Trident Production with Nuclear Targets*

G. Reading Henry
Lyman Laboratory, Harvard University, Cambridge, Massachusetts
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Trident production (production of a charged lepton pair by a charged lepton incident on a nuclear target) is discussed in the approximation of vanishing lepton mass. A compact formula is given for the trident differential cross section, valid for complex nuclear targets, taking into account nuclear recoil and inelasticities. The elastic, no-recoil approximation is also discussed.

I. INTRODUCTION

TRIDENT production (production of a charged lepton pair by a charged lepton incident on a nuclear target) has been discussed by several authors as a test of quantum electrodynamics at small distances.[1] Trident production is interesting for several reasons. It offers the possibility of exploring the time-like region of the photon propagator. Alternatively, in the pure-muon case it may be regarded as a way of looking for a hypothetical particle which might couple only to the muon ("explaining" the electron-muon mass difference). Again in the pure-muon case, trident production is sensitive to the statistics of the muon.

In this paper a fairly compact formula is given for the trident differential cross section, valid for complex targets and taking into account nuclear recoil and inelasticities. The lepton mass is neglected throughout. The result is stated in terms of the Drell-Walecka inelastic nuclear form factors,[2] which implies use of the first Born approximation in the interaction at the nucleus. Radiative corrections are not discussed.

In Sec. III the method of calculation is outlined, and various systematics of the calculation are noted. In Sec. IV the detailed formulas for the cross section are given, and in Sec. V these formulas are specialized to the no-recoil, elastic approximation. Concluding remarks are made in Sec. VI.

II. NOTATION

A typical diagram for trident production is shown in Fig. 1. The four-momentum[3] of the incident lepton is denoted by p_1, those of the outgoing leptons by p_2 and p_3, while p_4 denotes the out-going antilepton momentum (p_4 is the experimentally observed momentum of the μ^+ or e^+). The four-momentum transfer to the nucleus is $q=p_2+p_3+p_4-p_1$. The energies and chiralities of the particles are E_1, E_2, E_3, E_4, and k_1, k_2, k_3, k_4, respectively. The chirality is either plus or minus one; right-handed leptons and left-handed antileptons have positive chirality. The symbol $\epsilon_{\alpha\beta\rho\sigma}$ is the four-dimensional totally antisymmetric tensor, with $\epsilon_{1234}=+1$.

Diagram 1 is explicity shown (Fig. 1); by attaching the upper end of the nuclear photon to the points 2, 3, 4, the diagrams 2, 3, 4 are created, respectively. If the leptons are all electrons or all muons, exchange diagrams must be considered: Diagrams 5 through 8 are obtained from diagrams 1 through 4, respectively, by interchanging p_2 and p_3.

The notation for the inelastic nuclear form factors follows Ref. 2; for spinors and gamma matrices, the notation of Källén[4] is adopted.

III. METHOD OF CALCULATION

Each amplitude corresponding to a given Feynman diagram is explicitly calculated (using traces) for a given set of leptonic chiralities[5]; the amplitudes are then added together and squared. Summations over chiralities may then be performed if desired.

The trident production cross section for definite chiralities, in terms of the Drell-Walecka form factors, is given by

$$d\sigma_{\mathrm{pol}}=\frac{E_1(Ze)^2W_{\mu\nu}}{q^4(-P\cdot p_1)}\frac{\Omega^4}{(2\pi)^{16}}\mathcal{L}_\mu\mathcal{L}_\nu^\star d^3p_2d^3p_3d^3p_4, \qquad (1)$$

where Ω is the normalization volume. Equation (4) of Ref. 2 defines $W_{\mu\nu}$; because of current conservation,

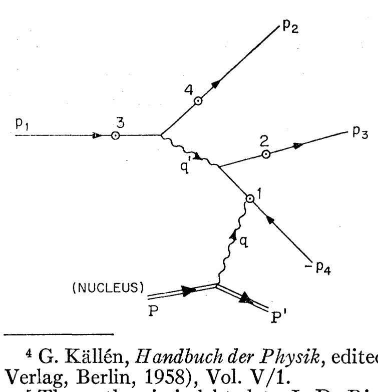

FIG. 1. Feynman diagram for trident production. Diagram 1 is shown; others are generated by connecting the nuclear photon to points 2, 3, and 4 instead of 1.

