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Some Comments on the Decays of η (550)*

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Various decay modes of the $\eta(550)$ are discussed. The relations, through SU_3 and the Gell-Mann–Sharp–Wagner model, between the η-decay modes and the modes $\eta\rightarrow\pi\pi\gamma$, $\pi^0\rightarrow\gamma\gamma$ are investigated, taking into account η-η^* mixing. The present experimental values for the neutral branching ratios plus the shape of the $\eta\rightarrow\pi^+\pi^-\pi^0$ Dalitz plot are shown to require a 30% $|\Delta\mathbf{I}|=3$ contribution to the $\eta\rightarrow 3\pi$ amplitude. The connection between a possible charge asymmetry in $\eta\rightarrow\pi^+\pi^-\pi^0$ and branching ratio $\Gamma_{\eta\rightarrow\pi^0 e^* e^*}/\Gamma_\eta^{\text{all}}$ is investigated in the framework of a model proposed earlier by several authors. It is shown that there is no conflict between the existing data and this model. The Dalitz-plot distribution of $\eta\rightarrow\pi^+\pi^-\pi^0$ is discussed under various assumptions about the properties of the interaction responsible for the decay.

1. INTRODUCTION

THE decay properties of the $\eta(550)$ meson are of great interest because they provide an insight into the SU_3 properties, isospin selection rules, and charge-conjugation behavior of the strong and electromagnetic (e.m.) interactions. For example, the decays $\eta\rightarrow 2\gamma$ and $\eta\rightarrow\pi\pi\gamma$ contain information on octet and singlet couplings, and are mutually related by the Gell-Mann–Sharp–Wagner[1] (GSW) model. These decay modes may be employed to obtain information on the total η-decay rate[2,3] and on octet-singlet mixing.[4] Conversely, as soon as we know the experimental value of the η lifetime, we may be led to a better understanding of the radiative meson decays.[5]

The decay $\eta\rightarrow 3\pi$ gives us the rare and as yet unique opportunity of testing for the presence of a $|\Delta\mathbf{I}|=3$ transition in the strong interactions. (We assume, of course, $|\Delta\mathbf{I}|\leq 1$ for e.m. interactions.) The experimental information now available[6] on the decay modes of $\eta(550)$ indicates the presence of a rather large amount of $|\Delta\mathbf{I}|=3$. Other more qualitative and less direct evidence points to a small admixture.[7]

Further, the existence of a C violation in strong[8] or e.m.[9] interactions may reveal itself through a charge asymmetry in $\eta\rightarrow 3\pi$. The isospin properties of such a C-violating interaction cannot be deduced from one's knowledge of the branching ratio $\Gamma_{K_L\rightarrow\pi^0\pi^0}/\Gamma_{K_L\rightarrow\pi^+\pi^-}$, or the $\eta\rightarrow 3\pi$ Dalitz plot, alone. As has been argued before,[10] the selection rule $|\Delta\mathbf{I}|=0$ for the C-violating interaction does not exclude a 30% admixture of $|\Delta\mathbf{I}|=\frac{3}{2}$ in $K_L\rightarrow 2\pi$, because cancellation between the C-violating part of the $|\Delta\mathbf{I}|=\frac{1}{2}$ amplitude and the mass matrix is possible,[11] and this effect would tend to make the $|\Delta\mathbf{I}|=\frac{3}{2}$ mode relatively more important. Further, in a recently proposed model of C violation in e.m. interactions,[12] the selection rule $|\Delta\mathbf{I}|=0$ was suggested, thus allowing $|\Delta\mathbf{I}|=0$ and 1, but not $|\Delta\mathbf{I}|=2$ in $K_L\rightarrow 2\pi$ and in $\eta\rightarrow 3\pi$. Thus, a large deviation from the rule $|\Delta\mathbf{I}|=\frac{1}{2}$ in $K_L\rightarrow 2\pi$ combined with no $|\Delta\mathbf{I}|=2$ in $\eta\rightarrow 3\pi$ would be positive evidence for the correctness of this point of view. It may be noted, though, that a $|\Delta\mathbf{I}|=0$ transition in $\eta\rightarrow 3\pi$ is highly suppressed because of angular-momentum barrier effects.

Next, assuming that C invariance is violated in $\eta\rightarrow\pi^+\pi^-\pi^0$,[13] one can ask about the isospin behavior of the relevant interaction. A criterion for deciding this question, valid under certain general and very plausible assumptions, is suggested below.

In Sec. 2 the consequences of SU_3 and the GSW model are investigated, following the methods introduced by Dalitz and Sutherland.[4] In Sec. 3 we discuss

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[5] For a detailed discussion of the application of the GSW model and SU_6 to the radiative meson decays, see J. Yellin, Phys. Rev. **147**, 1080 (1966); Ph.D. thesis, University of Chicago, 1965 (unpublished).

[6] G. DiGiugno *et al.*, Phys. Rev. Letters **16**, 767 (1966); F. S. Crawford, L. Lloyd, and E. C. Fowler, *ibid.* **16**, 909 (1966).

