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Classical Scattering of Neutral Mesons. II*

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In Paper I, the scattering of neutral scalar and vector mesons by nucleons was calculated on the basis of a classical action-at-a-distance theory and compared with the corresponding field-theoretical results. In both theories the postulated vector and tensor coupling terms of the vector mesons with the nucleons were analogous to those of electrodynamics. Recently, a more general form of interaction was proposed; for monopole singularities it introduces a pseudovector coupling term which does not satisfy an equation of continuity. In this paper, the scattering of neutral spin-1 mesons by heavy particles is calculated using all three coupling terms, on the basis of the field-theoretical as well as the action-at-a-distance equations of motion of the particles. Cross sections due to the pseudovector coupling alone are examined in detail. Unlike the results of Paper I, the new field-theoretical and action-at-a-distance results show significant differences; furthermore, the behavior of the cross sections is found to depend critically on the magnitudes of the physical constants characterizing the meson field and the particles, exhibiting various resonance-like features.

I. INTRODUCTION

IN Paper I[1] the scattering of neutral scalar and vector mesons by nucleons was calculated on the basis of a classical theory of point particles developed from the point of view of action at a distance.[2] The resultant cross sections were compared with those obtained using a field-theoretical approach by Harish-Chandra[3] and Bhabha.[4]

The motivation for the development of the action-at-a-distance equations was to obtain a theory free from the divergence difficulties inherent in the field-theoretical approach, closely analogous to the development familiar from electrodynamics.[5] While the predictions following from the two approaches are identical in electrodynamics,[6] this is not the case in meson theory. However, the differences found in I are too small to allow an experimental distinction.

In both the field-theoretical and the action-at-a-distance calculations, the interactions of the vector mesons with the nucleons were formulated in analogy with the electromagnetic interactions, and thus the source densities were required to satisfy an equation of continuity. It was shown subsequently that for fields of nonzero rest mass and spin this restriction is more stringent than required by the invariance properties of the fields[7]; using a less restrictive condition, a more general set of equations was proposed introducing new interactions. Corresponding equations of motion were derived for arbitrary multipole singularities in neutral, charged, and charge-symmetric fields. In a subsequent paper,[8] forms of the multipole moments compatible with these equations for vector and pseudovector (spin-one) fields were established.

In this paper we restrict our attention to monopole and dipole singularities of neutral meson fields of spin one. Two monopole interaction terms are used, a vector one which was introduced by Bhabha,[4] and a pseudovector one which does not satisfy the equation of continuity.[8] The dipole interaction term used is also due to Bhabha.[9]

The radiation reaction terms following from these interactions (which were obtained by the method of Harish-Chandra[10]) introduce third-order derivatives of the multipole moments; any of the possible multipole interaction terms proposed[8] would lead to higher order derivatives and result in effects whose evaluation is prohibitive.

Cross sections are calculated for the scattering of neutral spin-one mesons by heavy particles; for convenience we shall refer to these particles as nucleons, without, however, implying identification with any currently known particles. These cross sections show significant qualitative as well as quantitative differences in behavior as functions of meson energy for the field-theoretical and the action-at-a-distance equations. Furthermore, their behavior is found to depend critically on the magnitudes of the physical constants characterizing the mesons and nucleons. The results obtained are instructive because they show the wide variety of features associated with a particular dynamical theory, features which without the availability of such a theory might be interpreted as being due to a correspondingly wide variety of physical phenomena.

The calculations presented here are entirely classical. It is of course clear that a theory of elementary particle

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[1] C. R. Mehl and P. Havas, Phys. Rev. **91**, 393 (1953); hereafter referred to as I. The results are summarized there in Table I, where, however, a factor 2 was omitted erroneously in all cross sections for incoming longitudinal mesons.

[2] P. Havas, Phys. Rev. **87**, 309 (1952).

[3] Harish-Chandra, Proc. Indian Acad. Sci. **21**, 135 (1945).

[4] H. J. Bhabha, Proc. Roy. Soc. (London) **A172**, 384 (1939).

[5] For references to the earlier literature see I.

[6] P. Havas, Phys. Rev. **74**, 456 (1948).

[7] P. Havas, Phys. Rev. **113**, 732 (1959).

[8] P. Havas, Phys. Rev. **116**, 202 (1959).

[9] H. J. Bhabha, Proc. Roy. Soc. (London) **A178**, 314 (1941).

[10] Harish-Chandra, Proc. Roy. Soc. (London) **A185**, 269 (1946).

interactions must be quantum mechanical. In recent years much work has been done on quantum mechanical alternatives to field theory[11]; however, a satisfactory quantum theory of action at a distance is still lacking, and thus the problems considered in this paper can as yet be approached only from the classical end. It has been shown, however, that the results obtained in I as well as numerous other results of the classical theory of action at a distance are the classical limit of the quantum theory of radiation damping.[12] Thus the similarities revealed between these theories could be exploited to anticipate the results of a future quantum theory of action at a distance by calculations based on the quantum theory of radiation damping.

The classical action-at-a-distance theory used here was developed in close analogy to classical field theory. It should be noted, however, that both classical and quantum theory also provide the possibility of action-at-a-distance formalisms which do not necessarily have a field-theoretical counterpart.[13] These have not been developed sufficiently, however, to establish whether they can be of importance in elementary particle physics.

II. THE EQUATIONS OF MOTION

A neutral spin-one meson field is described by the potentials U_μ and the field strengths $U_{[\mu\nu]}$, where

$$U_{[\mu\nu]}=\partial_\mu U_\nu-\partial_\nu U_\mu. \tag{1}$$

The potentials satisfy the field equations [Eqs. (23) and (24) of Ref. 7]

$$\Box U_\mu+\chi^2U_\mu=4\pi(\rho_\mu+\chi^{-2}\partial_\mu\partial^\alpha\rho_\alpha), \tag{2}$$

with the restriction

$$\chi^2\partial^\alpha U_\alpha=4\pi\partial^\alpha\rho_\alpha. \tag{3}$$

In general we are using the same notation as in I. The constant χ characterizes the rest mass of the meson and the vector ρ_μ describes the source densities. For a source at z_ρ due to a monopole-dipole singularity, ρ_μ is given by

$$\rho_\mu=\int_{-\infty}^{\infty}S_{,\mu}\delta(s_0)\delta(s_1)\delta(s_2)\delta(s_3)d\tau +\partial^\alpha\int_{-\infty}^{\infty}S_{\alpha,\mu}\delta(s_0)\delta(s_1)\delta(s_2)\delta(s_3)d\tau, \quad s_\rho\equiv x_\rho-z_\rho, \tag{4}$$