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[1] See, for example, J. D. Bjorken and S. D. Drell, Phys. Rev. **114**, 1368 (1959); T. Yamamoto, Progr. Theoret. Phys. (Kyoto) **27**, 223 (1963); M. C. Chen, Phys. Rev. **127**, 1844 (1962); E. Johnson, *ibid.* **140**, B1005 (1965); S. J. Brodsky and S. C. C. Ting, *ibid.* **145**, 1018 (1966).

[2] S. D. Drell and J. D. Walecka, Ann. Phys. (N. Y.) **28**, 18 (1964).

[3] We use a metric such that $a_\mu=(\mathbf{a},a_4)=(\mathbf{a},ia_0)$; $a_\mu b_\mu=a\cdot b=\mathbf{a}\cdot\mathbf{b}-a_0b_0$. We also define $a_\mu^\star=(\mathbf{a}^*,ia_0^*)$ and $\tilde{a}_\mu=(-\mathbf{a},ia_0)$.

[4] G. Källén, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. V/1.

[5] The author is indebted to J. D. Bjorken for suggesting this approach.

terms in $W_{\mu\nu}$ proportional to q_μ and q_ν vanish and we may write

$$W_{\mu\nu}=W_1(q^2,q\cdot P)\delta_{\mu\nu}+(1/M_T^2)W_2(q^2,q\cdot P)P_\mu P_\nu\,, \quad (2)$$

where M_T is the mass of the target nucleus. Finally, $\mathcal{L}_\mu$ is the matrix element of the leptonic electromagnetic current, $\langle p_2,p_3,p_4|j_\mu|p_1\rangle$; it may be written as the sum of the contributions from the various Feynman diagrams:

$$\mathcal{L}_\mu=\sum_{i=1}^{4}(L_i)_\mu+\delta\sum_{i=5}^{8}(L_i)_\mu\,, \quad (3)$$

where $\delta=0$ if both electrons and muons are present, and in the all-muon or all-electron case, $\delta=-1$ for Fermi-Dirac statistics and $\delta=+1$ for Bose-Einstein statistics.

If the incident lepton is unpolarized and the final lepton polarizations are not observed, we are interested in the cross section averaged over the initial spin and summed over final spins:

$$d\sigma_{\text{unpol}}=\frac{1}{2}\sum_{\substack{k_1,k_2,k_3,k_4=\pm1\\ \text{all combinations}}} d\sigma_{\text{pol}}\,. \quad (4)$$

Now we wish to evaluate the amplitudes $(L_i)_\mu$. It turns out to be sufficient to evaluate explicitly only L_1, from which the others are easily derived. We have (see Fig. 1):

$$(L_1)_\mu=(i/4)(2\pi)^4\Omega^{-2}(p_1-p_2)^{-2}(p_2+p_3-p_1)^{-2}\delta_{k_1k_2}\delta_{k_3k_4}$$
$$\bar{u}(\mathbf{p}_2)\gamma_\alpha(1-k_1\gamma_5)u(\mathbf{p}_1)\bar{u}(\mathbf{p}_3)$$
$$\times\gamma_\alpha i(\not{p}_2+\not{p}_3-\not{p}_1)\gamma_\mu(1-k_4\gamma_5)u(-\mathbf{p}_4)\,. \quad (5)$$

The δ symbols in Eq. (5) occur because, in the zero-mass limit, the amplitude of a diagram vanishes if particles on the same lepton line have different chiralities; it is therefore sufficient to use two projection operators instead of four. In order to evaluate L_1 by trace techniques, we multiply by a phase factor $A/|A|$, where

$$A=\bar{u}(\mathbf{p}_1)\gamma_4u(\mathbf{p}_2)\bar{u}(-\mathbf{p}_4)\gamma_4u(\mathbf{p}_3)\,. \quad (6)$$

It is convenient to define $(M_1)_\mu$ corresponding to $(L_1)_\mu$:

$$(L_1)_\mu A/|A|\equiv\tfrac{1}{2}(2\pi)^4e^3(E_1E_2E_3E_4)^{-1/2}\Omega^{-2}(M_1)_\mu\,. \quad (7)$$

