Aside from testing the accuracy of the expansion against the exact two-body T matrix as shown in Table I, we also test the sensitivity of the three-body solution to the expansion. This is done in two ways. First, we compare the required coupling strength G with the three-term expansion against that with the five-term expansion for a fixed binding energy. The difference is within a few percent. For the second test we take a simple function for the inhomogeneous term $\phi(p,q)$ and solve for $\Psi(p,q)$ using a five-term expansion for the two-body T matrix. Having obtained $\Psi(p,q)$, we substitute it into the right-hand side of the original Faddeev equation (27) with the kernel given now by the exact T matrix. This gives a function $\bar{\Psi}(p,q)$ which is generally within 2% of the solution obtained by the five-term expansion.

The above results indicate that the Faddeev equation, along with the expansion method, is a practical tool for calculating the three-body wave function as well as the energy levels. For the two-body interaction in the higher orbital states, the expansion of Ref. 5 can be applied along with our treatment of the off-shell T matrix. Finally, we remark that the scattering of a two-body bound state by a third particle can also be handled in essentially the same way as above except that the kernel of the integral equation will be complex and the numerical solution will involve the inversion of a complex matrix.

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Example of an "Inelastic" Bound State*

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An example is given of a bound state which occurs in a channel with a repulsive Born approximation. The bound state occurs because of the attraction provided at low energy by three-particle intermediate states.

I. INTRODUCTION

FOR some time, physicists have speculated that inelastic channels in particle collisions might give rise to resonances or bound states. Such a mechanism is well known in nuclear physics,[1] and it has been applied to particle physics by several authors, including, for instance, Cook and Lee.[2] These authors performed a matrix N/D calculation to see if the higher nucleon resonances might be driven by the opening of the ρN channel. The interest in such a mechanism stems from the fact that particle exchange on the left is not always attractive in the channels where resonances are known to occur. Put in modern terminology, there is sometimes a breakdown in naive bootstrap philosophy, which assumes that elastic unitarity and the crossing matrix are sufficient principles for the prediction of resonances. The inelasticity mechanism is invoked as a cure for the breakdown of bootstrap theory.

Unfortunately, all past attempts to assess the effects of inelasticity have been marred by an enormous number of approximations and simplifications. It has never been clear whether it was the inelasticity or the approximations which produced the resonances. In the present paper, we wish to correct this situation by presenting a model calculation in which the dynamics are carefully evaluated, without important approximations other than the exclusion of states involving more than three particles. Specifically, our scattering amplitudes will have the hallowed properties of analyticity, crossing symmetry, and unitarity

We study a reaction in which the single-particle exchange poles provide a repulsive force. Corresponding to this, when we construct a scattering amplitude satisfying crossing and elastic unitarity (one-meson approximation), no bound state appears. This is in agreement with naive bootstrap theory. However, when we construct a scattering amplitude satisfying crossing and two- and three-particle unitarity (two-meson approximation), a bound state appears when the coupling is sufficiently strong. The bound state can only be a result of inelasticity because we have the elastic calculation for comparison. In addition, the development of a *bound* state indicates that inelasticity can affect the low energy properties of scattering amplitudes. Contrary to popular belief, inelastic effects are not limited to energies where three-particle phase space is large.

II. THE MODEL

The model we study is the charged scalar static model. This model has a spin-zero source which can exist in either positive (p) or neutral (n) charge states,

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[1] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chaps. VIII–X.
[2] L. F. Cook, Jr., and B. W. Lee, Phys. Rev. **127**, 283 (1962); **127**, 297 (1962).

and which emits charged mesons (π^+ and π^-) in S waves with conservation of charge. There are two elastic scattering amplitudes: $A_+(\omega)$, which refers to π^+p and π^-n scattering, and $A_-(\omega)$, which refers to π^-p and π^+n scattering. ω is the meson energy. We are principally interested in the amplitude A_-.

