phys. Letters 22, 208 (1966).

[7] H. R. Rubinstein, Phys. Letters 22, 210 (1966).

[8] C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Letters 7, 61 (1963).

weaker relations by allowing the three quarks to have different electromagnetic properties. On the other hand, as has been the case in other quark-model calculations, we find that in some cases properties of baryons and mesons can now be related.

In Sec. II we are concerned with the electromagnetic masses. By including only two-body quark-quark forces, but allowing for spin dependence, we find nine mass formulas connecting the eighteen charge states of the baryon octet and decuplet. One of them is the Coleman-Glashow formula.[9] No relations connecting meson and baryon masses may be obtained without further assumptions about the quark-quark and quark-antiquark forces.

In Sec. III we study the magnetic moments of the baryons and the magnetic dipole transition amplitudes for baryons and mesons. We find a number of places where the hypothesis of U-spin invariance of the electromagnetic interaction may be tested.

In Sec. IV we present a theory of photoreactions. Sum rules are obtained that are independent of U-spin invariance; some of these may soon be tested experimentally.

II. RELATIONS BETWEEN ELECTROMAGNETIC MASSES

We assume that in the absence of electromagnetic interactions all particles within the same isospin multiplet have equal masses. In the "independent quark model" the electromagnetic mass splittings are then given by one- and two-body interaction terms as

$$M=C+\sum(\delta m_i+D_{ij}{}^S),$$

where C is the common value of the masses when all interactions are absent and δm_i are the one-particle contributions to the shifts, i.e., the change in mass of the quarks themselves. The functions $D_{ij}{}^S$ are the two-body interaction terms, S denoting the relative spin state of the particles i and j. $D_{ij}{}^S$ and δm_i contain contributions from both strong and electromagnetic symmetry-breaking interactions. We assume, more generally than before,[10] that the electromagnetic forces may have a spin dependence. However, we retain the assumption that there are no contributions from three-body interactions.

There are eighteen distinct baryon masses and nine relations among them under these assumptions. The baryon masses are explicitly (suppressing C):

$$
\begin{aligned}
N^{*++}&=3\delta m_{\mathcal{P}}+3D_{\mathcal{P}\mathcal{P}}{}^1,\\
N^{*+}&=2\delta m_{\mathcal{P}}+\delta m_{\mathcal{N}}+D_{\mathcal{P}\mathcal{P}}{}^1+2D_{\mathcal{P}\mathcal{N}}{}^1,\\
N^{*0}&=\delta m_{\mathcal{P}}+2\delta m_{\mathcal{N}}+2D_{\mathcal{P}\mathcal{N}}{}^1+D_{\mathcal{N}\mathcal{N}}{}^1,\\
N^{*-}&=3\delta m_{\mathcal{N}}+3D_{\mathcal{N}\mathcal{N}}{}^1,\\
Y_1{}^{*+}&=2\delta m_{\mathcal{P}}+\delta m_\lambda+D_{\mathcal{P}\mathcal{P}}{}^1+2D_{\mathcal{P}\lambda}{}^1,\\
Y_1{}^{*0}&=\delta m_{\mathcal{P}}+\delta m_{\mathcal{N}}+\delta m_\lambda+D_{\mathcal{P}\mathcal{N}}{}^1+D_{\mathcal{P}\lambda}{}^1+D_{\mathcal{N}\lambda}{}^1,\\
Y_1{}^{*-}&=2\delta m_{\mathcal{N}}+\delta m_\lambda+D_{\mathcal{N}\mathcal{N}}{}^1+2D_{\mathcal{N}\lambda}{}^1,\\
\Xi^{*0}&=\delta m_{\mathcal{P}}+2\delta m_\lambda+2D_{\mathcal{P}\lambda}{}^1+D_{\lambda\lambda}{}^1,\\
\Xi^{*-}&=\delta m_{\mathcal{N}}+2\delta m_\lambda+2D_{\mathcal{N}\lambda}{}^1+D_{\lambda\lambda}{}^1,\\
\Omega^-&=3\delta m_\lambda+3D_{\lambda\lambda}{}^1,\\
p&=2\delta m_{\mathcal{P}}+\delta m_{\mathcal{N}}+D_{\mathcal{P}\mathcal{P}}{}^1+\tfrac{1}{2}D_{\mathcal{P}\mathcal{N}}{}^1+\tfrac{3}{2}D_{\mathcal{P}\mathcal{N}}{}^0,\\
n&=\delta m_{\mathcal{P}}+2\delta m_{\mathcal{N}}+\tfrac{1}{2}D_{\mathcal{P}\mathcal{N}}{}^1+D_{\mathcal{N}\mathcal{N}}{}^1+\tfrac{3}{2}D_{\mathcal{P}\mathcal{N}}{}^0,\\
\Sigma^+&=2\delta m_{\mathcal{P}}+\delta m_\lambda+D_{\mathcal{P}\mathcal{P}}{}^1+\tfrac{1}{2}D_{\mathcal{P}\lambda}{}^1+\tfrac{3}{2}D_{\mathcal{P}\lambda}{}^0,\\
\Sigma^0&=\delta m_{\mathcal{P}}+\delta m_{\mathcal{N}}+\delta m_\lambda+D_{\mathcal{P}\mathcal{N}}{}^1\\
&\qquad+\tfrac{1}{4}D_{\mathcal{P}\lambda}{}^1+\tfrac{1}{4}D_{\mathcal{N}\lambda}{}^1+\tfrac{3}{4}D_{\mathcal{P}\lambda}{}^0+\tfrac{3}{4}D_{\mathcal{N}\lambda}{}^0,\\
\Sigma^-&=2\delta m_{\mathcal{N}}+\delta m_\lambda+D_{\mathcal{N}\mathcal{N}}{}^1+\tfrac{1}{2}D_{\mathcal{N}\lambda}{}^1+\tfrac{3}{2}D_{\mathcal{N}\lambda}{}^0,\\
\Lambda&=\delta m_{\mathcal{P}}+\delta m_{\mathcal{N}}+\delta m_\lambda+\tfrac{3}{4}D_{\mathcal{P}\lambda}{}^1\\
&\qquad+\tfrac{3}{4}D_{\mathcal{N}\lambda}{}^1+D_{\mathcal{P}\mathcal{N}}{}^0+\tfrac{1}{4}D_{\mathcal{P}\lambda}{}^0+\tfrac{1}{4}D_{\mathcal{N}\lambda}{}^0,\\
\Xi^0&=\delta m_{\mathcal{P}}+2\delta m_\lambda+\tfrac{1}{2}D_{\mathcal{P}\lambda}{}^1+D_{\lambda\lambda}{}^1+\tfrac{3}{2}D_{\mathcal{P}\lambda}{}^0,\\
\Xi^-&=\delta m_{\mathcal{N}}+2\delta m_\lambda+\tfrac{1}{2}D_{\mathcal{N}\lambda}{}^1+D_{\lambda\lambda}{}^1+\tfrac{3}{2}D_{\mathcal{N}\lambda}{}^0.
\end{aligned}
$$