Now we have

$$(M_1)_\mu=[(p_1\cdot\tilde{p}_2)(p_4\cdot\tilde{p}_3)]^{-1/2}(p_1-p_2)^{-2}(p_2+p_3-p_1)^{-2}$$
$$\times\tfrac{1}{4}\text{Tr}\{\gamma_\alpha(1-k_1\gamma_5)\not{p}_1\gamma_4\not{p}_2\}(-\tfrac{1}{4})\text{Tr}$$
$$\times\{\gamma_\alpha(\not{p}_2+\not{p}_3-\not{p}_1)\gamma_\mu(1-k_4\gamma_5)\not{p}_4\gamma_4\not{p}_3\}\delta_{k_1k_2}\delta_{k_3k_4}\,. \quad (8)$$

These traces are evaluated in the following section.

An exactly analogous procedure may be used to evaluate the other amplitudes, except that the exchange diagrams must be multiplied by the phase $B/|B|$, where

$$B=\bar{u}(\mathbf{p}_1)\gamma_4u(\mathbf{p}_3)\bar{u}(-\mathbf{p}_4)\gamma_4u(\mathbf{p}_2)\,. \quad (9)$$

It is easily shown that $B/|B|=e^{i\theta}A/|A|$, where θ is given in Eqs. (14). Note that θ is defined only when $k_1=k_2=k_3=k_4$, since in all other cases either the normal, the exchange, or all diagrams give no contribution.

Defining the seven other M_i as in Eq. (7) [M_i with $i>4$ being defined in terms of $(L_i)_\mu B/|B|$], it is easy to show that they may be expressed in terms of $(M_i)_\mu$ [Eqs. (11)].

IV. DETAILED RESULTS

Using the results of the last section, Eq. (1) may be rewritten:

$$d\sigma_{\text{pol}}=\frac{Z^2\alpha^4W_{\mu\nu}}{4\pi^4q^4(-P\cdot p_1)}\mathfrak{M}_\mu\mathfrak{M}_\nu^\star\frac{d^3p_2}{E_2}\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}, \quad (10)$$

where

$$\mathfrak{M}_\mu=\sum_{i=1}^{4}(M_i)_\mu+\delta e^{-i\theta}\sum_{i=5}^{8}(M_i)_\mu\,, \quad (11)$$

with

$$(M_1)_\mu=[(p_1\tilde{p}_2)(p_4\cdot\tilde{p}_3)]^{-1/2}(p_1-p_2)^{-2}(p_5)^{-2}\delta_{k_1k_2}\delta_{k_3k_4}$$
$$\times\{E_2p_{1\alpha}+E_1p_{2\alpha}+i(p_1\cdot p_2)\delta_{\alpha4}+ik_1\epsilon_{\alpha4\rho'\sigma'}p_{1\rho'}p_{2\sigma'}\}$$
$$\times\{p_{5\alpha}[E_3p_{4\mu}+E_4p_{3\mu}+i(p_3\cdot p_4)\delta_{\mu4}+ik_4\epsilon_{\mu4\beta\rho}p_{4\beta}p_{3\rho}]$$
$$+p_{5\mu}[E_3p_{4\alpha}+E_4p_{3\alpha}+i(p_3\cdot p_4)\delta_{\alpha4}+ik_4\epsilon_{\alpha4\beta\rho}p_{4\beta}p_{3\rho}]$$
$$-\delta_{\mu\alpha}[E_3(p_4\cdot p_5)+E_4(p_3\cdot p_5)-E_5(p_3\cdot p_4)$$
$$-ik_4\epsilon_{4\beta\rho\sigma}p_{5\beta}p_{4\rho}p_{3\sigma}]+i\epsilon_{\rho\alpha\beta\mu}p_{5\beta}$$
$$\times[ik_4\{E_3p_{4\rho}+E_4p_{3\rho}+i(p_3\cdot p_4)\delta_{\rho4}\}-\epsilon_{\rho4\sigma\tau}p_{4\sigma}p_{3\tau}]\}\,; \quad (12a)$$

where

$$p_5=p_2+p_3-p_1\,. \quad (12b)$$

If

$$(M_1)_\mu\equiv F_\mu(p_1,p_2,p_3,p_4,k_1,k_2,k_3,k_4)\,,$$

then

$$(M_2)_\mu=F_\mu^\star(p_2,\,p_1,\,-p_4,\,-p_3,\,k_1,\,k_2,\,k_3,\,k_4)\,, \quad (13)$$
$$(M_3)_\mu=F_\mu(-p_4,\,p_3,\,p_2,\,-p_1,\,k_4,\,k_3,\,k_2,\,k_1)\,,$$