We denote the one-meson solutions for $A_\pm$ by $M_\pm$. They satisfy crossing and elastic unitarity, and were originally given by Castillejo, Dalitz, and Dyson (CDD).[3] In the present paper, we choose the one meson solutions which have no CDD poles:

$$M_-(\omega)=-\frac{g^2\omega^{-1}}{1-\alpha(\omega)},$$
$$\alpha(\omega)=-\frac{2\omega g^2}{\pi}\int_\mu^\infty \frac{d\omega_1 k_1 u^2(\omega_1)}{4\pi\omega_1(\omega_1^2-\omega^2-i\epsilon)}, \qquad (1)$$
$$M_+(\omega)=M_-(-\omega-i\epsilon).$$

Here g is the meson-source coupling constant, μ is the meson mass, $k=[\omega^2-\mu^2]^{1/2}$ is the meson momentum, and $u^2(\omega)$ is the cutoff function. For sufficiently large g, $\alpha(\mu)<-1$, so that M_+ has a bound state B. This is reasonable in view of the attractive character of n exchange in π^+p scattering. On the other hand, M_- never has a bound state, which is the conventional conclusion from the repulsive character of the direct n pole in π^-p scattering. (The contribution of B^{++} exchange to π^-p scattering is attractive, but never sufficiently so to overcome the repulsive direct n pole and produce a bound state in M_-.)

At this point, we introduce the relation between $M_\pm$ and the one-meson phase shifts $\delta_\pm$:

$$\frac{ku^2(\omega)}{4\pi}M_\pm(\omega)=e^{i\delta_\pm(\omega)}\sin\delta_\pm(\omega). \qquad (2)$$

We can then define the Omnes functions $\Delta_\pm(z)$:

$$\Delta_\pm(z)=\exp\left[\frac{z}{\pi}\int_\mu^\infty \frac{d\omega\ \delta_\pm(\omega)}{\omega(\omega-z)}\right]. \qquad (3)$$

It is possible to represent M_- in terms of these functions.[5] When the bound state B is present, the representation is

$$M_-(\omega)=-\frac{g^2\omega_B(\omega+\mu)}{\mu\omega(\omega+\omega_B)}\Delta_-(\omega+i\epsilon)\Delta_+(-\omega), \qquad (4)$$

where ω_B is the energy of the bound state. By examining the residue of M_- at $\omega=-\omega_B$, we can determine the meson–source–bound-state coupling constant g_B:

$$g_B{}^2=g^2\frac{\mu-\omega_B}{\mu}\Delta_+(\omega_B)\Delta_-(-\omega_B). \qquad (5)$$

Two-meson solutions for $A_\pm$ have been given by the author. There are two versions of the solutions: First, for the case that the bound state B is absent in M_+,[4] and, second, for the case when B is present.[5] These solutions do not have free parameters analogous to CDD parameters in them, and therefore they are the two-meson companions to $M_\pm$. We denote the two-meson approximations to $A_\pm$ by $T_\pm$. The $T_\pm$ satisfy the same crossing relations as $M_\pm$ [Eq. (1)], and they satisfy two- and three-particle unitarity. By this we mean that production and six point amplitudes are calculated which satisfy appropriate dispersion relations, and the amplitudes fit together with $T_\pm$ to form an unitary two- and three-particle scattering matrix. The distinction between the two versions of $T_\pm$ lies in the fact that $M_\pm$ are used to describe final state interactions in three-particle states. Consequently, in the state $\pi^+\pi^-n$, which is connected to $p\pi^-$ by a production amplitude, the π^-n system can coalesce into a bound state B^-. Therefore, the version of T_- for the case that B is present in M_+ includes an inelastic two particle cut coming from the π^+B^- state. The new cut appears automatically when the weak coupling forms of $T_\pm$ are analytically continued in g, and the enlarged scattering matrix remains unitary when the new channel appears. In the following calculation we shall need the form of T_- which holds when M_+ has a bound state.

