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(to zero order in e),⁴

$$
\begin{aligned} \left[j_{\mu}^{V,3}(x) + \frac{1}{3} \sqrt{3} j_{\mu}^{V,8}(x), \ j_0^{+}(y) \right] \\ &= j_{\mu}^{+}(x) \delta^{3}(x-y) + \frac{1}{e} \sum_{i=1}^{3} \frac{\partial}{\partial y_i} \left[\frac{\partial A_{\mu}}{\partial x_0}(x), j_i^{+}(y) \right]. \end{aligned} \tag{8}
$$

In an entirely similar manner we obtain corresponding commutation relations between two axial vector currents if we introduce, in direct analogy with the electromagnetic potential A_{μ} , a weak boson field W_{μ} which has the weak interaction current as a source.³ It should be clear that we can not assume in general that the equal-time commutator of $\partial A_{\mu}/\partial x_0$ with the space component of j_{μ} ⁺ vanishes, since this commutator ac-

We have left out of Eq. (8) a term

$$
-i\sum_{i=1}^3 A_i(y)\bigg[\frac{\partial A_\mu}{\partial x_0}(x),j_i+(y)\bigg],
$$

which is first order in e.

counts for the presence of gradients of $\delta^3(x-y)$ terms⁵ in the current-current commutator, Eq. (8).

This result can also be obtained readily if we consider, for example, the matrix elements of the divergence equations between a hadron state (a) and another hadron state (b, γ) containing a single photon. Applying the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula to the photon and keeping only first-order terms in e , we find Eq. (8) , showing explicitly how the compensation of the gradient of $\delta^3(x-y)$ terms occurs in the divergence equations.²

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Baryon-Meson Couplings in Broken $SU(6)_W$. II

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The baryon-meson vertex is studied within the framework of broken $SU(6)_W$ symmetry. The breaking is provided by a $W=1$ spurion transforming like the $I=0=Y$ member of the adjoint 35 representation of $SU(6)_W$ and which belongs to an $SU(3)$ octet and/or SU(3) singlet. The spin kinematic factor in this case is different from the $W=0$ spurion breaking. The present type of breaking forbids all decuplet decays of the type $D \to B+P$, and in this respect the present situation resembles that of the static $SU(6)_S$ in exact symmetry. The BBP couplings are related, in general, by 5 parameters which, under certain simplifying assumptions, reduce to a smaller number. Sum rules are listed for all the possibilities that might arise, and it is found that the predictions for the symmetry broken by a $W=1$ spurion belonging to an $SU(3)$ octet plus an $SU(3)$ singlet—both in the same 35 representation—are consistent with the present knowledge of the couplings.

I. INTRODUCTION

'N an earlier paper' (hereafter referred to as I) we have considered the baryon-pseudoscalar-meson vertex and the decuplet decays within the framework of exact and broken $SU(6)_W$ symmetry. In I we attributed the breaking of the $SU(6)_W$ symmetry to a *W*-spin scalar spurion having $I=0=Y$ and belonging to the adjoint 35-dimensional representation of $SU(6)_W$. However, this is not the only way in which the symmetry might be broken. For example, the symmetry may also be broken by a $W=1$ spurion which may belong to an $SU(3)$ octet and/or an $SU(3)$ singlet having $I=0=V$ and belonging to the 35-dimensional

adjoint representation of $SU(6)_W$. This type of breaking introduces a spin kinematic factor different from that for the $W=0$ spurion. Thus the two cases have to be considered quite independently. This has provided the motivation for the present paper, and we investigate in an exhaustive way the consequences following from the W -spin 1 spurion breaking the symmetry. In Sec. 2 we write down the interaction Lagrangian and outline the various interesting possibilities. In Sec. 3 the sum rules are listed for various cases, followed by discussions in Sec. 4.

II. BARYON-MESON VERTEX

The $SU(3)$ octet and singlet spurions with $W=1$ which transform like the $I=0=Y$ member and belong to the adjoint 35-dimensional representation may be

SECT The presence of a gradient of $\delta^3(x-y)$ term in the vacuum expectation value of the equal-time commutator of the time component with the space component of the electromagnetic current appears to have been first noticed by T. Gotô and T. Imanura, Progr. Theoret. Phys. (Kyoto) 14, 396 (1955). I am indebted to Professor G. Kallen for calling my attention to this reference. See also J. Schwinger, Phys. Rev. Letters 3, ²⁹⁶ (1959).