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[8] J. Prentki and M. Veltman, Phys. Letters **15**, 88 (1965); L. B. Okun' (unpublished); L. B. Okun', in *High Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965), p. 939; T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965).

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[10] J. Prentki, invited paper at the Oxford International Conference on Elementary Particles, 1965 (unpublished).

[11] S. Weinberg, Phys. Rev. **110**, 782 (1958).

[12] T. D. Lee, Phys. Rev. **140**, B957 (1965) and paper given at the Oxford International Conference on Elementary Particles, 1965 (unpublished).

[13] C. Baltay *et al.*, University of California Radiation Laboratory Report No. UCRL-16693 (unpublished); Phys. Rev. (to be published); E. C. Fowler, Bull. Am. Phys. Soc. **11**, 380 (1966); C. Baltay *et al.*, Phys. Rev. Letters **16**, 1224 (1966). The latter group reports a charge asymmetry of 7.2±2.8% in $\eta\rightarrow\pi^+\pi^-\pi^0$. The CERN group reports a charge asymmetry of 0.3±1.1%. [See G. Finocchiaro *et al.*, in Proceedings of the International Conference on High Energy Physics, Berkeley, California, 1966 (to be published).] Another result for the asymmetry reported at Berkeley by a Rutherford-Saclay collaboration is −6±4%.

TABLE I. Branching ratios for the main decay modes of $\eta^\circ(550)$. We have used the charged/neutral ratio of Rosenfeld *et al.*, and also the ratios $\Gamma_{\eta\to\pi^+\pi^-\gamma}/\Gamma$(all charged modes), $\Gamma_{\eta\to\pi^+\pi^-\pi^0}/\Gamma$(all charged modes) from that compilation. The neutral branching ratios are from the experiment of DiGiugno *et al.* (Ref. 6).

Decay mode	Branching ratio (%)
$\eta \to \pi^+\pi^-\pi^0$	25
$\eta \to \pi^+\pi^-\gamma$	5.5
$\eta \to 2\gamma$	29
$\eta \to \pi^0 2\gamma$	26
$\eta \to \pi^0\pi^0\pi^0$	14.5

$\eta \to 3\pi$ and $\eta \to \pi^0 e^+e^-$ phenomenologically, employing a model introduced by several authors.[14–17]

2. TOTAL DECAY RATE

No direct measurement of the $\eta^\circ(550)$ lifetime exists. To obtain an estimate of the total rate we must use SU_3 to connect one of the η partial decay rates to another rate that is experimentally known. The experimental $\eta^0(550)$ branching ratios are shown in Table I.

As is well known, $\eta \to 2\gamma$ and $\eta \to \pi\pi\gamma$ (the latter is assumed to go through $\eta \to \rho\gamma$) may be related to $\pi^0 \to 2\gamma$ and $\omega \to \pi^0\gamma$ by SU_3,[3,5] and to each other by the Gell-Mann–Sharp–Wagner (GSW) model. In addition to the formulas relevant to this discussion, we will also derive, for completeness, some formulas given elsewhere.[4]

The following assumptions are essential: (i) The physical $\eta^{0*}(960)$ and $\eta^0(550)$ particles are mixtures of SU_3 singlet and octet states; (ii) The sum of the squares of the mixing coefficients is 1.

Most of the processes considered below have effective coupling constants which depend on two independent, SU_3-invariant interactions. Some of these processes are related to each other by the GSW model. This leads us to the assumption[4] (iii) The GSW model may be used to relate the *ratios* of SU_3-invariant couplings.

In accordance with the above we write

$$\eta^{0*} = \eta_1 \cos\alpha + \eta_8 \sin\alpha; \tag{2.1}$$

$$\eta^0 = -\eta_1 \sin\alpha + \eta_8 \cos\alpha, \tag{2.2}$$

where we use $\tan\alpha = 0.19$, obtained from the observed η,η^* masses. We are interested in processes involving one pseudoscalar meson and two photons, or one pseudoscalar meson, a photon, and a vector meson. We assume that the e.m. field has the usual octet transformation properties. For the vector mesons we employ the nonet representation suggested by SU_6 and corresponding to a φ-ω mixing angle θ, with $\tan\theta \cong 0.8$.

[14] S. Glashow and C. Sommerfield, Phys. Rev. Letters **15**, 78 (1965).
[15] Y. Fujii and G. Marx, Phys. Letters **17**, 75 (1965).
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[17] B. Barrett *et al.*, Phys. Rev. **141**, 1342 (1966).

TABLE II. The first column shows the amplitude for the given process as a function of the octet and singlet couplings of the pseudoscalar nonet to vector mesons, and also as a function of the 0^- singlet octet mixing angle α. The second column shows the phase space integrated over the simplest momentum dependence of the amplitude. The top eight lines show the predicted branching ratio relative to $\omega \to \pi\gamma$. The bottom three lines show the same versus $\pi \to 2\gamma$. The experimental numbers are taken from Rosenfeld *et al.* (Ref. 18), except for $\Gamma_{\pi^0\to\gamma\gamma}$, which is taken from Stamer *et al.* (Ref. 18).