T_- has the form

$$T-(\omega)=-\frac{g^2\omega^{-1}}{[1+\omega C(-\omega)][1-\omega C(-\omega)]^{-1}-\alpha(\omega)}, \qquad (6)$$

where

$$C(-\omega)=\frac{1}{\pi}\int_{\mu+\omega_B}^\infty \frac{d\omega_1\rho_-{}^B(\omega_1)}{\omega_1(\omega_1-\omega-i\epsilon)}+\frac{1}{\pi}\int_{2\mu}^\infty \frac{d\omega_1\rho_-{}^T(\omega_1)}{\omega_1(\omega_1-\omega-i\epsilon)}$$
$$+\frac{1}{\pi}\int_{2\mu}^\infty \frac{d\omega_1\rho_+(\omega_1)}{\omega_1(\omega_1+\omega)}. \qquad (7)$$

The weight functions are

$$\rho_-{}^B(\omega)=\frac{g_B{}^2\omega_B{}^2\bar{k}u^2(\bar{\omega})\Delta_-{}^2(-\omega)\,|\,\Delta_-(\bar{\omega})\,|^2}{2\pi\omega\bar{\omega}^2\Delta^2{}_-(-\omega_B)},$$

$$\rho_-{}^T(\omega)=\frac{g^4\omega_B{}^2\Delta_-{}^2(-\omega)}{8\pi^3\mu^2\omega}\int_\mu^{\omega-\mu} d\omega_1 k_1 k_{-1} u^2(\omega_1)u^2(\omega_{-1})$$
$$\times\left|\frac{(\omega_1-\mu)}{\bar{\omega}_1\omega_{-1}}\Delta_+(\omega_1)\Delta_-(\omega_{-1})\right|^2, \qquad (8)$$

$$\rho_+(\omega)=\frac{g^4\omega\omega_B{}^2(\omega+\mu)^2}{16\pi^3\mu^2(\omega+\omega_B)^2}\Delta^2{}_+(-\omega)\int_\mu^{\omega-\mu} d\omega_1 k_1 k_{-1} u^2(\omega_1)$$
$$\times u^2(\omega_{-1})\left|\frac{\Delta_-(\omega_1)\Delta_-(\omega_{-1})}{\omega_1\omega_{-1}}\right|^2,$$

[3] L. Castillego, R. Dalitz and F. Dyson, Phys. Rev. **101**, 453 (1956).

[4] J. B. Bronzan, J. Math. Phys. **7**, 1351 (1966).

[5] J. B. Bronzan, J. Math. Phys. **8**, 6 (1967).

where $\bar{\omega}=\omega-\omega_B$, $\bar{\omega}_1=\omega_1-\omega_B$, $\omega_{-1}=\omega-\omega_1$, $\bar{k}=[\bar{\omega}^2-\mu^2]^{1/2}$ and $k_{-1}=[\omega_{-1}^2-\mu^2]^{1/2}$. Note that when we set $C=0$, the one-meson solution is recovered. ρ_-^B gives the contribution of π^+B^- intermediate states to T_-, ρ_-^T gives the contribution of $\pi^+\pi^-n$ intermediate states, and ρ_+ gives the contribution of $\pi^+\pi^+n$ exchange. π^-p intermediate states and π^+p exchange contribute to α, as in the one meson solution.