Here $\mathcal{P}$ and $\mathcal{N}$ denote the isodoublet quarks and λ the strange quark, while p, n, Λ represent the physical baryons. The fact that the $\mathcal{P}$-$\mathcal{P}$, $\mathcal{N}$-$\mathcal{N}$, and λ-λ two-body interactions appearing in these expressions are pure spin-triplet interactions is a consequence of the fact that the baryon wave functions are assumed to be fully symmetric in the quark indices, i.e., are assigned to the **56** representation of $SU(6)$. From the above expressions we find nine relations among the physical masses. These relations are

$$N^{*++}-N^{*-}=3(N^{*+}-N^{*0}),\tag{1a}$$

$$N^{*+}-N^{*0}=Y_1{}^{*+}-Y_1{}^{*-}-\Xi^{*0}+\Xi^{*-},\tag{1b}$$

$$N^{*+}+N^{*-}-2N^{*0}=Y_1{}^{*+}+Y_1{}^{*-}-2Y_1{}^{*0},\tag{1c}$$

$$N^{*+}-N^{*0}=p-n,\tag{1d}$$

$$Y_1{}^{*+}+Y_1{}^{*-}-2Y_1{}^{*0}=\Sigma^++\Sigma^--2\Sigma^0,\tag{1e}$$

$$p-n=\Sigma^+-\Sigma^--\Xi^0+\Xi^-,\tag{1f}$$

$$\Omega^--N^{*-}=3(\Xi^{*-}-Y_1{}^{*-}),\tag{1g}$$

$$\Xi^{*-}-Y_1{}^{*-}=\Xi^--\Sigma^-,\tag{1h}$$

$$
\begin{aligned}
&-\tfrac{1}{4}(N^{*++}+N^{*+}+N^{*0}+N^{*-})\\
&\quad+\tfrac{1}{3}(Y_1{}^{*+}+Y_1{}^{*0}+Y_1{}^{*-})+\tfrac{1}{2}(\Xi^{*0}+\Xi^{*-})-\Omega\\
&\quad=3\Lambda+\tfrac{1}{3}(\Sigma^++\Sigma^0+\Sigma^-)-(p+n)-(\Xi^0+\Xi^-).
\end{aligned}\tag{1i}
$$

Most of these relations have appeared before in the literature, in various group-theoretic discussions of mass

[9] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

[10] H. R. Rubinstein, Phys. Rev. Letters **17**, 41 (1966).

formulas.[11] This is because our assumption that the baryon masses are given by a sum of single-quark terms and two-body terms is equivalent to keeping only terms which transform like the representations **1**, **35**, **405** in the $SU(6)$ mass operator. We would like to discuss this correspondence in some detail.

Let us first review the group-theoretic approach to a physical mass spectrum. Having identified a set of states which "fill" one or more irreducible representations of a symmetry group, one writes the mass operator for the multiplet as a sum of terms, each of which possesses definite transformation properties under the group operations. Each term may be associated with a "spurion," a fictitious particle having the same transformation properties. The most general mass formula has as many terms as there are physical particles in the multiplet, and it is possible to invert the equations for the physical particle masses to solve for the magnitude of each of the terms.[12] When this is done for the baryon octet and decuplet in the $SU(3)$ theory, for example, it is found that the octet terms are larger than the terms which transform like a **27**.

We obtain a mass formula when we set the contribution of some spurion equal to zero. Two things which make a symmetry attractive are: (i) finding that some spurions contribute much less than others which might have been expected to have the same strength, and (ii) finding that the same spurions are important in the mass spectra of particles belonging to different multiplets.[13]

When we are dealing with a set of states which include the decuplet resonances, we are confronted by a situation in which all of the masses are not accurately known. The electromagnetic mass differences between decuplet resonance states are either not known at all or have been measured within rather large experimental errors. In this situation, we may do one of two things. We may neglect electromagnetism, by assuming that the quark dynamics which is responsible for the mass differences is isospin invariant; in the group-theoretic approach this is equivalent to keeping only spurions transforming like $I=0$. In this case we obtain mass formulas involving "average" masses, where the particular average over the masses of the charge states in each isotopic multiplet is not clearly defined. This procedure was followed in Refs. 6 and 7. On the other hand, we can include $I\neq 0$ spurions, which in the quark model has the very natural meaning that we are including electromagnetic effects in the quark dynamics. We then obtain mass formulas which are a bit ahead of the experimental situation, but which leave no ambiguities about which charge states are involved. This is the procedure we have followed here.

In our example, we are treating the baryon states which are assigned to the **56** representation of $SU(6)$. The most general mass formula contains terms which transform like members of the representations contained in the reduction of the direct product:

$$\mathbf{56}\otimes\mathbf{56}=\mathbf{1}\oplus\mathbf{35}\oplus\mathbf{405}\oplus\mathbf{2695}.$$

The representations on the right-hand side contain a number of spurions with different transformation properties. We demand, of course, the conservation of angular momentum, charge and hypercharge. This restricts us to spurions which carry spin zero, $I_3=0$, and $Y=0$. The list of the available spurions is contained in Table I below.