Amplitude	Phase space factor	Experiment
$A(\eta^{0*} \to \omega\gamma) = A\cos\alpha + M\sin\alpha$	0.28	
$A(\eta^{0*} \to \rho^0\gamma) = 3A\cos\alpha + 3M\sin\alpha$	0.223	<1 MeV
$A(\eta \to \rho\gamma) = -3A\sin\alpha + 3M\sin\alpha$	2.16×10^{-4}	
$A(\eta \to \omega\gamma) = -A\sin\alpha + M\cos\alpha$	$\sim10^{-6}$	
$A(\omega \to \pi\gamma) = 3(3)^{1/2}M$	1	1.3 MeV
$A(\phi \to \eta\gamma) = (2)^{1/2}(A\sin\alpha + 2M\cos\alpha)$	0.865	<0.3 MeV
$A(\phi \to {}^{0*}\gamma) = (2)^{1/2}(-A\cos\alpha + 2M\sin\alpha)$	0.38×10^{-2}	
$A(\phi \to \pi\gamma) = 0$	2.3	
$A(\pi \to 2\gamma) = (3)^{1/2}M'$	1	6.6 MeV
$A(\eta \to 2\gamma) = -A'\sin\alpha + M'\cos\alpha$	67.3	
$A(\eta^{0*} \to 2\gamma) = A'\cos\alpha + M'\sin\alpha$	358.2	<0.6 MeV

The SU_3-invariant couplings are

$$A\eta_1(VE) + M(PVE)$$

and (2.3)

$$M'\eta_1(EE) + A'(PEE),$$

where P, V, and E represent the pseudoscalar octet, the vector nonet, and the e.m. octet, respectively. In Table II, the second column gives the phase-space factor with respect to $\omega \to \pi\gamma$. The last three lines of column 2 show the phase space relative to $\pi \to \gamma\gamma$. The third column gives the scanty experimental information available.

Using the GSW model, one finds[4]

$$A'/M' = 2(A/M). \tag{2.4}$$

The quantities M and M' can be determined from $\omega \to \pi^0\gamma$ and $\pi^0 \to 2\gamma$, respectively. From the experimental upper limit on $\Gamma_{\eta^*\to\rho\gamma}$ we derive the condition[4]

$$|A/M| < 3. \tag{2.5}$$

The processes $\varphi \to \eta\gamma$ is not very sensitive to $|A/M|$, so that the bound derived from the experimental upper limit on $\Gamma_{\varphi\to\eta\gamma}$ is probably less trustworthy. One finds

$$A/M < -0.36, \tag{2.6}$$

corresponding to a branching ratio of 9% for $\varphi \to \eta\gamma$. The value $A/M = 3$ gives a branching ratio of 16%. In the following we will assume

$$-2 \le A/M \le 2 \tag{2.7}$$

and compute the total η width from this. Some support for this assumption comes from the observed $\eta \to \pi\pi\gamma/\eta \to 2\gamma$ branching ratio, as explained below. We find first, for $A/M = -2$, -1, 0, 1, and 2 using

$$\Gamma_{\eta\to\pi\pi\gamma} = 90.3[1 - (A/M)\tan\alpha]^2 \text{ eV}, \tag{2.8}$$

the values

$$\Gamma_{\eta\to\pi\pi\gamma}=172,\,128,\,90,\,59,\,34.5\text{ eV}, \qquad (2.9)$$

and from these

$$\Gamma_\eta^{\text{Total}}=3127,\,2327,\,1640,\,1072,\,627\text{ eV (from }\omega\to\pi\gamma).$$

Employing $A'/M'=2A/M$ and the known $\pi^0\to 2\gamma$ rate,[18] we have

$$\Gamma_{\eta\to 2\gamma}=143[1-(2A/M)\tan\alpha]^2\text{ eV} \qquad (2.10)$$

and

$$\Gamma_{\eta\to 2\gamma}=442,\,272,\,143,\,66,\,8.2\text{ eV}, \qquad (2.11)$$

and now

$$\Gamma_\eta^{\text{total}}=1525,\,940,\,490,\,187,\,28\text{ eV}. \qquad (2.12)$$

The above results disagree by a factor between 2 and 20. To get somewhat more insight into the situation, we compute the $\eta\to 2\gamma$ rate in the GSW model, using $\Gamma_{\omega\to\pi^0\gamma}$ as input. Thus we first determine the $\omega\pi^0\gamma$ coupling constant:

$$g_{\omega\pi^0\gamma}=3\sqrt{3}M=0.946\times 10^{-3}\text{ MeV}^{-1}. \qquad (2.13)$$