A bound state of T_- occurs if the denominator of Eq. (6),

$$D(\omega)=\frac{1+\omega C(-\omega)}{1-\omega C(-\omega)}-\alpha(\omega)\,, \tag{9}$$

increases through zero between $\omega=0$ and $\omega=\mu$. (A zero of D between $\omega=-\mu$ and $\omega=0$ is a bound state in the T_+ channel. We know that such a bound state exists, analogous to B in M_+.) In the one meson approximation, $C=0$, D is always greater than 1 between $\omega=0$ and $\omega=\mu$, so M_- has no bound state. However, if C is large, D develops a pole, and it can then increase through zero (see Fig. 1). The conditions for T_- to have a bound state are

$$\begin{aligned}&\mu C(-\mu)>1\,,\\ &\frac{1+\mu C(-\mu)}{1-\mu C(-\mu)}-\alpha(\mu)>0\,,\end{aligned} \tag{10}$$

These inequalities can be satisfied only if $\alpha(\mu)<-1$, which is the condition that M_+ have a bound state B. Therefore, the bound state in T_- cannot develop until M_+ has developed a bound state. A sufficient condition for T_- to develop a bound state is that

$$C(-\mu)\xrightarrow[g^2\to\infty]{}+\infty\,.$$

In the next section we shall show that this behavior occurs.

We point out that the pole of D (see Fig. 1) is a CDD pole, since it corresponds to a zero of T_-. It is induced by the coupling to the inelastic channels, and is an example of a dynamically determined CDD pole of the type noted by Bander, Coulter, and Shaw.[6]

III. THE BOUND STATE IN T_-

We have seen that T_- has a bound state if

$$C(-\mu)\xrightarrow[g^2\to\infty]{}+\infty\,.$$

In order to establish this, we first examine the behavior of the ρ's for finite ω as $g^2\to\infty$. We observe from Fig. 2 that $\delta_\pm(\omega)$ remain finite as $g^2\to\infty$. Although $\delta_-(\infty)$ changes from 0 to $-\pi$ as $g^2\to\infty$, this affects only the asymptotic form of $\Delta_-(\omega)$, and at present we are studying finite ω. We conclude that $\Delta_\pm(\omega)$ approach finite limits as $g^2\to\infty$ for finite ω. Next, we observe

[6] M. Bander, P. Coulter, and G. Shaw, Phys. Rev. Letters **14**, 270 (1965).

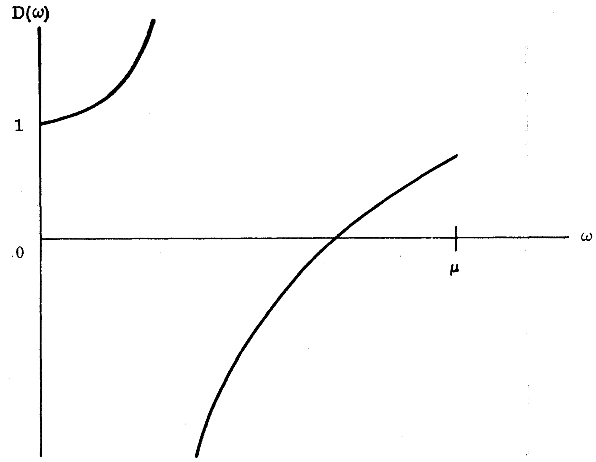

FIG. 1. The denominator function $D(\omega)$ when T_- has a bound state. $\omega C(-\omega)$ is a monotonically increasing function of ω, so D has at most one pole.

that near $\omega=0$, $\alpha(\omega)\approx-g^2\omega\beta$, with $\beta>0$. Thus, for large g^2,

$$\omega_B\approx 1/g^2\beta\,. \tag{11}$$

From Eq. 5, we observe that for large g^2

$$g_B^2\approx g^2\,. \tag{12}$$

These remarks suffice to determine that for finite ω and large g^2, ρ_-^B decreases like g^{-2}, and ρ_-^T and ρ_+ are independent of g^2. Therefore,

$$C(-\mu)=C+\frac{1}{\pi}\int_{\omega_0}^{\infty}\frac{d\omega_1}{\omega_1}\times\left[\frac{\rho_-^B(\omega_1)+\rho_-^T(\omega_1)}{\omega_1-\mu}+\frac{\rho_+(\omega_1)}{\omega_1+\mu}\right], \tag{13}$$

where ω_0 is a large energy chosen so that we may use high energy forms of the quantities appearing in the integrand. C is a positive constant which is independent of g^2 for large g^2. Evidently, if $C(-\mu)$ is to increase with g^2, this increase is to be found in the high-energy integral of Eq. (13).