We note that there are eighteen spurions in all, and thus the most general mass operator gives us eighteen linear equations for the baryon masses in terms of these eighteen spurion contributions. Our quark model leads us to the hypothesis that we can neglect the contribution of the three-quark interaction to the baryon mass differences, and that we can treat the two-quark contribution in lowest order. This hypothesis is equivalent to setting the contributions of the spurions belonging to the **2695** representation of $SU(6)$ equal to zero. We see from Table I that there are nine such spurions; hence we obtain nine mass formulas, Eqs. (1a)–(1i) above. To repeat, once the masses of the decuplet states are better known, it will be possible to invert the eighteen equations and to solve for the contributions of the eighteen spurions directly. We predict that the ordering of the magnitudes of the spurion contributions which has already been found to emerge experimentally for the $I=0$ spurions[12]

$$\mathbf{35}>\mathbf{405}>\mathbf{2695}$$

should persist for the $I\neq 0$ spurions. Our prediction is based on our model, in which this ordering—which from group theory is completely mysterious—is attributed to a hierarchy of interactions (three-body forces negligible compared to one-body and two-body forces, and one-body forces more effective than two-body forces) which

TABLE I. $SU(6)$ Spurions contributing to the baryon masses.

$SU(6)$ multiplet	$SU(3)$ representations and isotopic spins of available spurions. ($S=I_3=Y=0$)
1	(1,0)
35	(8,0)+(8,1)
405	(1,0)+(8,0)+(8,1)+(27,0)+(27,1)+(27,2)
2695	(8,0)+(8,1)+(27,0)+(27,1)+(27,2)+(64,0) +(64,1)+(64,2)+(64,3)

[11] T. K. Kuo and T. Yao, Phys. Rev. Letters **14**, 79 (1965); see also R. Faustov, Nuovo Cimento **45A**, 145 (1966).

[12] H. Harari and M. A. Rashid, Phys. Rev. **143**, 1354 (1966).

[13] H. Harari and H. J. Lipkin, Phys. Rev. Letters **14**, 570 (1965). A major conclusion of this work is that no *simple* assumption about the transformation property of the symmetry breaking interaction leads to successful mass relations: many spurions are nonzero, and not the same ones for mesons as for baryons. The quark model with one- and two-body matrix elements, on the other hand, seems to have the right degree of complexity. The model avoids the wrong mass formulas against which simple group-theoretic models founder.

should not depend on the presence or absence of electromagnetism.

Let us now return to the mass formulas (1a)–(1i). The first six involve relations among electromagnetic mass differences of particles belonging to the same isotopic multiplet.[14] Equation (1f), the last of these, is the Coleman-Glashow formula,[9] known to be well satisfied. Equations (1a)–(1e) all involve decuplet electromagnetic mass differences; their experimental determination is still at an early stage, but preliminary measurements are in general agreement. Equations (1g)–(1i) are refinements, in the sense discussed above, of the nonelectromagnetic relations already obtained in Ref. 6. Equation (1g) is a weaker form of the equal spacing law, Eq. (1h) is a well-known $SU(6)$ result, and Eq. (1i) is an interesting relation which equates the deviation from equal spacing in the decuplet to the violation of the Gell-Mann–Okubo formula in the octet. The right-hand side of Eq. (1i) is $+26\pm1$ MeV, while the left-hand side is almost surely positive, with magnitude not yet well known.

The masses of the pseudoscalar and vector mesons can be treated by the same model as the baryons. All masses, however, involve the interaction between a quark and an antiquark. Without additional assumptions there are no relations among the meson masses, nor are there relations connecting meson masses and baryon masses.

III. MAGNETIC MOMENTS AND MAGNETIC DIPOLE TRANSITIONS

A. Magnetic Moments of the Spin-$\frac{1}{2}$ Baryons

It has been shown by Thirring[15] that the magnetic moments of the baryons can be deduced in the quark model assuming (i) no orbital contributions and (ii) a magnetic moment for each quark proportional to its electric charge. We wish to relax the second assumption. We define

$$\mu_{\mathcal{P}}=\tfrac{2}{3}\mu_0{}^{\mathcal{P}};\quad \mu_{\mathfrak{N}}=-\tfrac{1}{3}\mu_0{}^{\mathfrak{N}};\quad \mu_\lambda=-\tfrac{1}{3}\mu_0{}^{\lambda},\tag{2}$$

where, to repeat, $\mathcal{P}$, $\mathfrak{N}$, λ denote the isodoublet and strange quarks, respectively. (We shall denote the physical proton, neutron, and lambda by p, n, Λ.) If the quarks are ordinary spin-$\frac{1}{2}$ objects obeying the Dirac equation, their magnetic moments should be measured in "quark magnetons":

$$\mu_0{}^i=e\hbar/2M_ic;\quad i=\mathcal{P},\,\mathfrak{N},\,\lambda.\tag{3}$$

It seems natural to assume that $\mu_0{}^{\mathcal{P}}$ and $\mu_0{}^{\mathfrak{N}}$ are equal but that they are different from $\mu_0{}^{\lambda}$. One then gets, besides the famous relation

$$\mu_p/\mu_n=-\tfrac{3}{2}\tag{4}$$

the following sum rule

$$\mu_\Lambda+3\mu_{\Sigma^+}=(8/3)\mu_p.\tag{5}$$

If, in addition, the assumption $\mu_0{}^{\mathcal{P}}=\mu_0{}^{\mathfrak{N}}=\mu_0{}^{\lambda}$ is made, one of course regains the well-known $SU(3)$ relations[9]

$$\mu_{\Sigma^+}=\mu_p,\tag{6}$$

$$\mu_\Lambda=\tfrac{1}{2}\mu_n.\tag{7}$$

The present experimental values are[16] (in units of the *nuclear* Bohr magneton):

$$\mu_{\Sigma^+}=\begin{cases}1.5\pm1.1\\4.3\pm1.5,\end{cases}\tag{8a}$$

$$\mu_\Lambda=-0.69\pm0.13.\tag{8b}$$

Our model predicts that relations (4) and (5) should be better satisfied than relations (6) and (7). The experimental error on the Σ^+ moment is too big for a definite test of these assumptions. However, we can use (8) to obtain a preliminary estimate of the magnitude of the U-spin violation, defining the parameter

$$\epsilon=(\mu_{\mathfrak{N}}-\mu_\lambda)/\mu_{\mathfrak{N}},\tag{9}$$

in terms of which

$$\mu_\Lambda/\mu_n=(1-\epsilon)/2,\tag{10a}$$

and

$$\mu_{\Sigma^+}/\mu_p=1-\tfrac{1}{9}\epsilon.\tag{10b}$$

Using only (10a) for a quantitative estimate, we get

$$\epsilon=0.28\pm0.14.\tag{11}$$

In the following subsection we will present a number of additional results in which a possible U-spin violation is parametrized by ϵ.[17]