where $S_{,\mu}$ and $S_{\mu,\nu}$ represent the monopole and the dipole moments, respectively. The equations of motion of a point particle described by (4) are (see Theorem I of Ref. 8[14])

$$\dot{A}_\mu=S^{,\sigma}\partial_\mu U_\sigma-S^{\alpha,\sigma}\partial_{\mu\alpha}U_\sigma, \tag{5}$$

$$\dot{B}_{\mu\nu}=S_{,\mu}U_\nu-S_{,\nu}U_\mu-S_{\mu,\sigma}\partial_\nu U^\sigma+S_{\nu,\sigma}\partial_\mu U^\sigma -S_{\alpha,\mu}\partial^\alpha U_\nu+S_{\alpha,\nu}\partial^\alpha U_\mu+v_\nu A_\mu-v_\mu A_\nu, \tag{6}$$

with

$$A_\mu=(m+S_{,\sigma}U^\sigma-S_{\alpha,\sigma}\partial^\alpha U^\sigma)v_\mu-B_{\mu\sigma}\dot{v}^\sigma +S_{,\sigma}v^\sigma U_\mu-S_{,\mu}v^\sigma U_\sigma +S_{\mu,\sigma}\dot{U}^\sigma-S_{\alpha,\sigma}v^\sigma\partial^\alpha U_\mu+S_{\alpha,\mu}\partial^\alpha U_\sigma v^\sigma, \tag{7}$$

where m is the mass of the particle (nucleon). A dot over the quantity denotes differentiation with respect to the proper time τ, and $v^\mu\equiv\dot{z}^\mu$ is the four-velocity. A_μ is the four-momentum of the nucleon and $B_{\mu\nu}$ is an antisymmetric tensor which can be interpreted as the intrinsic angular momentum of the nucleon if it is subject to the conditions

$$v^\mu B_{\mu\nu}=0,\quad B^{\mu\nu}\dot{B}_{\mu\nu}=0. \tag{8}$$

It follows from Theorems I and II of Ref. 8 that we can choose a monopole moment of the form

$$S_{,\mu}=g_1v_\mu+fS_\mu,\quad S^\mu=\epsilon^{\mu\alpha\beta\gamma}B_{\alpha\beta}v_\gamma, \tag{9}$$

and a dipole moment defined by

$$S_{\mu,\nu}=g_2B_{\mu\nu}, \tag{10}$$

where f, g_1, and g_2 are coupling constants which, for convenience, were not given explicitly in Ref. 8. $\epsilon^{\mu\alpha\beta\gamma}$ is a pseudotensor antisymmetric in each pair of indices, with $\epsilon^{0123}=-\epsilon_{0123}=1$. $B_{\mu\nu}$ and S_ν satisfy the equations

$$S^\nu B_{\mu\nu}=0,\quad v^\mu S_\mu=0,\quad S^\mu\dot{S}_\mu=0. \tag{11}$$

Thus the translational and rotational equations of motion (5) and (6) can be written

$$\frac{d}{d\tau}\{[m+fS^\sigma U_\sigma-g_2B^{\alpha\sigma}\partial_\alpha U_\sigma]v_\mu-B_{\mu\sigma}\dot{v}^\sigma+g_1U_\mu-fS_\mu v^\sigma U_\sigma+g_2[B_{\mu\sigma}\dot{U}^\sigma+B_{\alpha\mu}\partial^\alpha v^\sigma U_\sigma]\} =[g_1v^\sigma+fS^\sigma]\partial_\mu U_\sigma-g_2B^{\alpha\sigma}\partial_{\mu\alpha}U_\sigma, \tag{12}$$

and

$$\dot{B}_{\mu\nu}=f[S_\mu U_\nu-S_\nu U_\mu+(v_\mu S_\nu-v_\nu S_\mu)v^\sigma U_\sigma]+g_2[B_{\nu\sigma}\partial_\mu U^\sigma-B_{\mu\sigma}\partial_\nu U^\sigma+B_{\alpha\mu}\partial^\alpha U_\nu-B_{\alpha\nu}\partial^\alpha U_\mu+v_\nu B_{\mu\sigma}\dot{U}^\sigma-v_\mu B_{\nu\sigma}\dot{U}^\sigma +(B_{\alpha\mu}v_\nu-B_{\alpha\nu}v_\mu)\partial^\alpha U_\sigma v^\sigma]-[v_\mu B_{\nu\sigma}\dot{v}^\sigma-v_\nu B_{\mu\sigma}\dot{v}^\sigma]. \tag{13}$$

[11] See, e.g., G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1961).
[12] J. E. Chatelain and P. Havas, Phys. Rev. **129**, 1459 (1963); R. L. Knight, Utah State University thesis, 1964 (unpublished).
[13] For a review of the various approaches see P. Havas, in *Statistical Mechanics of Equilibrium and Non-Equilibrium*, edited by J. Meixner (North-Holland Publishing Company, Amsterdam, 1965), p. 1.
[14] Note that in Eq. (C) of that theorem as well as in Eqs. (1), (15), (68), and (C′) of Ref. 8 we should have $_\mu$ (i.e., $dA_\mu/d\tau$), but, apparently due to defective type, the dot is missing in some or all of these expressions in many copies of the Physical Review.

The retarded and advanced solutions of Eqs. (2) and (3) are

$$U_\mu{}^r=\left(\frac{S_{,\mu}}{\kappa}\right)_r+\partial^\alpha\left(\frac{S_{\alpha,\mu}}{\kappa}\right)_r+\frac{1}{\chi^2}\partial_{\mu\sigma}\left(\frac{S^{,\sigma}}{\kappa}\right)_r+\frac{1}{\chi^2}\partial_{\mu\alpha\sigma}\left(\frac{S^{\alpha,\sigma}}{\kappa}\right)_r-\chi\int_{-\infty}^{\tau_r}S_{,\mu}\frac{J_1(\chi s)}{s}d\tau-\chi\partial^\alpha\int_{-\infty}^{\tau_r}S_{\alpha,\mu}\frac{J_1(\chi s)}{s}d\tau$$
$$-\frac{1}{\chi}\partial_{\mu\sigma}\int_{-\infty}^{\tau_r}S^{,\sigma}\frac{J_1(\chi s)}{s}d\tau-\frac{1}{\chi}\partial_{\mu\alpha\sigma}\int_{-\infty}^{\tau_r}S^{\alpha,\sigma}\frac{J_1(\chi s)}{s}d\tau, \quad (14)$$