and

$$(M_4)_\mu=F_\mu^\star(p_3,\,-p_4,\,p_1,\,-p_2,\,k_4,\,k_3,\,k_2,\,k_1)\,.$$

From M_1 through M_4 one obtains M_5 through M_8 respectively, by the exchanges $p_2\leftrightarrow p_3$ and $k_2\leftrightarrow k_3$. Finally, the angle θ is determined by

$$\cos\theta=\frac{1}{2}\frac{(p_4\cdot\tilde{p}_3)(p_1\cdot\tilde{p}_2)+(p_1\cdot\tilde{p}_3)(p_4\cdot\tilde{p}_2)-(p_1\cdot p_4)(\tilde{p}_2\cdot\tilde{p}_3)}{[(p_1\cdot\tilde{p}_2)(p_4\cdot\tilde{p}_3)(p_1\cdot\tilde{p}_3)(p_4\cdot\tilde{p}_2)]^{1/2}}$$

and (14)

$$\sin\theta=\frac{1}{2}\frac{ik_1\epsilon_{\alpha\beta\rho\sigma}p_{1\alpha}\tilde{p}_{2\beta}\tilde{p}_{3\rho}p_{4\sigma}}{[(p_1\cdot\tilde{p}_2)(p_4\cdot\tilde{p}_3)(p_1\cdot\tilde{p}_3)(p_4\cdot\tilde{p}_2)]^{1/2}}\,.$$

In Eq. (12a), summation over the repeated index α is implied. This dot product has been carried out, and the substitution (12b) made, but the resulting expression is rather long. In any computer program written to evaluate Eq. (12a), it would probably be easier to include the sum on α in the program. If polarizations are not observed, Eq. (12a) should be substituted in Eq. (4). Of the sixteen different combinations of chiralities, only six are nonzero, and only two of these involve both the normal and exchange graphs.

V. SPECIALIZATION TO THE NO-RECOIL CASE

Although Eqs. (10) through (14) will probably be required in making detailed comparisons between theory and experiment, very great simplifications occur if the nucleus may be regarded as the source of a static Coulomb field, and such an approximation may be useful in estimating counting rates, etc. In this case, Eqs. (10) through (14) become

$$d\sigma_{\rm pol}=\frac{Z^2\alpha^4|F(q^2)|^2}{4\pi^4q^4E_1}|\mathfrak{M}_4|^2\delta(q_0)\frac{d^3p_2}{E_2}\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}, \qquad (10')$$

where

$$\mathfrak{M}_4\sum_{i=1}^{4}(M_i)_4+\delta e^{-i\theta}\sum_{i=5}^{8}(M_i)_4 \qquad (11')$$

with

$$\begin{aligned}(M_1)_4=i\{&[(p_1\cdot\tilde{p}_2)(\tilde{p}_4\cdot p_3)]^{-1/2}(p_1-p_2)^{-2}(p_5)^{-2}\delta_{k_1k_2}\delta_{k_3k_4}\\ &\times[E_2p_{1\alpha}+E_1p_{2\alpha}+i(p_1\cdot p_2)\delta_{\alpha 4}+ik_1\epsilon_{\alpha 4\beta\rho'}p_{1\beta}p_{2\rho'}]\\ &\times[-(\tilde{p}_4\cdot p_3)p_{5\alpha}-(\tilde{p}_4\cdot p_5)p_{3\alpha}\\ &\qquad+(p_3\cdot p_5)\tilde{p}_{4\alpha}-k_4\epsilon_{\alpha\rho\sigma\tau}p_{5\rho}\tilde{p}_{4\sigma}p_{3\tau}]\}, \qquad (12a')\end{aligned}$$

where, as before,

$$p_5=p_2+p_3-p_1. \qquad (12b')$$

If

$$(M_1)_4\equiv G(p_1,p_2,p_3,p_4,k_1,k_2,k_3,k_4),$$

then

$$(M_2)_4=G(p_2,\,p_1,\,-p_4,\,-p_3,\,-k_1,\,-k_2,\,-k_3,\,-k_4), \qquad (13')$$

$$(M_3)_4=G(-p_4,\,p_3,\,p_2,\,-p_1\ k_4\ k_3,\,k_2,\,k_1),$$

and

$$(M_4)_4=G(p_3,\,-p_4,\,p_1,\,-p_2,\,-k_4,\,-k_3,\,-k_2,\,-k_1).$$

From M_1 through M_4, M_5 through M_8 are obtained, respectively, by the exchanges $p_2\leftrightarrow p_3$ and $k_2\leftrightarrow k_3$. The angle θ is still given by Eqs. (14).