The rate $\Gamma_{\eta\to 2\gamma}$ is then given by

$$\Gamma_{\eta\to 2\gamma}=(g^2/64\pi)M_\eta^3, \qquad (2.14)$$

with

$$g=\frac{eM\cos\alpha}{15}\left\{6-12\frac{A}{M}\tan\alpha\right\}=0.36\times 10^{-5}\times\left(6-12\frac{A}{M}\tan\alpha\right)\text{ MeV}^{-1}; \qquad (2.15)$$

$$e=(4\pi/137)^{1/2}=0.303.$$

In the case $A/M=0$ we find $g=2.17\times 10^{-5}$ MeV^{-1}, and $\Gamma_{\eta\to 2\gamma}=386$ eV, a factor 2.7 larger than found via $\pi^0\to 2\gamma$. This discrepancy is independent of A/M. The ratio $\Gamma_{\eta\to\pi\pi\gamma}/\Gamma_{\eta\to 2\gamma}$, however, is dependent on A/M. We have

$$\frac{\Gamma_{\eta\to\pi\pi\gamma}}{\Gamma_{\eta\to 2\gamma}}=0.233\left(\frac{1-(A/M)\tan\alpha}{1-2(A/M)\tan\alpha}\right)^2. \qquad (2.16)$$

Experimentally this ratio is 0.19, corresponding to $A/M\simeq -1$. The values $A/M=1$, 2 give 0.4 and 1.6, respectively.

The situation is now as follows: Using the rate $\omega\to\pi^0\gamma$ as input, the rate for $\pi^0\to 2\gamma$ can be computed in two ways:

$$\Gamma_{\omega\to\pi^0\gamma}\xrightarrow{\text{SU}_3}\Gamma_{\eta\to\pi\pi\gamma}\xrightarrow{\text{Exp.}}\Gamma_{\eta\to 2\gamma}\xrightarrow{\text{SU}_3}\Gamma_{\pi^0\to\gamma\gamma}\sim 19\text{ eV}; \qquad [1]$$

$$\Gamma_{\omega\to\pi^0\gamma}\xrightarrow{\text{GSW}}\Gamma_{\pi^0\to 2\gamma}\sim 19\text{ eV}. \qquad [2]$$

These two methods give identical results but disagree by a factor 3 with the experimental result[18] $\Gamma_{\pi^0\to\gamma\gamma}=6.6\pm 3.3$ eV. It might be remarked that the GSW model, applied to compute $\omega\to\rho\pi$ from $\omega\to\pi^0\gamma$, gives a result which is a factor of 2 too low.[5]

On the whole, the situation is not very satisfying. It is not possible to pin down which experimental number it is that disagrees with the theory. For instance, $\Gamma_{\pi\to 2\gamma}$ or $\Gamma_{\eta\to\pi\pi\gamma}$ may be low for some unknown reason. The situation might become clearer if new experimental results for any of the decay rates depending on A/M become available. (It would be quite helpful in this connection if the limits on the $\eta^{0*}\to\rho\gamma$ rate, the $\varphi\to\eta\gamma$ rate, or the η lifetime itself, could be improved.)

From the above we conclude that the η width will be somewhere between 3000 and 100 eV. Whether or not the GSW model predicts correctly the $\eta\to\pi\pi\gamma$ / $\eta\to 2\gamma$ ratio[5] is a question that can be answered only if the ratio of singlet to octet coupling is known. Clearly, any order-of-magnitude estimate of η decay parameters involving $\eta\to 2\gamma$ or $\eta\to\pi\pi\gamma$ cannot be trusted. A typical example is the estimate of C-violating effects in $\eta\to\pi\pi\gamma$. An estimate of the strength of the $\pi^+\pi^-$ D-wave amplitude relative to the P wave cannot be of any interest if the P wave itself is not understood. Further, the GSW model, while it may correctly prescribe the form of the P-wave amplitude, does not apply to the D wave.

3. $\eta\to 3\pi$

We now discuss some phenomenological aspects of the 3π decay of the η.[19] The partial decay rate $\eta\to\pi^+\pi^-\pi^0$ is given by

$$\Gamma_{\eta\to\pi^+\pi^-\pi^0}=\frac{g^2}{64\pi^3M}\int_{y_-}^{y_+}dy\int_{x_-}^{x_+}dx\,F(y,x), \qquad (3.1)$$

where

$$y_-=m-M/3,$$
$$y_+=M/6-1.5m^2/M,$$
$$x_\pm=\pm|\mathbf{p}|[(M^2-m^2-2ME_0)^2-4m^4]^{1/2}/2(M^2+m^2-2ME_0),$$

and

$$y=E_0-M/3,\quad x=\tfrac{1}{2}(E_+-E_-),\quad |\mathbf{p}|=(E_0^2-m^2)^{1/2},$$

$M=M_\eta$, $m=M_\pi$, $E_{0,\pm}=$ pion energies. The segments of the Dalitz plot are the areas bounded by the lines $E_0=E_+$, $E_0=E_-$, $E_-=E_+$ (Fig. 1). The function $F(y,x)$,

[18] P. Stamer *et al.*, Phys. Rev. **151**, 1108 (1966). This experiment gives $\tau_{\pi^0}=(1.0\pm 0.5)\times 10^{-16}$ sec, using emulsion techniques. This compares well with the older value of von Dardel *et al.*, Phys. Rev. Letters 4, 51 (1963), who, from their counter experiment, give $\tau_{\pi^0}=(1.05\pm 0.18)\times 10^{-16}$ sec. The reader is referred to Table 6 of the paper of Stamer *et al.*, for comparison with other experiments, some of which give rather different results. Note that the Rosenfeld compilation gives $\tau_{\pi^0}=(1.78\pm 0.26)\times 10^{-16}$ sec. [A. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965)]. In the text we use the Stamer value.