and

$$U_\mu{}^a=-\left\{\left(\frac{S_{,\mu}}{\kappa}\right)_a+\partial^\alpha\left(\frac{S_{\alpha,\mu}}{\kappa}\right)_a+\frac{1}{\chi^2}\partial_{\mu\sigma}\left(\frac{S^{,\sigma}}{\kappa}\right)_a+\frac{1}{\chi^2}\partial_{\mu\alpha\sigma}\left(\frac{S^{\alpha,\sigma}}{\kappa}\right)_a-\chi\int_{\tau_a}^{\infty}S_{,\mu}\frac{J_1(\chi s)}{s}d\tau-\chi\partial^\alpha\int_{\tau_a}^{\infty}S_{\alpha,\mu}\frac{J_1(\chi s)}{s}d\tau\right.$$
$$\left.-\frac{1}{\chi}\partial_{\mu\sigma}\int_{\tau_a}^{\infty}S^{,\sigma}\frac{J_1(\chi s)}{s}d\tau-\frac{1}{\chi}\partial_{\mu\alpha\sigma}\int_{\tau_a}^{\infty}S^{\alpha,\sigma}\frac{J_1(\chi s)}{s}d\tau\right\}, \quad (15)$$

where $\kappa=s_\rho v^\rho$. In field theory, the potential U_μ equals the potential of an external field incident on the nucleon, denoted by $U_\mu{}^{\text{in}}$, plus the "modified radiation field" potential $U_\mu{}^{\text{rad}}$ defined by [10,15]

$$U_\mu{}^{\text{rad}}=\tfrac{1}{2}\{U_\mu{}^r-U_\mu{}^a\}_0-\tfrac{1}{2}\left\{\chi\int_{-\infty}^{\infty}S_{,\mu}\frac{J_1(\chi s)}{s}d\tau+\chi\partial^\alpha\int_{-\infty}^{\infty}S_{\alpha,\mu}\frac{J_1(\chi s)}{s}d\tau\right.$$
$$\left.+\frac{1}{\chi}\partial_{\mu\sigma}\int_{-\infty}^{\infty}S^{,\sigma}\frac{J_1(\chi s)}{s}d\tau-\frac{1}{\chi}\partial_{\mu\alpha\sigma}\int_{-\infty}^{\infty}S^{\alpha,\sigma}\frac{J_1(\chi s)}{s}d\tau\right\}_0, \quad (16F)$$

where the subscript 0 indicates that the expression has to be evaluated at the position of the particle.

In action-at-a-distance theory, there is no meaning to the action of a particle on itself. A consistent theory can be developed using half-retarded, half-advanced rather than retarded interactions, and omitting certain integrals extended over the entire world line of the particle which appear in the field-theoretical equations of motion with time-symmetric interactions. A description of radiation effects can then be obtained by considerations similar to those used by Wheeler and Feynman in electrodynamics.[16] The final result is simply a modification of the radiation reaction term in the field-theoretical equations of motion with retarded interactions; this was discussed in detail in Ref. 2 for the case of scalar and vector mesons, and the arguments used there apply here without change. Thus for a theory of action at a distance U_μ has to be taken as the sum of the incident potential $U_\mu{}^{\text{in}}$ and a radiation reaction term $U_\mu{}^{\text{rad}}$ defined by

$$U_\mu{}^{\text{rad}}=\tfrac{1}{2}\{U_\mu{}^r-U_\mu{}^a\}_0. \quad (16A)$$

Using Eqs. (9), (10), (14), and (15), the expressions (16F) and (16A) can be evaluated by the method of Harish-Chandra.[10] Using the abbreviation $(ab)\equiv a_\alpha b^\alpha$, we obtain[17]

$$U_\mu{}^{\text{rad}}=g_1\left\{-\dot v_\mu-\chi\int_{-\infty}^{\tau}v_\mu\frac{J_1(\chi s)}{s}d\tau'\right\}+f\left\{\tfrac{1}{2}(\dot vS)v_\mu-\tfrac{1}{2}\dot S_\mu-\frac{1}{\chi^2}\left[v_\mu\langle(v\ddot v)(\dot vS)+2(\dot v\ddot S)+\tfrac{1}{3}(\ddot v\dot S)+(v\dddot S)\rangle+\dot v_\mu\langle(v\ddot S)-\tfrac{1}{3}(\ddot vS)\rangle\right.\right.$$
$$\left.\left.+\tfrac{2}{3}(v\dot S)\ddot v_\mu-\tfrac{1}{3}\langle(\dot v\ddot v)S_\mu+(\dot v)^2\dot S_\mu+\dddot S_\mu\rangle\right]-\chi\int_{-\infty}^{\tau}S_\mu\frac{J_1(\chi s)}{s}d\tau'+\int_{-\infty}^{\tau}S_\mu\frac{J_2(\chi s)}{s^2}d\tau'-\chi\int_{-\infty}^{\tau}s_\mu s_\sigma S^\sigma\frac{J_3(\chi s)}{s^3}d\tau'\right\}$$
$$+g_2\left\{\tfrac{1}{6}\ddot v^\alpha B_{\alpha\mu}-\tfrac{1}{2}v^\alpha\ddot B_{\alpha\mu}+\chi^2\int_{-\infty}^{\tau}s^\alpha B_{\alpha\mu}\frac{J_2(\chi s)}{s^2}d\tau'\right\}, \quad (17F)$$

for the field-theoretical case, and a similar expression, with $\int_{-\infty}^{\tau}$ replaced by $\tfrac{1}{2}[\int_{-\infty}^{\tau}-\int_{\tau}^{\infty}]$ throughout, for the action at a distance case.

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[15] Here and in the following Eqs. (F) refer to field theory, and Eqs. (A) to action-at-a-distance theory.
[16] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **17**, 157 (1945); compare also Ref. 6, and P. Havas, Phys. Rev. **86**, 974 (1952).
[17] For details of the calculations see A. D. Craft, Lehigh University thesis, 1959 (unpublished).

III. CALCULATION OF SCATTERING CROSS SECTIONS

To calculate the scattering of neutral mesons, we must take for the potential U_μ^{in} describing the incident mesons a plane-wave solution of the empty-space field equations:

$$\begin{aligned} U_0^{\text{in}} &= \delta(k/\omega)\cos\varphi\sin(\omega\tau-\mathbf{k}\cdot\mathbf{R}),\\ U_1^{\text{in}} &= \delta\cos\theta_1\sin(\omega\tau-\mathbf{k}\cdot\mathbf{R}),\\ U_2^{\text{in}} &= \delta\cos\theta_2\sin(\omega\tau-\mathbf{k}\cdot\mathbf{R}),\\ U_3^{\text{in}} &= \delta\cos\theta_3\sin(\omega\tau-\mathbf{k}\cdot\mathbf{R}),\\ k &\equiv |\mathbf{k}| = (\omega^2-\chi^2)^{1/2}. \end{aligned} \tag{18}$$

Here $\cos\theta_i$ are the direction cosines of the polarization vector and φ is the angle between the directions of polarization and of propagation. No wave can be propagated for $\omega<\chi$.