B. Magnetic Moments of the Spin-$\frac{3}{2}$ Baryons

The magnetic moment of the Ω^- will ultimately be measured; the magnetic moments of the other members of the decuplet will probably not be measured since they are unstable with respect to the strong interactions. We wish to point out that the quark model gives the prediction

$$\mu_{\Omega^-}=3\mu_\Lambda,\tag{12}$$

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[14] Equations (1a)–(1f) are all equivalent to relations given in Ref. 10. The seventh equation there, $\Xi^{*-}-\Xi^{*0}=\Xi^--\Xi^0$, is lost when the electromagnetic interaction is made spin-dependent.

[15] W. Thirring, Notes from the Internationale Universitaetswochen fuer Kernphysik, Austria, March 1966 (unpublished).

[16] The two measurements of the Σ^+ moment are in V. Cook, T. Ewart, G. Masek, R. Orr, and E. Platner, Phys. Rev. Letters **17**, 223 (1966); A. D. McInturff and C. E. Roos, *ibid.* **13**, 246 (1964). The Λ moment is an average of several experiments given in A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965).

[17] The detection of a nonzero electromagnetic form factor for the neutral K-meson would be another indication of U-spin violation. We might imagine $F_{K^0}(t)=F_{\mathfrak{N}}(t)-F_\lambda(t)$, where $F_{\mathfrak{N}}(t)$ and $F_\lambda(t)$ are quark electric form factors; if the heavier λ quark has smaller spatial extension, we arrive at a picture of the K^0 with negatively charged outer shell and positively charged center. In the U-spin limit $F_{K^0}(t)=0$ because K^0 and $\bar{K}^0$ are in the same U-spin multiplet. A possible experiment to detect this form factor would be $e^+e^-\rightarrow K_1K_2$, in electron-positron colliding beams.

where we define the magnetic moment of a spin-$\frac{3}{2}$ object to be the expectation value of M_z, the z projection of the magnetic moment operator, in the state with $S_z=+\frac{3}{2}$. Using (8b) and (12) we predict

$$\mu_{\Omega^-}=-2.1\pm0.4 \text{ nuclear magnetons}, \tag{13a}$$

while using the U-spin prediction (7) and (12) gives

$$\mu_{\Omega^-}=-2.86 \text{ nuclear magnetons}, \tag{13b}$$

with a small uncertainty because (4) is not exact.

An additional prediction of this model is that the electric quadrupole moment and magnetic octupole moment of the Ω^- should vanish. This is a direct consequence of the spin-$\frac{1}{2}$ nature of the quarks, and of our hypothesis that the photons couple to individual quarks. A measurement of these higher moments of the Ω^- would be extremely interesting.

C. Electromagnetic Transitions from Spin-$\frac{3}{2}$ Baryons to Spin-$\frac{1}{2}$ Baryons

The electromagnetic decays of the spin-$\frac{3}{2}$ decuplet baryons into a spin-$\frac{1}{2}$ octet baryon and a photon may be experimentally measurable. The U-spin hypothesis places severe restrictions on these amplitudes, and predicts two rates to be zero. The quark-model relations among amplitudes are weaker, but the dependence of the amplitudes on the U-spin–violation parameter ϵ defined in Eq. (9) is such that the quark-model predictions are not very different from the exact U-spin predictions. However, the quark model makes two strong predictions about the individual decays: (i) It predicts that the electric quadrupole transition should not contribute since this is a "two-quark" effect,[18] and (ii) it predicts the absolute rates of the decays in terms of the nucleon magnetic moment. We present a detailed derivation of these decay rates below.

We begin by exhibiting the relations among amplitudes which follow from the quark model when we express these amplitudes as sums over single-quark transition amplitudes. Defining the transition moments according to the normalization (M_z couples to the z component of the magnetic field)

$$\mu(N^{*+},p)=\langle p, S_z=\tfrac{1}{2}|M_z|N^{*+}, S_z=\tfrac{1}{2}\rangle, \tag{14}$$

etc., we can express these moments in terms of the proton magnetic moment and the parameter ϵ, defined in Eq. (9), which characterizes the U-spin violation. For example,

$$\mu(N^{*+},p)=\tfrac{2}{3}\sqrt{2}\mu_p, \tag{15a}$$

$$\mu(Y_1^{*+},\Sigma^+)=\tfrac{2}{3}\sqrt{2}(1-\tfrac{1}{3}\epsilon)\mu_p. \tag{15b}$$

(We have again used $\mu_{\mathcal{P}}=-2\mu_{\mathcal{N}}$.)[19] The other six transition moments can be expressed in terms of these two. We define the proportionality constants α_{DB} by

$$\mu(D,B)=\alpha_{DB}\mu(N^{*+},p), \tag{16}$$

where D, B are the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ baryons, respectively, and we present the values of α_{DB} in Table II.

We may now calculate the absolute rates for these decays, following the same principles which were used by Becchi and Morpurgo[20] to calculate successfully the rate for $\omega\to\pi^0\gamma$. The assumption that the vertex function varies little in spite of the presence of mass differences is discussed in Ref. 20; this approximation should be much less severe here where the mass differences are smaller.