For large positive or negative ω,

$$\mathrm{Re}\,\alpha(\omega)\approx+\frac{g^2\gamma}{\omega}\,,\quad \gamma>0\,. \tag{14}$$

We assume that $u^2(\omega)\approx\eta\omega^{-n}$ for large ω, where $n>1$. Examination of Fig. 2 verifies that $\Delta_+(\infty)$ is a finite positive number when $g^2\to\infty$, so from Eq. (4) we have

$$\Delta_-(\omega)\xrightarrow[g^2\to\infty]{}-\frac{\omega\mu\beta M_-(\omega)}{\Delta_+(\infty)}\approx\frac{g^2\mu\beta}{\Delta_+(\infty)\{1-g^2[\gamma/\omega-iu^2(\omega)\theta(\omega)/4\pi]\}}\,. \tag{15}$$

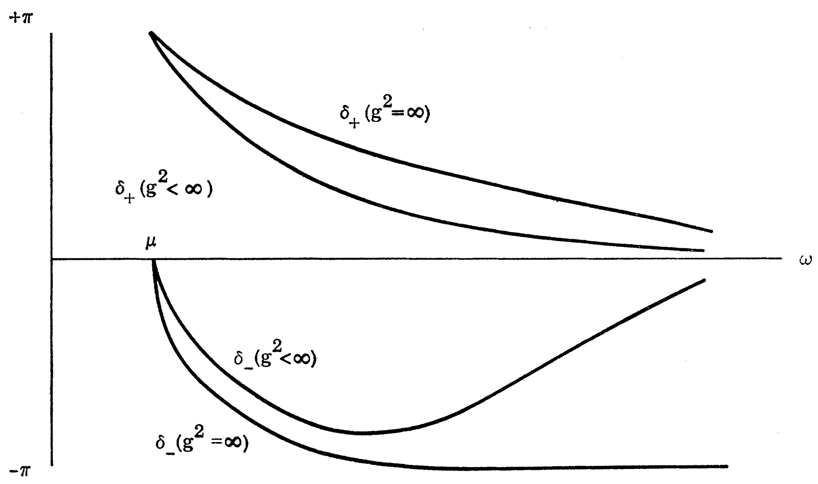

FIG. 2. The one meson phase shifts for finite and infinite g^2.

This form is valid for large g^2 and ω, and demonstrates that the asymptotic behavior of $\Delta_-(\omega)$ changes as $g^2 \to \infty$.

We are now able to evaluate the remaining integral in Eq. (13). We first examine the ρ_-^B term, replacing all the terms in the definition of ρ_-^B by their values at $\omega=\infty$, except $\Delta_-(\omega)$, for which we use Eq. (15). We let $x=\omega_1/g^2$ be the variable of integration.

$$C_-^B \equiv \frac{1}{\pi}\int_{\omega_0}^{\infty} \frac{d\omega_1 \rho_-^B(\omega_1)}{\omega_1(\omega_1-\mu)} \tag{16a}$$

$$\approx \frac{\mu^4\beta^2\eta}{2\pi^2 g^{2n}\Delta_+^4(\infty)} \times \int_{\omega_0/g^2}^{\infty} \frac{x^{n-2}dx}{[x+\gamma]^2[(x-\gamma)^2x^{2n-2}+\eta^2/16\pi^2g^{4n-4}]}. \tag{16b}$$

For large g, the dominant contribution to the integral comes from the region around $x=\gamma$, and C_-^B vanishes like g^{-2}. Thus, the contribution of the π^+B^- state vanishes for large g^2, and if T_- is to have a bound state, it is solely a three-particle effect.