⁺ On leave of absence from Roorkee University, Roorkee, India. ¹ S. N. Gupta, Phys. Rev. 151, 1235 (1966). All other references are listed in this paper.

written in the usual notation² as We now have

$$
V_a^{\alpha'} = (\sigma_3 p_3)_i^{i'} Y_A^{A'}
$$
 (2.1)

and

$$
S_{\alpha}{}^{\alpha'} = (\sigma_3 p_3) i^i \delta_A{}^{A'}, \qquad (2.2)
$$

respectively; Y is given by

$$
Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} . \tag{2.3}
$$

For generality we assume that these two spurions belong to two different 35 representations. The interaction Lagrangian may then be written as

$$
L = g_1 \psi_{\alpha\beta\gamma} \psi^{\alpha\beta'\gamma'} [\phi_\delta \gamma V_{\gamma'}{}^\delta + V_\delta \gamma \phi_{\gamma'}{}^\delta]
$$

+
$$
g_2 \psi_{\alpha\beta\gamma} \psi^{\alpha\beta'\gamma'} \phi_{\beta'}{}^\beta V_{\gamma'}{}^\gamma + g_3 \psi_{\alpha\beta\gamma} \psi^{\alpha\beta\gamma} \phi_\delta \gamma' V_{\gamma'}{}^\delta
$$

+
$$
f_1 \psi_{\alpha\beta\gamma} \psi^{\alpha\beta'\gamma'} [\phi_\delta \gamma S_{\gamma'}{}^\delta + S_\delta \gamma \phi_{\gamma'}{}^\delta]
$$

+
$$
f_2 \psi_{\alpha\beta\gamma} \psi^{\alpha\beta'\gamma'} \phi_{\beta'}{}^\beta S_{\gamma'} \gamma + f_3 \psi_{\alpha\beta\gamma} \psi^{\alpha\beta\gamma} \phi_\delta \gamma' S_{\gamma'}{}^\delta , \quad (2.4)
$$

where $\psi^{\alpha\beta\gamma}$ and $\phi_{\alpha}{}^{\beta}$ were defined earlier.¹ For the interaction Lagrangian (2.4) it appears that the couplings are now related through 6 parameters. However, it is apparent that the term with f_3 does not contribute to any process, since $Tr(\phi S) = Tr(\phi) = 0$. On further analysis we find that the interaction (2.4) forbids all decuplet-octet pseudoscalar-meson decays, as the spin kinematic factor vanishes in this case.³ This is analogous to the situation one meets in exact $SU(6)_s$ symmetry, where again the decuplet decays are forbidden. Incidentally, in the broken $SU(3)$ symmetry there are seven parameters connecting the twelve baryon-meson vertices, and five sum rules result. No quantitative estimates can, however, be given for these, because of the present lack of knowledge about the seven independent couplings. Nor can one reduce the seven parameters to a smaller number. In contrast to this, in our present model the twelve independent baryon-meson vertices are connected by only five parameters and, as we show at the end of this section, it is possible, under certain simplifying assumptions, to further reduce the number of parameters; this enables us to estimate most of the coupling strengths. We may also remark that the term with g_3 contributes only to the four baryon- η vertices.

$$
\sqrt{2}G(p p \pi^0) = \frac{2}{3}g_1 + (5/9)g_2 + \frac{2}{3}f_1 + (4/9)f_2,
$$

\n
$$
\sqrt{2}G(\Xi^- \Xi^- \pi^0) = -\frac{2}{3}g_1 - (4/9)g_2 - \frac{2}{3}f_1 + \frac{1}{6}f_2,
$$

\n
$$
\sqrt{2}G(\Sigma^0 \Sigma^+ \pi^-) = -\frac{4}{3}g_1 - (5/9)g_2 - \frac{4}{3}f_1 - (5/18)f_2,
$$

\n
$$
\sqrt{6}G(\Sigma^0 \Lambda \pi^0) = (11/18)f_2,
$$

\n
$$
\sqrt{6}G(p p \eta) = 2g_1 - \frac{2}{3}g_2 + 2f_1 + \frac{1}{9}f_2 + 12g_3,
$$

\n
$$
\sqrt{6}G(\Lambda \eta) = 4g_1 - \frac{2}{3}g_2 + (11/18)f_2 + 12g_3,
$$

\n
$$
\sqrt{6}G(\Sigma^0 \Sigma^0 \eta) = 4g_1 + \frac{2}{3}g_2 + (11/18)f_2 + 12g_3,
$$