We begin with the most general relativistic gauge-invariant amplitude; it has two terms, corresponding to the possibility of magnetic dipole and electric quadrupole transitions. We will make this identification later, after first giving the general result. The transition amplitude is (with the usual normalization factors suppressed):

$$\bar{u}_\mu(p)[a\gamma_\nu/M+bp_\nu/M]\gamma_5u(q)F_{\mu\nu}, \tag{17a}$$

$$F_{\mu\nu}=k_\mu\epsilon_\nu-k_\nu\epsilon_\mu, \tag{17b}$$

where $u_\mu(p)$ is a 16-component Rarita-Schwinger spinor for the spin-$\frac{3}{2}$ baryon of momentum p and mass M, q, and m are the momentum and mass of the spin-$\frac{1}{2}$ baryon, k_μ, ϵ_μ are the momentum and polarization vectors of the photon, and $k+q=p$. We have arranged for a and b to be dimensionless. As in many cases, the average over spin projections for the Rarita-Schwinger spinors involves only the *symmetric* sum:

$$\sum_{r=1}^{4}\tfrac{1}{2}[u_\mu^{(r)}(p)\bar{u}_\nu^{(r)}(p)+u_\nu^{(r)}(p)\bar{u}_\mu^{(r)}(p)] =\frac{p\!\!\!/+M}{2M}\frac{2}{3}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{M^2}\right), \tag{18}$$

where in our metric $g_{if}=-\delta_{if}$ for i, $f=1$, 2, 3. Averaging over the four spin states of the spin-$\frac{3}{2}$ particle and summing over the spins and polarizations of the final particles, we obtain for the decay rate

$$\Gamma=\frac{k_c^3}{12\pi M^2}[|a|^2(3+\lambda^2)+(\mathrm{Re}\,a^*b)(3-\lambda)\times(1-\lambda)+|b|^2(1-\lambda)^2], \tag{19a}$$

where

$$\lambda=m/M, \tag{19b}$$

and

$$k_c=M(1-\lambda^2)/2 \tag{19c}$$

is the momentum in the center-of-mass system.[21]

[18] C. Becchi and R. Morpurgo, Phys. Letters **20**, 864 (1965).

[19] Equation (15a) is also an $SU(6)$ result when the photon is assumed to transform like a member of the **35**. It was first derived by M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).

[20] C. Becchi and R. Morpurgo, Phys. Rev. **140**, B687 (1965).

[21] The amplitude for the electromagnetic decay of a spin-$\frac{3}{2}$ baryon into a spin-$\frac{1}{2}$ baryon of *opposite* parity may be represented by (17a) with γ_5 removed, whereupon the decay rate is given by (19a) with the substitution $\lambda\to-\lambda$.

TABLE II. The constants α_{DB} and the rates $\Gamma(D\to B\gamma)$.

D B	N^{*+} p	N^{*0} n	Y_1^{*0} Λ	Y_1^{*+} Σ^+	Y_1^{*0} Σ^0	Y_1^{*-} Σ^-	Ξ^{*0} Ξ^0	Ξ^{*-} Ξ^-
α_{DB}	1	-1	$\sqrt{3}/2$	$1-\frac{1}{3}\epsilon$	$\frac{1}{2}-\frac{1}{3}\epsilon$	$-\frac{1}{3}\epsilon$	$-1+\frac{1}{3}\epsilon$	$\frac{1}{3}\epsilon$
$\Gamma(D\to B\gamma)$ (MeV)	0.40	0.40	0.26	0.12	0.025	0.0012	0.17	0.0016

The quark model makes two predictions about the parameters a and b: It says first that the transition should be purely magnetic dipole. To identify a and b with $M1$ and $E2$ transition amplitudes, we rewrite (17) in the center-of-mass frame $p=(M,0)$:

$$u_{ix}^\dagger(p)(\sigma_j)_{xy}u_y(q)T_{ij}\,, \tag{20a}$$

$$T_{ij}=\frac{v}{M}(\epsilon_ik_j-\epsilon_jk_i)+\frac{w}{M^2}(\epsilon_ik_jk_0+\epsilon_jk_ik_0-2\epsilon_0k_ik_j)\,, \tag{20b}$$

$$v=[a(3+\lambda)+b(1-\lambda)]/(2+2\lambda)\,, \tag{20c}$$

$$w=(a+b)/(1+\lambda)^2. \tag{20d}$$

In Eq. (20a) the indices x and y take on the values 1, 2; u_{ix} is the six-component form of the Rarita-Schwinger field at rest. By inspecting Eq. (20b) we see that v and w are associated with the $M1$ and $E2$ transitions, respectively. We obtain a pure $M1$ transition if $w=0$, i.e., if $a+b=0$ and $v=a$.

Substituting (20c) and (20d) into Eq. (19a), we obtain the decay rate in terms of v and w:

$$\Gamma=\frac{k_c^3(1+\lambda)^2}{12\pi M^2}[|v|^2+3(k_c^2/M^2)|w|^2]. \tag{21}$$

We note that there is no "cross term" in Eq. (21) since the $M1$ and $E2$ transitions do not interfere in the total rate.

The quark model also tells us the strength of the $M1$ transition. We have only to translate Eq. (14) into a statement about v. This is

$$2v_{DB}/M\sqrt{6}=\mu(D,B). \tag{22}$$

Thus, the quark-model predictions (22) and $w=0$ give finally

$$\Gamma(D\to B+\gamma)=\alpha_{DB}^2k_c^3(1+\lambda)^2\mu_p^2/9\pi\,, \tag{23}$$

where we have used Eqs. (15) and (16) to rewrite $\mu(D,B)$. The evaluation of (23) using physical masses gives the results listed in Table II. We have used the central value of (11), $\epsilon=0.28$, in obtaining the results.

Attention is drawn to the rates for the decays of the resonances carrying negative charge, $Y_1^{*-}\to\Sigma^-\gamma$ and $\Xi^{*-}\to\Xi^-\gamma$. These rates vanish in the U-spin limit. (The argument is trivial: the states with negative charge in the decuplet have $U=\frac{3}{2}$, in the octet have $U=\frac{1}{2}$ and the photon has $U=0$, so the transitions do not occur.) We see that with the value of ϵ given by (11) the departure from the U-spin selection rule is extremely small.