We next examine the contribution of ρ_-^T. In the Appendix we show that the integral in Eq. (8) is bounded from below by $\lambda g^{2(2-n)}\omega^{-n+1}$ when $\omega > 3g^2\gamma > \omega_0$. λ is independent of g^2 and ω. Thus, for $\omega > 3g^2\gamma > \omega_0$,

$$\rho_-^T(\omega) > \frac{\lambda g^{2(4-n)}}{8\pi^3\Delta_+^2(\infty)[1+\gamma g^2/\omega]^2\omega^n}. \tag{17}$$

Using x as variable again, we have

$$C_-^T \equiv \frac{1}{h}\int_{\omega_0}^{\infty} \frac{d\omega_1 \rho_-^T(\omega_1)}{\omega_1(\omega_1-\mu)} > \frac{\lambda g^{6-4n}}{8\pi^4\Delta_+^2(\infty)} \times \int_{3\gamma}^{\infty} \frac{dx}{x^n(x+\gamma)^2}. \tag{18}$$

The contribution of ρ_+ is positive, so for large g we have

$$C(-\mu) > C_0 + C_1 g^{4(\frac{3}{2}-n)};\quad C_0, C_1 > 0. \tag{19}$$

We conclude that for $n<\frac{3}{2}$, the two meson amplitude T_- can have a bound state. (The condition $n<\frac{3}{2}$ is sufficient for a bound state, but probably not necessary.) On the other hand, for $1<n$, (as we have assumed), the model is conventional in the sense that the one meson phase shifts approach multiples of π at infinity. Thus, for $1<n<\frac{3}{2}$, the theory is conventional, and T_- can have a bound state.

It is worth mentioning that for large g^2, the bound state B moves to the origin and nearly cancels the source pole there [see Eqs. (11) and (12)]. This is the way unitarity is maintained when g^2 is large. However, it would be erroneous to conclude that the net effect of the direct n pole and exchanged B^{++} pole is no longer repulsive when g^2 is large. This is evident from the fact that M_- never can have a bound state. Even in the limit $g^2\to\infty$, the bound state in T_- must be interpreted as a three-particle effect which occurs despite repulsive Born terms.

APPENDIX

We wish to obtain a lower bound for the integral

$$I=\int_{\mu}^{\omega-\mu} d\omega_1 k_1 k_{-1} u^2(\omega_1)u^2(\omega_{-1}) \times \left|\frac{\omega_{-1}-\mu}{\bar{\omega}_{-1}\omega_1}\Delta_+(\omega_{-1})\Delta_-(\omega_1)\right|^2, \tag{A1}$$

which is valid for large ω and g. We assume that $\omega/2 > g^2\gamma$. Then

$$I > I_1 = \int_{g^2\gamma}^{\omega/2} d\omega_1 k_1 k_{-1} u^2(\omega_1)u^2(\omega_{-1}) \times \left|\frac{(\omega_{-1}-\mu)}{\omega_{-1}\omega_1}\Delta_+(\omega_{-1})\Delta_-(\omega_1)\right|^2. \tag{A2}$$

We further assume that g is so large that $g^2\gamma > \omega_0$, where ω_0 is still the energy above which the asymptotic forms of Δ_- and u^2 may be used. This energy is independent of g^2, so clearly g^2 can be chosen large enough to put $g^2\gamma > \omega_0$. Then in Eq. (A2) we may use

$$|\Delta_-(\omega_1)|^2 = \frac{g^4\mu^2\beta^2}{\Delta_+^2(\infty)[(1-g^2\gamma/\omega)^2 + g^4\eta^2/16\pi^2\omega^{2n}]}. \quad \text{(A3)}$$