We now substitute $U_\mu = U_\mu^{\text{in}} + U_\mu^{\text{rad}}$ with Eq. (17F) or the corresponding action-at-a-distance equation into Eqs. (12) and (13). The resulting integro-differential equations of motion are of such mathematical complexity that exact solutions of scattering problems cannot be obtained. The usual method for handling such problems is to assume the incoming field to be sufficiently weak that only small perturbations are induced in the nucleon variables,[4] and to retain only terms of first order in the amplitudes of the perturbations and the incoming field. The resultant motions are then considered to induce a scattered wave described by the retarded potential calculated at a point at a sufficiently large distance from the nucleon that only the first-order terms in $1/r$ are significant. The energy flows due to the scattered field and to the incoming field are calculated from the energy-momentum tensor, and the scattering cross sections are computed from the ratio of these energy flows.

In previous calculations of scattering from particles with a dipole moment[4,9,18,19] the small amplitude approximation resulted in a complete decoupling of the translational and rotational equations of motion; consequently, independent cross sections were obtained for monopole and dipole scattering. However, in our problem the equations cannot be uncoupled and thus the calculations are considerably more complicated.

If we omit the radiation damping terms, we can readily find solutions of the coupled equations of motion (5) and (6) in which the nucleon performs small-amplitude three-dimensional vibrations with the S_1 and S_2 components of the pseudovector S_μ oscillating with small amplitude about the constant component S_3. If the radiation damping is included, we shall assume, as usual, the motion of the nucleon variables to differ from that of the simpler problem only in the amplitudes and phases. Therefore we try the solutions

$$\begin{aligned} S_0 &= \epsilon_3 B\sin(\omega\tau+\alpha_3),\\ S_1 &= \eta_1\sin(\omega\tau+\beta_1),\\ S_2 &= \eta_2\sin(\omega\tau+\beta_2),\\ S_3 &= B, \end{aligned} \tag{19}$$

and

$$\begin{aligned} z_0 &= \tau,\\ z_1 &= -(\epsilon_1/\omega)\cos(\omega\tau+\alpha_1),\\ z_2 &= -(\epsilon_2/\omega)\cos(\omega\tau+\alpha_2),\\ z_3 &= -(\epsilon_3/\omega)\cos(\omega\tau+\alpha_3), \end{aligned} \tag{20}$$

where B is a constant and the α's and β's are phase constants. All terms appearing in the equations of motion which are quadratic in the amplitudes ϵ_1, ϵ_2, ϵ_3, η_1, η_2, and δ will be neglected.

From Eq. (9) we have

$$B_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\rho}v^\alpha S^\beta. \tag{21}$$

Therefore we can replace the various components of the tensor $B_{\mu\nu}$ in the equations of motion by the appropriate components of the products of the vectors v^α and S^β. Defining the spin I of the nucleon by

$$B_{\mu\nu}B^{\mu\nu} = 2I^2, \tag{22}$$

we have

$$S_\mu S^\mu = -B^2 = -2B_{\mu\nu}B^{\mu\nu} = -4I^2.$$

We consider only nonrelativistic velocities and thus $\tau\approx t$ and $s^2 = s_\rho s^\rho \approx s_0{}^2 = (t-t')^2$. All integrals in the equations of motion are transformed to integrations over s with the change of variables $t'=t-s$, $dt'=-ds$. The resulting integrals are readily evaluated.[17]

Substituting our expressions for the incoming field and the motion of the nucleon variables into the equations of motion we obtain ten simultaneous algebraic equations which determine the conditions imposed on the amplitudes and the phase constants in order that (19) and (20) be solutions in the first-order approximation. These conditions relate the amplitude ratios ϵ_j/δ, η_j/δ and the phase constants α_j, β_j to the frequency, polarization, and direction of propagation of the incoming field. The calculations are straightforward though very tedious and the resultant expressions are extremely long, and will not be presented here.[17]

If f is set equal to zero, the equations of motion uncouple and the results reduce to those obtained earlier both in field theory[4,9] and in action-at-a-distance theory.[18,19] The solutions for $f\neq 0$ are of the same type for f coupling alone as for this coupling together with g_1 and g_2 coupling.[17] Consequently the principal features of the pseudovector coupling of interest to us can be discussed by setting g_1 and g_2 equal to zero, which results in considerable simplification in the mathematical expressions. However, the complete expressions can be found in Ref. 17.

In order to compute the scattering cross sections, we

[18] C. R. Mehl, Lehigh University thesis, 1954 (unpublished).
[19] R. C. Majumdar, S. Gupta, and S. K. Trehan, Progr. Theoret. Phys. (Kyoto) **12**, 31 (1954).

need only the lowest order term in $1/r$ of the retarded potential. Then we get from Eq. (14)[17]

$$U_0{}^r=\frac{2fI}{r}\frac{\omega^2-\chi^2}{\chi^2}\sum_{j=1}^{3}\epsilon_j[\cos\Psi_j'\cos\Psi_3'-\delta_j{}^3]\sin(\omega t+\alpha_j-\mathbf{k}\cdot\mathbf{r})+\frac{f}{r}\frac{\omega(\omega^2-\chi^2)^{1/2}}{\chi^2}\sum_{j=1}^{2}\eta_j\cos\Psi_j'\sin(\omega t+\beta_j-\mathbf{k}\cdot\mathbf{r}),\quad(23)$$

and

$$U_i{}^r=\frac{2fI}{r}\frac{(\omega^2-\chi^2)^{1/2}}{\omega}\sum_{j=1}^{3}\epsilon_j\left\{\cos\Psi_i'\left[\frac{\omega^2-\chi^2}{\chi^2}(\cos\Psi_j'\cos\Psi_3'-\delta_i{}^3\delta_j{}^3)-(1-\delta_i{}^3)\delta_j{}^3\frac{\omega^2}{\chi^2}\right]+\delta_i{}^3(1-\delta_j{}^3)\cos\Psi_j'\right\}$$
$$\times\sin(\omega t+\alpha_j-\mathbf{k}\cdot\mathbf{r})+\frac{f}{r}\sum_{j=1}^{2}\eta_j\left[\frac{\omega^2-\chi^2}{\chi^2}\cos\Psi_i'\cos\Psi_j'+(1-\delta_i{}^3)\delta_j{}^i\right]\sin(\omega t+\beta_j-\mathbf{k}\cdot\mathbf{r}),\quad(24)$$

where $i=1, 2, 3$, the Ψ_i' are the angles between the propagation vector $\mathbf{k}$ and the x_i axes, and $\delta_j{}^i$ is the Kronecker δ.