The transition rates are not negligibly small. In particular, we remark that the Ξ^* width is only 7.5 ± 1.7 MeV, so the Ξ^{*0} should decay electromagnetically approximately one time in forty. The ratio

$$\rho=\Gamma(\Xi^{*-}\to\Xi^-\gamma)/\Gamma(\Xi^{*0}\to\Xi^0\gamma) \tag{24}$$

should therefore be measurable and it will be interesting to see whether it is indeed as small as our estimates suggest ($\rho=0.01$). It will also be interesting to study the angular distributions of these electromagnetic decays, to discover whether the $E2$ term is indeed absent.[22]

D. Electromagnetic Transitions from Vector Mesons to Pseudoscalar Mesons

One of the most exciting calculations performed with the quark model was the successful calculation of the decay rate $\Gamma(\omega\to\pi^0\gamma)$ by Becchi and Morpurgo.[20] The rate is obtained in terms of the nucleon magnetic moment under the assumption, which we have used consistently in this section, that the amplitude for the decay can be written as a sum over single-quark transition amplitudes. They also assume that the ω particle has no $\lambda\bar{\lambda}$ component. Using the relativistic expression for phase space, they obtain

$$\Gamma(\omega^0\to\pi^0\gamma)=\mu_p^2k^3/3\pi=1.2\text{ MeV}. \tag{25}$$

Here $\mu_p=2.79e/2M_p$ and k is the center-of-mass momentum. The experimental value is 1.4 ± 0.4 MeV which is in excellent agreement with (25).

Becchi and Morpurgo also present estimates of the other decay rates, assuming U-spin conservation. For completeness and in the spirit of the previous discussion, we wish to note the effect of the U-spin violation suggested by the quark model. For example consider the decays $K^{*0}\to K^0\gamma$ and $K^{*+}\to K^+\gamma$. The quark-model

[22] The γ-N-N^* vertex has been studied in photoproduction of pions near the N^* resonance by M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 and 309 (1963) and Ph. Salin, *ibid.* **28**, 1294 (1963). Our model [and $SU(6)$] predicts $v=0.65$, $w=0.0$. They find $v=1.07$, $w=0.36$. There have been a number of more recent investigations of this vertex, all pointing to a value of v somewhat larger than the $SU(6)$ prediction: J. Mathews, Phys. Rev. **137**, B444 (1965); R. H. Dalitz, D. G. Sutherland, *ibid.* **146**, 1180 (1966); A. Donnachie and G. Shaw, Ann. Phys. **37**, 333 (1966). A larger value of v would make the electromagnetic decays easier to observe.

prediction for the ratio of the rates is

$$r=\Gamma(K^{*0}\rightarrow K^0\gamma)/\Gamma(K^{*+}\rightarrow K^+\gamma)=[(\mu_{\mathcal{N}}+\mu_\lambda)/(\mu_{\mathcal{P}}+\mu_\lambda)]^2=[(2-\epsilon)/(1+\epsilon)]^2, \tag{26}$$

where we have used $\mu_{\mathcal{P}}=-2\mu_{\mathcal{N}}$ and Eq. (9). Using the value of ϵ provided by the present value of the Λ magnetic moment (11), we predict

$$r=1.8\pm0.5, \tag{27}$$

which is very different from the well-known U-spin prediction ($\epsilon=0$) that $r=4$. This prediction of our model would be particularly interesting to test, in view of the absence of uncertainties about phase space factors which might otherwise complicate the interpretation of the results.

However, the predicted absolute rates for these K^* decays are quite small:

$$(K^{*0}\rightarrow K^0\gamma)=(2-\epsilon)^2\mu_p{}^2k^3/27\pi=0.21\text{ MeV}, \tag{28a}$$

$$(K^{*+}\rightarrow K^+\gamma)=(1+\epsilon)^2\mu_p{}^2k^3/27\pi=0.11\text{ MeV}, \tag{28b}$$

and may be beyond detection.

Analogous vector meson decays involving the ω and φ and the X^0 and η are an interesting place to look for the detailed quark structure of these particles.[23] The "ω-φ mixing" problem is now commonly regarded as "explained" by the fact that the ω is made of nonstrange quarks while the φ is made of strange quarks only. We would like to suggest that the η-X^0 mixing problem may be resolved in the same way, with η made (approximately) of pure strange quarks. With this simple assignment we forbid the decays $\varphi\rightarrow\pi^0\gamma$, $\varphi\rightarrow X^0\gamma$, $\rho^0\rightarrow\eta\gamma$, and $\omega\rightarrow\eta\gamma$, none of which has ever been seen, while we predict[24]

$$\Gamma(X^0\rightarrow\rho^0\gamma)=\mu_p{}^2k^3/\pi=0.34\text{ MeV}, \tag{29a}$$

$$\Gamma(X^0\rightarrow\omega\gamma)=\mu_p{}^2k^3/9\pi=0.03\text{ MeV}, \tag{29b}$$

$$\Gamma(\varphi\rightarrow\eta\gamma)=4(1-\epsilon)^2\mu_p{}^2k^3/27\pi=0.24\text{ MeV}. \tag{29c}$$

[In (29c) we have used $\epsilon=0.28$.]

This simplified assumption about the quark structure of the η and X^0 can obviously be tested in many different experiments. (For example, we would expect more production of X^0 than of η in p-$\bar{p}$ annihilation.[25]) We do not regard the Gell-Mann–Okubo (GMO) formula as an argument against this classification since (i) the GMO formula for the pseudoscalar mesons does not arise in a natural way in the quark model, and (ii) once $\eta-X^0$ mixing is admitted, there is no reason to expect the GMO formula to be valid, and indeed the extent of mixing depends strongly on the power of the mass used in the mass formula.[26]

IV. PHOTOPRODUCTION OF MESONS AND BARYONS

We consider reactions of the type

$$\gamma+N\rightarrow M+B, \tag{30}$$

where M represents a pseudoscalar or vector meson, N a nucleon, and B any octet or decuplet baryon. The photon, in spite of having many of the properties of a vector meson, is clearly not a bound state of quarks. There is therefore a certain ambiguity in defining the dominant mechanism for reaction (30). One might assume that the photon is first transformed into a (virtual) vector meson which then interacts with the target nucleon via the reaction mechanism used for meson-baryon and baryon-baryon reactions. Since this intermediate meson is off the energy shell it is not clear however whether the simple additivity assumption of quark amplitudes still holds and what kind of form factor has to be applied to the meson-baryon vertex. On the other hand, in the spirit of the quark model, it seems more natural to assume that the photon interacts *directly* with each individual quark rather than via an intermediate virtual vector meson. We therefore would like to propose the following two mechanisms for reactions of type (30):