In the no-recoil case, the implied dot product in Eq. (12a′) and the substitution (12b′) are fairly easily carried out (see Appendix). Bjorken and Chen have also obtained the cross section in this approximation.[6]

VI. CONCLUSIONS

Equations (10) through (14) [and (10′) through (13′)] give the trident production cross section in terms of the various amplitudes involved. It should therefore be possible to use these equations to find experimental conditions maximizing the effect of the diagrams of interest. For example, to study time-like photon propagators, diagrams 3 and 4 (and 7 and 8) are relevant, while the others are not.

It should be emphasized that the Sec. IV equations do still apply when a complex nuclear target (eg., carbon) is used; inelastic effects, such as nuclear breakup and strong particle creation, are included by virtue of the form factors used. The formulas are no longer valid if the nucleus is initially polarized or if a selection of events is made on the basis of strong particles detected in the final state.[2]

ACKNOWLEDGMENTS

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APPENDIX

Equations (12a′) and (12b′) may be combined to give

$$\begin{aligned}(M_1)_4=i\{&E_1[(p_2\cdot p_3)\{(p_1\cdot\tilde{p}_4)-2(p_3\cdot\tilde{p}_4)\}-(p_1\cdot p_3)(p_2\cdot\tilde{p}_4)]+E_2[-(p_1\cdot p_3)\{(p_2\cdot\tilde{p}_4)+2(p_3\cdot\tilde{p}_4)\}+(p_1\cdot\tilde{p}_4)(p_2\cdot p_3)]\\ &+(p_1\cdot p_2)[E_3\{2(p_3\cdot\tilde{p}_4)+(p_2\cdot\tilde{p}_4)-(p_1\cdot\tilde{p}_4)\}+E_4\{(p_1\cdot p_3)-(p_2\cdot p_3)\}]\\ &-ik_1[\{2(p_3\cdot\tilde{p}_4)+(p_2\cdot\tilde{p}_4)-(p_1\cdot\tilde{p}_4)\}\mathbf{p}_3+\{(p_2\cdot p_3)-(p_1\cdot p_3)\}\mathbf{p}_4]\cdot[\mathbf{p}_1\times\mathbf{p}_2]\\ &-ik_4[(E_1+E_2)|p_1p_2\tilde{p}_4p_3|+(p_1\cdot p_2)(\mathbf{p}_1-\mathbf{p}_2)\cdot(\mathbf{p}_3\times\mathbf{p}_4)]+k_1k_2[(E_2-E_1)\{(p_1\cdot\tilde{p}_4)(p_2\cdot p_3)-(p_2\cdot\tilde{p}_4)(p_1\cdot p_3)\}\\ &\qquad+(p_1\cdot p_2)\{E_3(p_2\cdot\tilde{p}_4)+E_3(p_1\cdot\tilde{p}_4)-E_4(p_2\cdot p_3)-E_4(p_1\cdot p_3)\}]\}\delta_{k_1k_2}\delta_{k_3k_4}\\ &\qquad\times[(p_1\cdot\tilde{p}_2)(\tilde{p}_4\cdot p_3)]^{-1/2}(p_1-p_2)^{-2}(p_5)^{-2}\end{aligned}$$

where $|PQRS|$ is the determinant

$$\begin{vmatrix}P_1 & Q_1 & R_1 & S_1\\ P_2 & Q_2 & R_2 & S_2\\ P_3 & Q_3 & R_3 & S_3\\ P_0 & Q_0 & R_0 & S_0\end{vmatrix}=-i\epsilon_{\alpha\beta\rho\sigma}P_\alpha Q_\beta R_\rho S_\sigma.$$

[6] J. D. Bjorken and M. C. Chen (private communication) and Phys. Rev., this issue, 154, 1335 (1966).