To calculate the energy-flow terms necessary to get the cross sections, we must compute the time average of the energy-momentum tensor for the retarded and the incoming fields, respectively. This tensor is given by

$$4\pi T_\mu{}^\nu=U_{[\mu\sigma]}U^{[\sigma\nu]}+\tfrac{1}{4}\delta_\mu{}^\nu U_{[\sigma\rho]}U^{[\sigma\rho]}+\chi^2[U_\mu U^\nu-\tfrac{1}{2}\delta_\mu{}^\nu U_\sigma U^\sigma],\quad(25)$$

and its time average for a plane wave of the form

$$U_\mu=A_\mu\sin\omega_\alpha x^\alpha\quad(26)$$

equals[4]

$$-\frac{1}{8\pi}\omega_\mu\omega^\nu A^2.\quad(27)$$

The potential describing the incoming meson field is already of the form (26). The expression for the potential of the scattered field (23) and (24) can be put in this form with the amplitudes A_μ specified by rather lengthy expressions involving the phase constants α_j, β_j and the amplitudes of $\sin(\omega t+\alpha_j-\mathbf{k}\cdot\mathbf{r})$ and $\sin(\omega t+\beta_j-\mathbf{k}\cdot\mathbf{r})$.[17] As we wish to consider the effects of the longitudinal and transverse components of the retarded field separately, we must resolve the amplitudes of (23) and (24) into longitudinal and transverse components. Then we obtain for the energy flow into the solid angle $d\Omega'$ due to the longitudinal component

$$\frac{1}{8\pi}\omega(\omega^2-\chi^2)^{1/2}\left\{f^2\frac{\omega^2}{\chi^2}[\eta_1{}^2\cos^2\Psi_1'+\eta_2{}^2\cos^2\Psi_2']\right.$$
$$\left.+4f^2I^2\frac{\omega^2-\chi^2}{\chi^2}[\epsilon_1{}^2\cos^2\Psi_1'\cos^2\Psi_3'+\epsilon_2{}^2\cos^2\Psi_2'\cos^2\Psi_3'+\epsilon_3{}^2\sin^4\Psi_3']+Z^L\right\}d\Omega',\quad(28)$$

and due to the transverse component

$$\frac{1}{8\pi}\omega(\omega^2-\chi^2)^{1/2}\left\{f^2[\eta_1{}^2\sin^2\Psi_1'+\eta_2{}^2\sin^2\Psi^{2\prime}]\right.$$
$$\left.+4f^2I^2\frac{\omega^2-\chi^2}{\chi^2}[\epsilon_1{}^2\cos^2\Psi_1'\sin^2\Psi_3'+\epsilon_2{}^2\cos^2\Psi_2'\sin^2\Psi_3'+\epsilon_3{}^2\cos^2\Psi_3'\sin^2\Psi_3']+Z^T\right\}d\Omega'.\quad(29)$$

Z^L and Z^T represent expressions involving cross terms in the amplitudes and phases of (23) and (24). These terms vanish upon integration over the entire solid angle Ω' and hence will not be written out explicitly.[17]

The energy flow due to the incoming field defined by (18) is also computed using (27). If the incoming field is transverse, we have $\varphi=\pi/2$, and the energy flow equals

$$\frac{1}{8\pi}\omega(\omega^2-\chi^2)^{1/2}\delta^2.\quad(30)$$

If the field is longitudinal, we have $\varphi=0$, and the corresponding energy flow equals

$$\frac{1}{8\pi}\omega(\omega^2-\chi^2)^{1/2}\left(\frac{\chi\delta}{\omega}\right)^2.\quad(31)$$

Taking the appropriate ratios of (28) and (29) to (30) and (31), we obtain the four differential cross sections corresponding to all the possible combinations of a transverse or longitudinal incoming wave with a transverse or longitudinal scattered wave. Integrating over the entire solid angle, we get for the total cross sections

$$\sigma_{tt}=\frac{8\pi}{3}f^2\left\{\tfrac{4}{5}I^2\frac{\omega^2-\chi^2}{\omega^2}\left[2\frac{\epsilon_1^2+\epsilon_2^2}{\delta^2}+\frac{\epsilon_3^2}{\delta^2}\right]+\frac{\eta_1^2+\eta_2^2}{\delta^2}\right\}, \tag{32}$$

$$\sigma_{tl}=\frac{4\pi}{3}f^2\left(\frac{\omega}{\chi}\right)^2\left\{\tfrac{4}{5}I^2\frac{\omega^2-\chi^2}{\omega^2}\left[\frac{\epsilon_1^2+\epsilon_2^2}{\delta^2}+8\frac{\epsilon_3^2}{\delta^2}\right]+\frac{\eta_1^2+\eta_2^2}{\delta^2}\right\}, \tag{33}$$

$$\sigma_{lt}=\frac{8\pi}{3}f^2\left(\frac{\omega}{\chi}\right)^2\left\{\tfrac{4}{5}I^2\frac{\omega^2-\chi^2}{\omega^2}\left[2\frac{\epsilon_1^2+\epsilon_2^2}{\delta^2}+\frac{\epsilon_3^2}{\delta^2}\right]+\frac{\eta_1^2+\eta_2^2}{\delta^2}\right\}, \tag{34}$$

$$\sigma_{ll}=\frac{4\pi}{3}f^2\left(\frac{\omega}{\chi}\right)^4\left\{\tfrac{4}{5}I^2\frac{\omega^2-\chi^2}{\omega^2}\left[\frac{\epsilon_1^2+\epsilon_2^2}{\delta^2}+8\frac{\epsilon_3^2}{\delta^2}\right]+\frac{\eta_1^2+\eta_2^2}{\delta^2}\right\}. \tag{35}$$

The first index on σ denotes whether the incoming wave is transverse (t) or longitudinal (l) and the second index similarly specifies the scattered wave. The ϵ's and η's are different for incoming transverse and longitudinal waves, respectively; their explicit expressions are given in Ref. 17.