(i) The photon is *absorbed* by one of the quarks of the target proton, thus forming an intermediate excited state of the nucleon. The final state of reaction (30) is then reached through emission, after a certain time, of the meson M by either the same or another quark of the

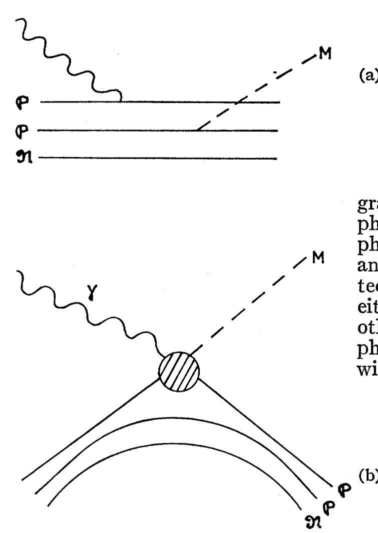

FIG. 1. Possible diagrams for the quark-photon vertex; (a) the photon is first absorbed, and the meson is emitted, after a while, by either the same or another quark; (b) the photon interacts directly with each quark.

[23] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965).

[24] These rates in fact do not differ substantially from those predicted by S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965) on the basis of nonet coupling and a 10° η-X^0 mixing angle.

[25] Present experimental results are ambiguous: the observed η production in π reactions argues against the assignment $\eta=(\lambda\bar{\lambda})$ (see Ref. 5) while the observed suppression of the decay $A_2\rightarrow\pi\eta$ is a point in its favor (see Ref. 24).

[26] A. J. Macfarlane and R. H. Socolow, Phys. Rev. **144**, 1194 (1966).

nucleon. This reaction mechanism is represented schematically in Fig. 1(a).

(ii) The *direct* photoproduction of the meson M on a single quark, i.e., the process

$$\gamma+q\rightarrow M+q' \tag{31}$$

is possible and the photon-baryon reaction amplitude (30) is given by the sum of all possible photon-quark amplitudes of the type (31). This is represented schematically in Fig. 1(b).

The reaction via a mechanism of type (i) clearly leads to an (approximately) isotropic distribution of the produced particles. In a reaction of type (ii) however, which is analogous to the meson-baryon reactions discussed earlier,[5] the meson will be scattered mainly in the forward direction. Experimentally it is well known that at sufficiently high energies all reactions of type (30) are strongly forward-peaked.[27] In this energy region the process (ii) thus seems to be predominant over the competing process (i). We will therefore concentrate on this second, direct mechanism for photoreactions at high energies and neglect possible contributions from the diagram in Fig. 1(a). Furthermore, we assume that in reaction (30) the total z component of the spin (or the total helicity) in the initial state (photon-baryon) is conserved. This assumption appears reasonable since all reactions are strongly forward peaked.

No further assumptions are made about the photon-quark vertex itself. The vertex in Fig. 1(b) may contain any diagram which is compatible with the peripheral nature of the production process. In particular, it may contain, among others, the diagram discussed above where the photon is first transformed into a virtual meson. We need not, however, assume that this is the dominant mechanism.

Using the known quark-model wave functions for the baryons, any reaction amplitude (30) can then be expressed in terms of quark photo-amplitudes like, for example,

$$\langle\gamma_1\mathcal{P}_{1/2}|\rho_1{}^+\mathfrak{N}_{1/2}\rangle,\quad \langle\gamma_1\mathcal{P}_{-1/2}|\rho_0{}^+\mathfrak{N}_{1/2}\rangle,\quad \langle\gamma_1\mathcal{P}_{-1/2}|K^+\lambda_{1/2}\rangle,$$

where $\mathcal{P}$, $\mathfrak{N}$, λ denote the three quarks and the indices denote either the z component of the spin or the helicity. These quark amplitudes are unknown parameters for which invariance under a particular symmetry like, e.g., U spin may or may not be assumed. The cross sections for any $p+\gamma$ reaction are then expressed in terms of squares of these amplitudes, averaging over all initial and summing over all final polarization states. Relations between different cross sections are obtained if it is possible to eliminate the squares of the amplitudes.

[27] Y. Eisenberg (private communication).

A. Relations for the Production of Pseudoscalar Mesons

The relations for pseudoscalar mesons are particularly simple since the cross sections can all be expressed in terms of one single amplitude.[28] For example the cross section for the reaction

$$\gamma+p\rightarrow\pi^++n$$

only depends on the amplitude $\langle\gamma_1\mathcal{P}_{-1/2}|\pi^+\mathfrak{N}_{+1/2}\rangle$. We find the following relations:

$$\bar\sigma(\gamma p\rightarrow\pi^+n):\bar\sigma(\gamma n\rightarrow\pi^+N^{*-}):\bar\sigma(\gamma p\rightarrow\pi^+N^{*0}) =25:24:8\,, \tag{32}$$

$$\bar\sigma(\gamma p\rightarrow K^+\Lambda):\bar\sigma(\gamma n\rightarrow K^+Y_1{}^{*-}): \bar\sigma(\gamma p\rightarrow K^+Y_1{}^{*0}):\bar\sigma(\gamma n\rightarrow K^+\Sigma^-): \bar\sigma(\gamma p\rightarrow K^+\Sigma^0)=27:16:8:2:1\,, \tag{33}$$

$$\bar\sigma(\gamma n\rightarrow K^0\Lambda):\bar\sigma(\gamma p\rightarrow K^0Y_1{}^{*+}):\bar\sigma(\gamma n\rightarrow K^0Y_1{}^{*0}): \bar\sigma(\gamma p\rightarrow K^0\Sigma^+):\bar\sigma(\gamma n\rightarrow K^0\Sigma^0)=27:16:8:2:1\,, \tag{34}$$

$$\bar\sigma(\gamma n\rightarrow\pi^-p):\bar\sigma(\gamma p\rightarrow\pi^-N^{*++}):\bar\sigma(\gamma n\rightarrow\pi^-N^{*+}) =25:24:8\,, \tag{35}$$

$$\bar\sigma(\gamma p\rightarrow\pi^0N^{*+})=\bar\sigma(\gamma n\rightarrow\pi^0N^{*0})\,. \tag{36}$$

The relation (36) also holds if π^0 is replaced by η or X^0.