[19] J. Prentki and M. Veltman, in *Preludes to Theoretical Physics*, edited by A. de-Shalit *et al.* (North-Holland Publishing Company, Amsterdam, 1966), p. 250; T. D. Lee, Phys. Rev. **139**, B1415 (1965).

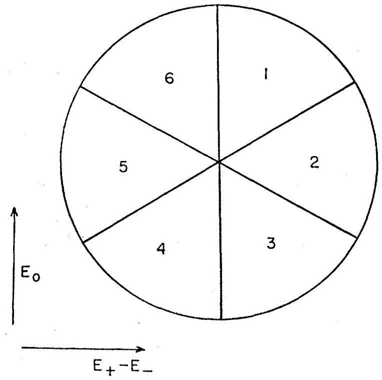

FIG. 1. Lines of zero amplitude in the Dalitz plot for $\eta\to\pi^+\pi^-\pi^0$ via a $|\Delta I|=0$ transition.

normalized so that the x,y independent part is 1, is the experimentally measured Dalitz-plot distribution and describes the decay including final state interactions. The dimensionless coupling constant g is supposedly of order $\alpha=1/137$, but turns out to be between 0.075 and 0.41 for $100<\Gamma_\eta<3000$ eV, one to two orders-of-magnitude larger.

There is no experimental evidence for any resonant structure of $F(y,x)$ over the range considered here, and we may expand $F(y,x)$ in a Taylor series:

$$F(y,x)=1+a_{10}y+a_{01}x+a_{20}y^2+a_{11}xy+a_{02}x^2+\cdots. \quad (3.2)$$

If C is conserved, $F(y,x)=F(y,-x)$. In integrating over the Dalitz plot, the terms odd in x and/or y do not contribute. Neglecting terms odd in x, the data give a good fit to[13]

$$F(y,x)=1-(0.96\pm0.08)y/y_+. \quad (3.3)$$

In view of the relation $F(y,x)=|A(y,x)|^2$, where $A(y,x)$ is the transition amplitude, we should have a contribution $a_{20}y^2$ if there exists a contribution $a_{10}y$. In this case the resulting quadratic term is $\geq 0.2(y^2/y_+^2)$. Of course, there could be additional contributions to a_{20}, but experimentally all y^2 terms seem to be very unimportant, their effects on the total rate being $\lesssim 15\%$.

Thus, *independent of C violation*, the total rate is determined up to 15% corrections by the energy-independent part of $F(y,x)$. This situation is very similar to the one encountered in $K^+\to 3\pi$ and $K^0\to 3\pi$, and in fact the structure of the $\eta\to 3\pi$ Dalitz plot is quite similar to the structure of $K_{02}\to\pi^+\pi^-\pi^0$. There one finds the good fit[20]

$$F(y,x)=1-(0.75\pm0.07)y/y_+. \quad (3.4)$$

This similarity is in agreement with the idea that the observed structure is due to 3π final-state interactions. The K_{02} data strongly suggest that the 3π system has isospin 1, and the similarity noted above supports the usual idea that the decay $\eta\to 3\pi$ obeys $|\Delta\mathbf{I}|=1$.

The transitions $|\Delta\mathbf{I}|=0$ and $|\Delta\mathbf{I}|=2$ violate C, and are thus antisymmetric in x. We consider them below. To lowest order in the e.m. interactions we can have $|\Delta\mathbf{I}|=1$, but not $|\Delta\mathbf{I}|=3$. The $\mathbf{I}=1$ state of 3 pions contains a totally symmetric part, corresponding to the isospin structure $(\boldsymbol{\pi}\cdot\boldsymbol{\pi})\boldsymbol{\pi}$, and a part that contains no symmetric piece $(\boldsymbol{\pi}\times\boldsymbol{\pi}\times\boldsymbol{\pi})$. Only the symmetric part contributes to the energy-independent piece of $F(y,x)$, and from the Dalitz-plot distribution[13] we conclude that up to 15% the rate is determined by the symmetric $|\Delta\mathbf{I}|=1$ amplitude. Using the energy-independent part alone, and including a factor 1.13 for the phase-space correction due to the pion mass splittings,

$$R\equiv\Gamma_{\eta\to\pi^0\pi^0\pi^0}/\Gamma_{\eta\to\pi^+\pi^-\pi^0}=1.69. \quad (3.5)$$

An experimental deviation of more than 10–15% from the above would indicate the presence of a $|\Delta\mathbf{I}|=3$ contribution. We can write R as a function of the ratio between the $|\Delta\mathbf{I}|=3$ and $|\Delta\mathbf{I}|=1$ energy-independent amplitudes[21]:

$$R=|\lambda\sqrt{2}-\sqrt{3}/\lambda\sqrt{3}+\sqrt{2}|^2\times1.13;$$
$$\lambda\equiv[A(\Delta I=3)/A(\Delta I=1)]. \quad (3.6)$$

The present experimental value $R\sim0.5$ gives $|\lambda|\geq0.31$. A value $R=1$ requires $|\lambda|\geq0.13$.