IV. DISCUSSION

The explicit form of the scattering cross sections follows from Eqs. (32) through (35) upon substitution of the expressions for the appropriate components of the ratios of amplitudes ϵ_j/δ and η_j/δ. For computational purposes, we also transform back into the system of units in which the velocity of light c and Planck's constant $\hbar$ appear explicitly. Introducing the dimensionless parameters

$$A=\tfrac{4}{3}f^2I\hbar c^{-1},\quad B=mc(\chi I\hbar)^{-1},\quad x=\omega(\chi c)^{-1}, \tag{36}$$

we can write the total cross sections in the form[20]

$$\begin{Bmatrix}\sigma_{tt}\\ \sigma_{tl}\\ \sigma_{lt}\\ \sigma_{ll}\end{Bmatrix}=12\pi\left(\frac{A}{\chi}\right)^2\left[\begin{Bmatrix}2\sin^2\theta_3\\ x^2\sin^2\theta_3\\ 2x^2\sin^2\Psi_3\\ x^4\sin^2\Psi_3\end{Bmatrix}F_1(x,A,B)+\begin{Bmatrix}4\sin^2\Psi_3\cos^2\theta_3\\ x^2\sin^2\Psi_3\cos^2\theta_3\\ 4x^2\sin^2\Psi_3\cos^2\Psi_3\\ x^4\sin^2\Psi_3\cos^2\Psi_3\end{Bmatrix}F_2(x,A,B)+\begin{Bmatrix}\cos^2\Psi_3\cos^2\theta_3\\ 4x^2\cos^2\Psi_3\cos^2\theta_3\\ x^2\sin^4\Psi_3\\ 4x^4\sin^4\Psi_3\end{Bmatrix}F_3(x,A,B)\right]. \tag{37}$$

In field theory

$$F_1(x,A,B)=\frac{A^2x^2(x^2+3)+1}{x^2[A^2x^2(x^2+3)+1]^2-16A^2}, \tag{38F}$$

$$F_2(x,A,B)=\frac{\frac{1}{5}(x^2-1)^2\{[Bx^2+2A(x^2-\frac{2}{5})]^2+(1/25)A^2(x^2+4)^2(x^2-1)^3+x^6\}}{\{[Bx^2+2A(x^2-\frac{2}{5})]^2+(1/25)A^2(x^2+4)^2(x^2-1)^3+x^6\}^2-4x^6[Bx^2+2A(x^2-\frac{2}{5})]^2}, \tag{39F}$$

$$F_3(x,A,B)=\frac{\frac{2}{5}(x^2-1)^2}{[Bx^2-\frac{2}{5}A]^2+(4/25)A^2(4x^2+1)^2(x^2-1)^3}, \tag{40F}$$

and in action-at-a-distance theory

$$F_1(x,A,B)=\frac{1}{x^2[A^2x^2(x^2+3)+1]-4A^2}, \tag{38A}$$

$$F_2(x,A,B)=\frac{\frac{1}{5}(x^2-1)^2[B^2x^4+(1/25)A^2(x+4)^2(x^2-1)^3+x^6]}{[B^2x^4+(1/25)A^2(x^2+4)^2(x^2-1)^3+x^6]^2-4B^2x^{10}}, \tag{39A}$$

$$F_3(x,A,B)=\frac{\frac{2}{5}(x^2-1)^2}{B^2x^4+(4/25)A^2(4x^2+1)^2(x^2-1)^3}. \tag{40A}$$

[20] In Ref. 17 the Eqs. (6-1) through (6-7) corresponding to our Eqs. (37)–(40) contain some errors and should be replaced by the latter. The figures have to be changed correspondingly.

These equations determine the cross sections as functions of the frequency of the incident meson field, and of its directions of polarization and propagation relative to the nucleon dipole, if the values of the constants characterizing the meson and the nucleon (which define the parameters A and B) are specified. However, our primary interest in this study is not in matching experimental results of known particles, but in a qualitative understanding of the behavior of particles characterized by pseudovector coupling. Therefore we are also concerned with the dependence of these cross sections on the constants themselves.

In order to study this dependence we simplify the expressions assuming the incoming transverse meson field to be unpolarized. We thus eliminate θ_3 by averaging over the transverse polarization. Then we get

$$\begin{Bmatrix}\sigma_{tt}'\\ \sigma_{tl}'\\ \sigma_{lt}'\\ \sigma_{ll}'\end{Bmatrix}=6\pi\left(\frac{A}{\chi}\right)^2\left[\begin{Bmatrix}2(1+\cos^2\Psi_3)\\ x^2(1+\cos^2\Psi_3)\\ 4x^2\sin^2\Psi_3\\ 2x^4\sin^2\Psi_3\end{Bmatrix}F_1(x,A,B)+\begin{Bmatrix}4\sin^4\Psi_3\\ x^2\sin^4\Psi_3\\ 8x^2\sin^2\Psi_3\cos^2\Psi_3\\ 2x^4\sin^2\Psi_3\cos^2\Psi_3\end{Bmatrix}F_2(x,A,B)+\begin{Bmatrix}\sin^2\Psi_3\cos^2\Psi_3\\ 4x^2\sin^2\Psi_3\cos^2\Psi_3\\ 2x^2\sin^4\Psi_3\\ 8x^4\sin^4\Psi_3\end{Bmatrix}F_3(x,A,B)\right]. \tag{41}$$

We can also eliminate Ψ_3 by assuming that all orientations of the nucleon dipole are equally probable. Hence, averaging over all values of Ψ_3, we get

$$\begin{Bmatrix}\bar\sigma_{tt}\\ \bar\sigma_{tl}\\ \bar\sigma_{lt}\\ \bar\sigma_{ll}\end{Bmatrix}=4\pi\left(\frac{A}{\chi}\right)^2\left[\begin{Bmatrix}2\\ x^2\\ 2x^2\\ x^4\end{Bmatrix}F_1(x,A,B)+\frac{1}{5}\begin{Bmatrix}16\\ 4x^2\\ 8x^2\\ 2x^4\end{Bmatrix}F_2(x,A,B)+\frac{1}{5}\begin{Bmatrix}1\\ 4x^2\\ 8x^2\\ 32x^4\end{Bmatrix}F_3(x,A,B)\right]; \tag{42}$$

thus now $\bar\sigma_{tl}=\frac{1}{2}\bar\sigma_{lt}$.

F_1 represents the contribution to the scattering due to the nucleon dipole motion, whereas F_2 and F_3 are a result of nucleon recoil. The latter two terms vanish in the infinite mass approximation ($B\rightarrow\infty$ as $m\rightarrow\infty$). Furthermore, F_1 is nonzero at the threshold frequency, $\omega=\chi c(x=1)$; hence the cross sections are nonzero at threshold for the pseudovector interaction in contrast to the usual monopole and dipole interactions.[1,3,4]

We note that in both theories the cross sections fall off at least as x^{-2} for large x. This is a necessary consequence of the fact that pseudovector coupling does not exist for zero rest mass fields ($\chi\rightarrow 0$, $x\rightarrow\infty$).