It is important to notice that in none of these relations invariance under any symmetry has been assumed. In particular, we did not make use of the usual U-spin classification of the photon. Its transformation properties under U spin or more generally under $SU(3)$ are irrelevant. If however the photon is assumed to be a scalar under U spin and if the photoreaction amplitude is invariant under U spin, then Eq. (32) and (33) are related through

$$\bar\sigma(\gamma p\rightarrow\pi^+N^{*0}):\bar\sigma(\gamma p\rightarrow K^+\Sigma^0)=16:1\,. \tag{37}$$

In particular, if U-spin invariance is assumed, one easily verifies that the reaction amplitudes satisfy the known U-spin relations derived by Levinson, Lipkin, and Meshkov.[8]

As discussed in the context of meson-baryon reactions,[5] the *experimental* cross sections are related to the quantities $\bar\sigma=\sum|M|^2$ for which our relations hold, through the relation

$$\sigma_{\text{experiment}}=\bar\sigma p_{\text{out}}\exp(-Kq^2)/p_{\text{in}}s=F\bar\sigma\,. \tag{38}$$

Here p_{out}, p_{in} denote the center-of-mass momenta of the outgoing and the incoming particles respectively, s denotes the square of the center-of-mass energy and q is the three-momentum transfer in the forward direction:

$$q=p_{\text{in}}-p_{\text{out}}\,.$$

The factor $\exp(-Kq^2)$ represents the *baryon* form-factor with the constant K determined from the ex-

[28] While completing this manuscript, we received a report by J. Kupsch, now published in Phys. Letters **22**, 609 (1966), with results similar to those in this section.

TABLE III. Kinematic corrections for the photoproduction cross sections of pseudoscalar mesons.

Reaction	$Q=1.0$ GeV		$Q=1.5$ GeV		$Q=2.0$ GeV	
	F^{-1} [GeV^2]	p^{lab} [GeV/c]	F^{-1} [GeV^2]	p^{lab} [GeV/c]	F^{-1} [GeV^2]	p^{lab} [GeV/c]
$\gamma N \to \pi N$	4.37	1.84	6.69	3.08	9.52	4.59
$\gamma N \to \pi N^*$	7.30	2.54	9.76	3.94	12.80	5.61
$\gamma N \to K\Lambda$	8.53	3.16	11.13	4.68	14.36	6.47
$\gamma N \to K\Sigma$	9.73	3.38	12.29	4.94	15.54	6.77
$\gamma N \to KY_1^*$	14.01	3.95	16.19	5.61	19.36	7.55

TABLE IV. Kinematic corrections for the photoproduction cross sections of vector mesons.

Reaction	$Q=1.0$ GeV		$Q=1.5$ GeV		$Q=2.0$ GeV	
	F^{-1} [GeV^2]	p^{lab} [GeV/c]	F^{-1} [GeV^2]	p^{lab} [GeV/c]	F^{-1} [GeV^2]	p^{lab} [GeV/c]
$\gamma N \to K^*\Sigma$	16.78	4.60	18.60	6.37	21.72	8.41
$\gamma N \to K^*\Lambda$	14.66	4.35	16.79	6.08	20.02	8.09
$\gamma N \to K^*Y_1^*$	24.52	5.25	24.81	7.12	27.36	9.27
$\gamma N \to \rho N^*$	15.07	4.33	17.14	6.07	20.34	8.06

perimental data on elastic meson-baryon and baryon-baryon scattering. This is discussed in some detail in Ref. 5. The other factors are the known phase-space corrections. Related cross sections should, as usual, be compared at the same Q values, where Q is the kinetic energy of the outgoing particles in the center-of-mass system. In Table III we list the factor F^{-1}, for three different Q values, by which the *experimental* data should be multiplied before being compared with our predictions. Note that in all of these relations the kinematic corrections are usually quite small, the mass differences between initial and final state being in most cases the same for the related reactions. These relations therefore should provide a sensitive test of the model.

B. Relations for the Production of Vector Mesons

The cross sections for vector mesons depend in general on more than one amplitude and the elimination of the unknown parameters is possible only in a few cases. Some examples are

$$\bar{\sigma}(\gamma n \to K^{*+}\Sigma^-) = 2\bar{\sigma}(\gamma p \to K^{*+}\Sigma^0)\,, \tag{39}$$

$$\bar{\sigma}(\gamma p \to K^{*+}\Lambda) = 3\bar{\sigma}(\gamma p \to K^{*+}\Sigma^0) + 3\bar{\sigma}(\gamma p \to K^{*+}Y_1^{*0})\,, \tag{40}$$

$$\bar{\sigma}(\gamma n \to \rho^+ N^{*-}) = 3\bar{\sigma}(\gamma p \to \rho^+ N^{*0})\,, \tag{41}$$

$$\bar{\sigma}(\gamma p \to K^{*0}\Sigma^+) = 2\bar{\sigma}(\gamma n \to K^{*0}\Sigma^0)\,. \tag{42}$$

Again, in these relations no assumption about U-spin invariance or the classification of the photon has been made. If, however, we assume invariance under U spin, we get the additional relation

$$\bar{\sigma}(\gamma p \to \rho^+ N^{*0}) = 2\bar{\sigma}(\gamma p \to K^{*+}Y_1^{*0})\,. \tag{43}$$

In Table IV we give the phase-space and form-factor corrections for the cross sections appearing in relations (39) to (43) at three different Q values.

Unfortunately there is as yet very little experimental information available for a detailed comparison of these relations with experiment.[27] Since our relations provide a rather sensitive test of the quark model and the dynamical assumption (i), an accurate measurement of these cross sections seems desirable.

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