Here we wish to point out that at present there are no theoretical or experimental reasons for believing that an interaction with $|\Delta\mathbf{I}|=3$, and of e.m. strength, is not present. It is a very ugly possibility, but one can say nothing else.[22] No data on nuclear transitions give information on this point, and the relation,

$$\Delta^{++}-\Delta^-=3(\Delta^+-\Delta^0), \quad (3.7)$$

based on $|\Delta\mathbf{I}|\leq2$ for the masses of the $N^*(1238)$ $I=\frac{3}{2}$ multiplet, has been used with a certain success for other purposes,[23] but is not yet experimentally tested.

The experimental data indicate there may be a small C violation in $\eta\to\pi^+\pi^-\pi^0$.[13] If so, then the next important question is whether the C-violating amplitude obeys $|\Delta\mathbf{I}|=0$ or $|\Delta\mathbf{I}|=2$. A $|\Delta\mathbf{I}|=0$ amplitude, having an isospin structure $(\boldsymbol{\pi}\times\boldsymbol{\pi})\cdot\boldsymbol{\pi}$, is totally antisymmetric in all three pions. It does not contribute, therefore, to a_{01} or a_{11}. Let us now suppose that the

[20] J. S. Bell and J. Steinberger, paper given at the Oxford International Conference on Elementary Particles, 1965 (unpublished). Note that the mechanism giving $\Delta I=3$ in $\eta\to3\pi$ would give rise to a $\Delta I=\frac{7}{2}$ contribution of about 1% in K decay, which will be very hard to distinguish from the e.m.-induced $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{5}{2}$ admixtures of relative strength α.

[21] G. Feinberg and A. Pais, Phys. Rev. Letters **9**, 45 (1962). One of us (M.V.) wishes to acknowledge a helpful discussion with Professor C. N. Yang and Dr. Tsu-Teh Chou on the possible presence of a $|\Delta\mathbf{I}|=3$ amplitude in $\eta\to3\pi$.

[22] The authors are indebted to Professor C. A. Barnes, Dr. J. Weneser, and Dr. E. K. Warburton for several discussions concerning the possible presence of $|\Delta\mathbf{I}|=3$ transitions in nuclei. See also S. Weinberg and S. B. Treiman, Phys. Rev. **116**, 465 (1965); P. Kabir and N. Dombey, Phys. Rev. Letters **17**, 730 (1966).

[23] For a discussion of electromagnetic mass formulas in SU_3 and SU_6, see S. L. Glashow and R. Socolow, in *High Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965), p. 423.

TABLE III. Using the Hamiltonian (3.9) we show the number of events, normalized to 100, to be expected in each segment of the Dalitz plot, for various choices of the coupling constant. (See Fig. 1.) The world data, excluding the CERN results, are shown in the last column and comprise 3904 events.

Segment	$g_2=0$	$g_0=0$	$g_2=0.01g_0$	$g_2=-0.01g_0$	Experiment[a]
1	7.72	8.32	8.3	8.34	8.1
6	7.70	7.09	7.09	7.09	7.3
2	14.41	16.88	17.65	16.1	16.4
5	15.88	13.41	12.7	14.1	15.8
3	27.41	28.31	28.0	28.6	27.5
4	26.88	25.99	26.0	25.75	25.0
A_S[b]	−0.92	7.02	8.0	6.1	3.9
X_1[c]	0.55	3.55	3.0	4.1	3.3
X_2[d]	−7.47	3.47	4.9	2.0	0.6

[a] Including the following experiments reported at the Berkeley Conference: Columbia-Stony Brook, Rutherford-Saclay, Duke, and the earlier compilation reported by P. Franzini. The CERN results are not included because the populations, corrected for experimental bias, were not available.
[b] $A_S=N_1+N_2+N_3-N_4-N_5-N_6$.
[c] $X_1=N_1+N_3-N_4-N_6$.
[d] $X_2=N_2-N_5$.

series for $F(y,x)$ converges well; i.e., that $F(y,x)$ is a relatively smooth function without rapid variations in the energy range considered here ($\cong$84 MeV). If there are no resonances one customarily assumes, in accordance with other observations, that variations of $F(y,x)$ are small if one restricts oneself to energy ranges of the order of less than a few pion masses.This implies a rapid convergence of the Taylor series for $F(y,x)$. Thus, a $|\Delta\mathbf{I}|=0$ term can show up only if there are rapid variations or resonances associated with it. At this writing, there is no evidence for any P wave, or higher, $I=1$ di-pion enhancement in this region, and the nearest resonance of interest is the ρ. We will assume in the following that final-state interactions for the $|\Delta\mathbf{I}|=0$ and $|\Delta\mathbf{I}|=2$ modes are adequately described by inserting an intermediate ρ. (Fig. 2). This implies that the amplitudes for $|\Delta\mathbf{I}|=0$ and $|\Delta\mathbf{I}|=2$ are nearly in phase, since the induced phase shifts are rather small.