For small values of $f(A\ll 1)$, or weak coupling, the cross sections are proportional to f^4. For strong coupling they become independent of the coupling constant except near the threshold frequency at which the action-at-a-distance cross sections are proportional to f^4 for all values of f. The field-theoretical cross sections show a peculiar feature at threshold in that F_1, by Eq. (38F), exhibits a singularity at $A\equiv A_0=\frac{1}{2}$. This corresponds to a critical coupling constant given by

$$f=\frac{1}{2}\left(\frac{3}{2}\frac{c}{I\hbar}\right)^{1/2}\ (\mathrm{g\ cm})^{-1/2}. \tag{43}$$

The large numerical values for f, apparent from (43), are due to the way f is introduced into the equations of motion. The quantity to be compared with g_1 (which has dimensions of a charge) is $f\hbar$, which for the cases considered here is of the order of 10^{-11} to 10^{-7}.

The constant χ, which characterizes the meson field, is the classical analog of the quantum-mechanical constant $\mu c\hbar^{-1}$, where μ is the rest mass of the meson. Thus x and B correspond to the constants $\hbar\omega(\mu c^2)^{-1}$ and $m(\mu I)^{-1}$.

In order to establish a standard for numerical values for the cross sections we arbitrarily choose constants characteristic of the neutron and the π^0 meson. These are $I=\frac{1}{2}$, $m=1838m_0=1.67\times10^{-24}$ g, $\mu=264m_0=2.40\times10^{-25}$ g, where m_0 is the rest mass of the electron. This corresponds to $\chi=6.82\times10^{12}$ cm^{-1}, $B\equiv B_0=13.9$.

Using an electronic computer,[21] the behavior of the cross sections was studied for $A=10^{-4}A_0$, A_0 and 10^4A_0, which typify weak coupling, critical coupling, and strong coupling, respectively. The corresponding values for f are (assuming $I=\frac{1}{2}$ throughout)

$f=4.62\times10^{16}\ (\mathrm{g\ cm})^{-1/2}$, weak coupling,

$f=4.62\times10^{18}\ (\mathrm{g\ cm})^{-1/2}$, critical coupling,

$f=4.62\times10^{20}\ (\mathrm{g\ cm})^{-1/2}$, strong coupling.

For each A above, B was varied from $10^{-2}B_0$ to 10^2B_0 by factors of 10. The following figures present the more prominent features resulting from these calculations on a log-log scale for suitably selected values of B. We also calculated the cross sections for selected more extreme

[21] The authors wish to express their appreciation to the Technical Computing Department, A. O. Smith Corporation, Milwaukee, Wisconsin for the preliminary computations performed on their IBM 705 computer; and to Dr. R. L. Knight and D. W. Baltz for the final computations utilizing the C.D.C. 1604B computer at EG&G, Las Vegas, Nevada.

values of A and B, but no significant new features were found.

We have plotted "normalized" cross sections defined by

$$Q_{tt}=\frac{\bar{\sigma}_{tt}}{4\pi(A/\chi)^2},$$

$$Q_{tl}=\frac{\bar{\sigma}_{tl}}{4\pi(A/\chi)^2}=\tfrac{1}{2}Q_{lt},$$

and

$$Q_{ll}=\frac{\bar{\sigma}_{ll}}{4\pi(A/\chi)^2}.$$

Figures 1(a)–1(c) show the cross sections for weak coupling, Figs. 2(a)–2(c) those for critical coupling, and Figs. 3(a)–3(c) those for strong coupling. The action-at-a-distance and field-theoretical results coincide for weak coupling (small A). This is to be expected since F_1, F_2, and F_3 differ in the two theories only in terms containing A as a factor. Also, for $A=0$, $F_2(x)$ has a singularity at $x=B$ which corresponds to a meson energy equal to the rest energy of the nucleon divided by its spin. As a result, the cross sections exhibit a sharp, finite spike at $x=B$ for small A if B is greater than 1. For large B, a second maximum appears; both maxima move to larger values of x for increasing B.

The more prominent differences between the action-at-a-distance and the field-theoretical cross sections are due to the difference in dependence on the coupling constant at the threshold frequency, and therefore occur for small values of x with critical and strong coupling.

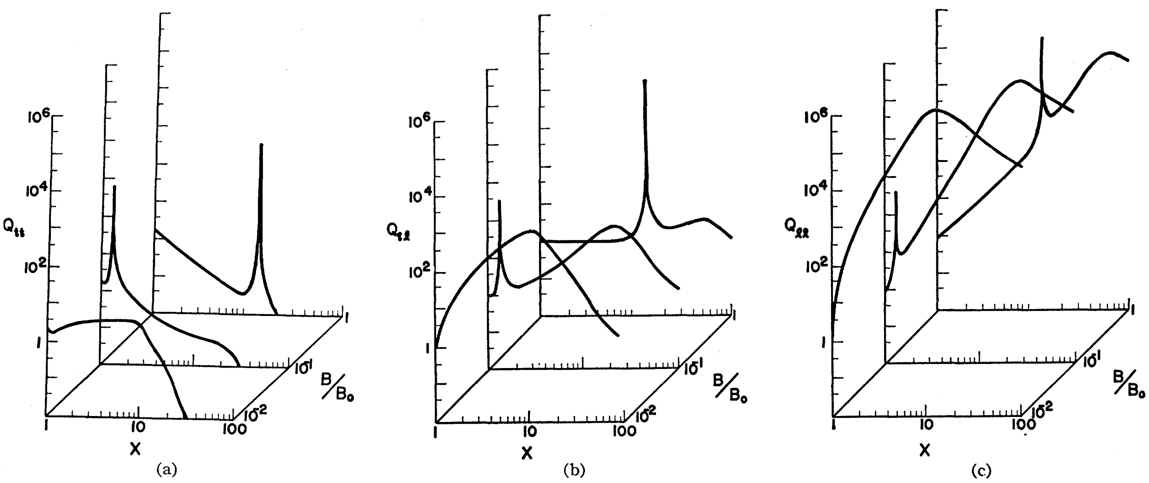

FIG. 1. Normalized cross sections for weak coupling ($A=10^{-4}A_0$) as a function of $x=\omega(\chi c)^{-1}$ for selected values of B/B_0 (defined in Sec. IV). The field-theoretical and action-at-a-distance results cannot be distinguished in this limit. The curves represent: (a) incident and scattered mesons both transverse, (b) incident transverse and scattered longitudinal mesons, (c) incident and scattered mesons both longitudinal.