We further assume that g is sufficiently large that

$$\eta^2/16\pi^2\gamma^2 < g^{4(n-1)}. \quad \text{(A4)}$$

Then

$$(1-g^2\gamma/\omega_1)^2 + g^4\eta^2/16\pi^2\omega_1^{2n} \leqslant 1\,, \quad (\omega_1 \geqslant g^2\gamma) \quad \text{(A5)}$$

and

$$|\Delta_-(\omega_1)|^2 \geqslant g^4\mu^2\beta^2/\Delta_+^2(\infty)\,, \quad (\omega_1 \geqslant g^2\gamma). \quad \text{(A6)}$$

Since $|\Delta_+(\omega)|^2$ has a nonzero lower bound κ which is independent of g,

$$I_1 \geqslant \frac{4^n g^4\mu^2\beta^2\kappa^2\eta^2}{\Delta_+^2(\infty)\omega^{n-1}} \int_{g^2\gamma}^{\omega/2} \frac{d\omega}{\omega_1^{n+1}} = \lambda g^{2(2-n)}\omega^{-n+1}\,, \quad (\omega > 3g^2\gamma > \omega_0)\,, \quad \text{(A7)}$$

where λ is independent of g and ω. The limit $3g^2\gamma$ is chosen so that the integral in Eq. (A7) may be bounded from below by an expression proportional to $(g^2\gamma)^{-n}$.

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Determination of the Nucleon-Nucleon Elastic-Scattering Matrix. V. New Results at 25 MeV

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Analyses of N-N experiments at 25 MeV have been hampered by a lack of complete scattering data, especially for the scattering states with isotopic spin $T=0$. In particular, there is an ambiguity in the single-energy $T=0$ solutions at 25 MeV. This ambiguity, which we discuss here in detail, is partly resolved by the addition of new (n,p) data. Some new (p,p) data have also been added. The resulting phases more closely resemble the values expected from potential models—with which they are compared. The new selection of data permits a determination of the pion-nucleon coupling constant ($g_\pi^2 = 14.3 \pm 1.3$), whereas the older selections did not. An investigation of the parabolic approximation for each of the phases indicates the extent to which one can believe the uncertainties as given by an error-matrix calculation. The energy-dependent analyses in this energy region have been improved by having the S phases extrapolate to the scattering length and effective-range expansions at low energies. The resulting phases give excellent fits to the data at 10 MeV as well as at 25 MeV. Experiments that would further improve the analysis at 25 MeV are suggested. The present results are in some disagreement with a recently released Dubna analysis at 23 MeV.

I. INTRODUCTION

IN previous papers in this series[1–4] we have published the results of energy-dependent and energy-independent phase-shift analyses in the energy range from 25 to about 350 MeV. Both (p,p) and (n,p) data were analyzed, and the isotopic spin $T=0$ and $T=1$ amplitudes were determined. However, whereas the (p,p) experiments are reasonably complete and give reliable values for the $T=1$ phase shifts, the (n,p) experiments are patently incomplete, and the $T=0$ scattering matrix obtained from our analyses must be considered with this fact in mind. For an incomplete data set, multiple phase solutions may exist. Even for a correct type of solution, the phases may be somewhat inaccurate, and the phase-shift uncertainties as given by an error matrix calculation may be grossly inaccurate. In particular, the least-squares sum (χ^2) hypersurface in the neighborhood of the solution minimum, that is for variations of a few standard deviations for each parameter, may not be parabolic. These statements are well illustrated in the present analysis of nucleon-nucleon data near 25 MeV.

In our previous analyses at 25 MeV,[3,4] we obtained $T=1$ and $T=0$ scattering matrices. However, we were unable to obtain a value for the pion-nucleon coupling constant g_π^2, and some of the $T=0$ phases, ϵ_1 in particular, had obviously misleading values and/or errors. Even for an energy as low as 25 MeV, triple-scattering parameters are needed for an accurate phase-shift analysis. These were incomplete for the (p,p) system and nonexistent for the (n,p) system.

Recently, additional experiments have been completed near 25 MeV that modify our previous results

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