A further fact of importance has to do with the convergence of the series for $F(y,x)$. Suppose that the C-violating interaction obeys $|\Delta\mathbf{I}|=0$. Barring strong P-wave, or higher, enhancements, C-violating effects will be very small. The e.m. interactions will induce a $|\Delta\mathbf{I}|=2$ part, i.e., a contribution to a_{01}. This contribution is obviously proportional to α, but because of the expected rapid convergence of the series for $F(y,x)$, its effects may be pronounced relative to the pure $|\Delta\mathbf{I}|=0$ part. Thus, a pure $|\Delta\mathbf{I}|=0$ C-violating interaction will not give rise to a well-determined interference pattern.

Conversely, a $|\Delta\mathbf{I}|=2$, C-violating amplitude contributes directly to a_{01}, and only small effects due to higher order terms in the series are expected. For an expression of the form

$$F(y,x)=1+a_{10}y+a_{01}x+a_{20}y^2+a_{11}xy+a_{02}x^2 + \text{terms even in } x$$

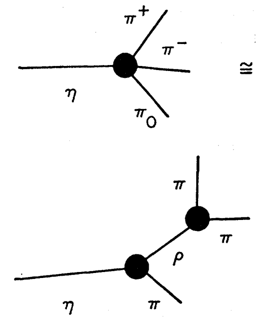

FIG. 2. Intermediate ρ model for $\eta\to\pi^+\pi^-\pi^0$.

one derives

$$A_{16}+A_{34}=A_{25}\,, \tag{3.8}$$

where $A_{ij}=N_j-N_i$, N_k being the number of events in sector k.

We conclude this section by giving, in Table II, the results of some calculations exhibiting the features mentioned above. As effective Hamiltonian we take

$$H_{\text{eff}}=g\eta(\boldsymbol{\pi}\boldsymbol{\pi})\pi^0+g'\partial_\mu\eta(\boldsymbol{\pi}\boldsymbol{\pi})\partial_\mu\pi^0+ig_0\eta(\boldsymbol{\rho}_\mu\partial_\mu\boldsymbol{\pi}) +ig_2\eta\rho_\mu{}^0\partial_\mu\pi^0+g_3\boldsymbol{\rho}_\mu(\partial_\mu\boldsymbol{\pi}\times\boldsymbol{\pi}) \tag{3.9}$$

where the boldface and upper indices refer to isospin. The $\pi\pi$ S-wave scattering phase shift is presumably very large, i.e., g and g' will have large imaginary parts. Apart from the magnitude of the interference effect, the phases of g and g' with respect to g_0 and g_2 (the last two are taken to be real in accordance with our assumption of ρ dominance in the relevant final states) are not of much relevance here, and we choose g and g' to be pure imaginary. The magnitude of g' with respect to g is fixed by the experimentally determined linear dependence on the π^0 energy, i.e., y. Further we will set $g_2=\pm0.01g_0$, and we fix g_0 so that an asymmetry of about 6% is obtained. One has then $g_0=6.5g$, $g_2=\pm0.065g$. The case $g_0=0$, $g_2\neq0$ is also shown. The combined data of Ref. 13 are listed in the last column of the table. The important quantities are X_1 and X_2, corresponding to the left- and right-hand side of Eq. (3.8). Note that the world data are only 1.75 standard deviations away from $X_1=X_2$, thus not allowing any conclusion with respect to the validity of Eq. (3.8).

4. RELATION BETWEEN $\eta\to3\pi$ AND $\eta\to\pi^0e^+e^-$

If one assumes the transition $\eta\to\rho\pi$ is responsible for C-violating effects in $\eta\to3\pi$, then one may try, using the GSW model, to make a prediction for the rate $\Gamma_{\eta\to\pi^0e^+e^-}$. Conversely, knowing the rate $\Gamma_{\eta\to\pi^0e^+e^-}$ gives us information about the asymmetry in $\eta^0\to\pi^+\pi^-\pi^0$.[17] Here we wish to show that such a connection depends very strongly on the momentum-transfer dependence of the $\eta\rho\pi$ vertex. Since the results have some practical consequences only if we assume $|\Delta\mathbf{I}|=0$ for this coupling, we will consider that case only.