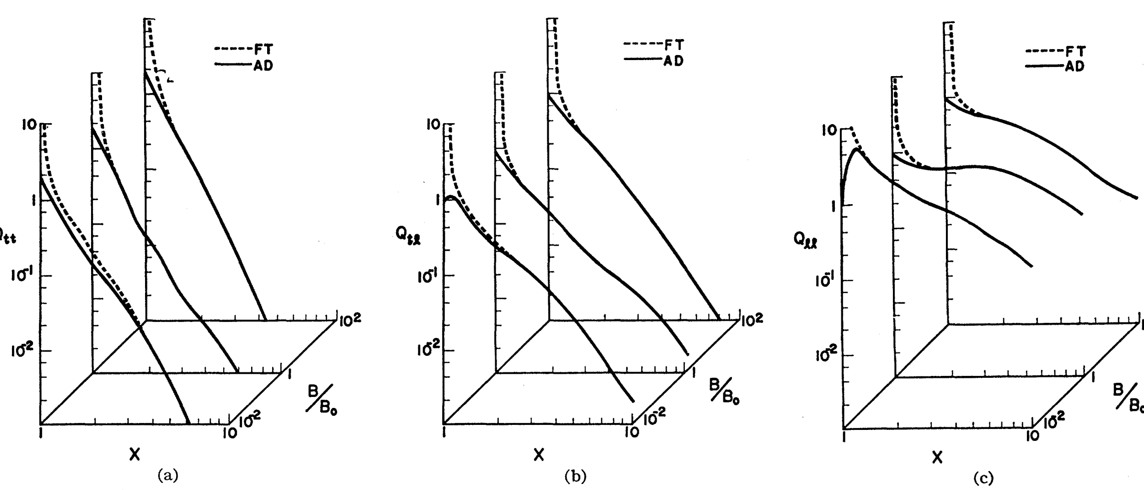

FIG. 2. Normalized cross sections for critical coupling ($A=A_0$) as a function of $x=\omega(\chi c)^{-1}$ for selected values of B/B_0 (defined in Sec. IV), for field theory (FT) and action-at-a-distance theory (AD). The FT curves go to infinity as $x\rightarrow 1$. The curves represent: (a) incident and scattered mesons both transverse, (b) incident transverse and scattered longitudinal mesons, (c) incident and scattered mesons both longitudinal.

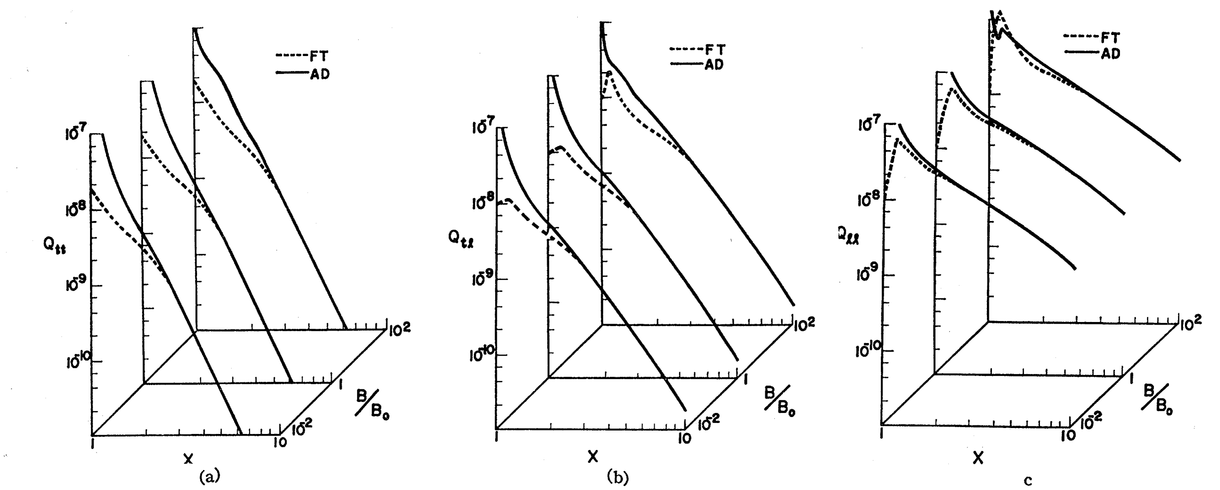

FIG. 3. Normalized cross sections for strong coupling ($A=10^4A_0$) as a function of $x=\omega(\chi c)^{-1}$ for selected values of B/B_0 (defined in Sec. IV), for field theory (FT) and action-at-a-distance theory (AD). For $x\rightarrow 1$, the AD curves approach 2 in case (a), and 1 in cases (b) and (c). The curves represent: (a) incident and scattered mesons both transverse, (b) incident transverse and scattered longitudinal mesons, (c) incident and scattered mesons both longitudinal.

For critical coupling, the field-theoretical cross sections are larger than the corresponding action-at-a-distance ones for small x. With two exceptions, the maximum values for the cross sections occur at the threshold frequency ($\omega=\chi c$); the exceptions being the action at a distance cross sections Q_{tl} and Q_{ll} for $B=10^{-2}B_0$. Fig. 2(b) depicts the rather anomalous behavior of Q_{tl} which from threshold passes through a minimum at $x=1.03$ and then reaches a maximum at $x=1.12$. In Fig. 2(c), Q_{ll} has a maximum at $x=1.17$ with $B=10^{-2}B_0$. For $B=B_0$, Q_{ll} has a weak broad secondary maximum at $x=2.70$ for both theories.

With strong coupling, the roles of the action-at-a-distance and the field-theoretical cross sections are essentially interchanged in that the former are the larger, though finite, at threshold. We also note the appearance of secondary maxima for the field-theoretical cross sections [Figs. 3(a) and 3(b)] and for the action-at-a-distance cross section in Fig. 3(c).

It is clear from the figures that in some instances the differences between the two theories are sufficiently large that an experimental distinction might be possible if scattering events were observable for mesons having the kind of interaction studied here. Of course, even if such mesons were available, the magnitudes of the various constants (and thus the choice of optimum observational conditions) would not be at our disposal. It should be noted, however, that quite apart from the difference between the two theories, the behavior of the cross sections depends critically on the values of these constants; curves obtained from the same dynamical theory show strikingly different behavior depending on these values, some displaying secondary maxima or (broad or narrow) resonance-like features.[22] Rather than dwell on the particular features resulting for particular values of the constants, it might be more appropriate to conclude that one should be very cautious in interpreting particular characteristics of curves obtained experimentally in terms of physical models, as they might not be representative properties of the interactions involved, but rather accidental features of a general expression for particular values of the parameters.

[22] These maxima do not result from any excitation of the nucleon. The possibility of excited states exists for the field-theoretical, but not for the action-at-a-distance equation of motion; however, it has not been taken into account in the calculations presented here. For the case of scalar and vector meson fields, see Ref. 18 and P. Havas and C. R. Mehl, in paper presented at the *Colloquium on Theoretical Physics in Honour of Professor P. A. M. Dirac* (National Research Council, Ottawa, 1955).