Furthermore, we will assume that the ρ meson couples to a conserved current. The matrix element for the process $\eta \to \pi e^+e^-$ is then given by

$$A_{\eta\to\pi^0 e^+ e^-}=\frac{e^2}{5}f(k^2)\frac{m_\rho^2}{k^2+m_\rho^2-im_\rho\Gamma_\rho}\times\frac{(\eta+\pi^0)_\mu}{m_\eta^2-m_\pi^2}(u_e\gamma^\mu u_e)\,, \quad (4.1)$$

where η_μ, $\pi_\mu{}^0$ are the η and π^0 four-momenta, $k=p_++p_-$ and $e=(4\pi/137)^{1/2}$. In our metric, k^2 is negative for time-like k. We take

$$f(k^2)\cong\lambda_1+\lambda_2(k^2/m_\rho^2)\,, \quad (4.2)$$

where λ_1 and λ_2 are dimensionless.[24] The case $\lambda_1=0$ corresponds to zero charge radius for the $\eta\pi^0\gamma$ vertex. It was demonstrated in Ref. 17 that with $\lambda_2=0$, $\lambda_1\neq 0$ the experimental limit of 1% on $\Gamma_{\eta\to\pi^0e^+e^-}/\Gamma_{\text{total}}$ leads to a maximum asymmetry of 3% in $\eta\to\pi^+\pi^-\pi^0$.[25] Let us denote the corresponding value of λ_1, leading to $\Gamma_{\eta\to\pi e^+e^-}/\Gamma_\eta{}^{\text{total}}=1\%$, by λ. Note that in this case the variation of amplitude over the Dalitz plot, necessary to obtain a nonzero contribution to a $|\Delta\mathbf{I}|=0$ transition, is obtained through the ρ propagator. It is easy to see that the functions $f(k^2)=\lambda$ or $f(k^2)=\lambda(k^2/m^2)$ produce almost equally large asymmetries. On the other hand, setting $\lambda_1=0$ one finds that a value $\lambda_2=13.5\lambda$ is needed to obtain the same branching ratio for $\eta\to\pi^0e^+e^-$ as with $\lambda_1=\lambda$, $\lambda_2=0$. Thus, if we assume a zero charge radius for the $\eta\pi\gamma$ vertex, the experimental limit on $\eta\to\pi^0e^+e^-$ [23] leads to an upper limit of about 10% on the charge asymmetry in $\eta\to\pi^+\pi^-\pi^0$.

5. CONCLUSION

In the foregoing the connection of the η decay modes, through SU_3, to other decays has been studied. We find that the branching ratio $\eta\to\pi^+\pi^-\gamma/\eta\to 2\gamma$ depends on the ratio of singlet (A) and octet (M) coupling, and as a result values of A/M around -1 are expected. This, in turn, suggests a rather large η width, somewhere between 500 and 2300 eV. There is no value of A/M that fits all the available data.

The known Dalitz distribution of the three pions in $\eta\to\pi^+\pi^-\pi^0$ leads to a branching ratio

$$R=\Gamma_{\eta\to\pi^0\pi^0\pi^0}/\Gamma_{\eta\to\pi^+\pi^-\pi^0}=1.7(\pm15\%)\,.$$

An experimental deviation from this prediction implies the presence of a $|\Delta\mathbf{I}|=3$ interaction with strength of order 0.01. The authors are aware of no theoretical or experimental arguments for or against the presence of such an interaction, except for its ugliness. A possible test is provided by the relation for the masses $\Delta^{++}-\Delta^-=3(\Delta^+-\Delta^0)$ of the $N^*(1238)$ multiplet. However, the similarity of the $\eta\to\pi^+\pi^-\pi^0$ and $K_{20}\to\pi^+\pi^-\pi^0$ Dalitz plots is evidence against the presence of a $|\Delta\mathbf{I}|=3$ transition.[20] We conclude that a direct experimental measurement of the branching ratio $\Gamma_{\eta\to3\pi^0}/\Gamma_{\eta\to\pi^+\pi^-\pi^0}$ is essential in this connection.

The isospin properties of a possible C-violating interaction lead to predictions for the asymmetry pattern in the $\eta\to\pi^+\pi^-\pi^0$ Dalitz plot. If the C-violating interaction allows $|\Delta\mathbf{I}|=2$, then the relation (3.8) must hold to good accuracy; otherwise we conclude that there is a considerable C-violating amplitude with $|\Delta\mathbf{I}|=0$ present. Further we find that, using the model of Ref. 17, we are unable to draw any useful information from the present experimental upper limit on $\Gamma_{\eta\to\pi e^+e^-}/\Gamma_\eta{}^{\text{total}}$, with respect to the magnitude of the asymmetry in $\eta\to\pi^+\pi^-\pi^0$.[25] The relation between $\eta\to3\pi$ and $\eta^*\to3\pi$ is independent of form-factor considerations, but on the other hand, many unknowns, such as different SU_3 properties, enter, and this problem is therefore not discussed here.

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[24] Of course, by introducing a function $f(k^2)$ as above we deviate from the spirit of the model of Ref. 17. In fact, the expression (4.1) is still perfectly general. Setting $\lambda_2=0$ in (4.2) gives the model of Ref. 17, while a nonzero λ_2 implies the presence of further structure not describable by the exchange of a ρ alone.

[25] The figure of 1% comes from L. Price and F. S. Crawford, Phys. Rev. Letters **15**, 123 (1965). We are now informed by Professor H. H. Bingham that the world data give an upper limit on the $\eta\to\pi^0e^+e^-$ branching ratio of 10^{-3}. The authors gratefully acknowledge a great number of helpful discussions with Professor Bingham. See C. Baglin and H. H. Bingham *et al.* (to be published).