\n
$$
\sqrt{6}G(\Xi^- \Xi^- \eta) = 6g_1 + g_2 - 2f_1 - (13/18)f_2 + 12g_3,
$$

\n
$$
\sqrt{2}G(p\Sigma^0 K^+) = \frac{1}{3}g_1 + (7/18)g_2 - \frac{2}{3}f_1 + \frac{1}{6}f_2,
$$

\n
$$
\sqrt{6}G(p \Lambda K^+) = g_1 + (1/18)g_2 - \frac{2}{3}f_1 - (13/9)f_2,
$$

\n
$$
\sqrt{2}G(\Xi^- \Sigma^0 K^-) = -\frac{1}{3}g_1 - (1/18)g_2 + \frac{2}{3}f_1 + (4/9)f_2,
$$

\n
$$
\sqrt{6}G(\Xi^- \Lambda K^-) = -g_1 - \frac{1}{2}g_2 + 2f_1 + \frac{1}{9}f_2.
$$

The following four possibilities may now arise:

(A) The $SU(3)$ octet and singlet spurions belong to two different 35 representations. In this case the twelve independent couplings are connected by five parameters and we get seven sum rules.

(B) The breaking is due to a $W=1$, $SU(3)$ singlet spurion only. In this case $g_1 = g_2 = g_3 = 0$. We now have two parameters connecting the twelve vertices, giving ten sum rules.

(C) The breaking is due to a $W=1$, $SU(3)$ octet spurion only. Then $f_1 = f_2 = 0$; there are now three parameters and nine sum rules result.

(D) Both the spurions belong to the same 35 representation. This implies $g_1 = f_1$, $g_2 = f_2$. We again have three parameters and nine sum rules; these are different from those in (C).

In case (A), to estimate the coupling strengths, we require experimental information for five vertices, which at present is lacking. In the two situations (C) and (D) we are able to express six vertices other than the η couplings in terms of the two known vertices $G(p p \pi^0)$ and $G(p \Lambda K^+)$. Three of the four η couplings can also be expressed in terms of these two known vertices and the fourth η coupling, say $G(p/p\eta)$. In contrast, all ten couplings are expressible in terms of the known ones in case (B).

III. SUM RULES

In this section we list all the sum rules which result under the four different possibilities outlined in the last section. We number the sum rules by the corresponding condition A, 8 etc. Thus we get:

² See Ref. 1; we follow the notation of this paper.
⁸ We thus note that the breaking by $W=1$ spurions does not yield any new information about the decuplet decays. A similar remark has also been made by H. Harari et al., Phys. Rev. 140, B1003 (1965). We, in fact, find that the structure of the $SU(3)$ terms for the decuplet decays is exactly the same as that for the $W\!=\!0$ spurion except for the spin factor which, however, vanishes in the present case.

Case (D).

IV. DISCUSSION

To make any quantitative estimates of the couplings for the set A, we require a knowledge of the strength for five vertices which, however, is not available at present, and so we are unable to compare these predictions with experiment.

For the rest of the cases (B, C, and D) we have made an estimate by using the couplings $g_{NN\pi}$ and g_{NNK} . We use the following relation to define the G 's:

$$
\frac{G^2(p p \pi^0)}{4 \pi} = \frac{g_{NN\pi}^2}{4 \pi} \left(\frac{1}{2M}\right)^2, \tag{4.1}
$$

where M is the mass of the baryon octet, which we take to be 1 BeV, and $(g_{NN\pi^2}/4\pi) \approx 15$ is the well-known nucleon-pion coupling constant. The $(N\Lambda K)$ coupling as determined on the basis of forward dispersion relations⁴ for KN scattering gives $(g_{N\Lambda K^2}/4\pi)$ 4.8 which gives $G(p\Lambda K^{+}) = -3.9 \text{ BeV}^{-1}$.

In Tables I, II, and III, we have listed the estimates for the various coupling constants for the cases 8, C, and D. The dimensionless coupling constants $(g^2/4\pi)$ are also listed in the last columns of these tables. Lusignoli et al.⁴ have put an upper limit on $(N\Sigma K)$ coupling; they give $(g_{\nu\Sigma K^2}/4\pi) \leq 3.2$. The estimates for the pion couplings' based on experiments give

$$
g_{\Xi\Xi\pi^2}/4\pi = 3.09
$$
,
\n $g_{\Sigma\Sigma\pi^2}/4\pi = 4.3$,
\n $g_{\Sigma\Lambda\pi^2}/4\pi = 11.42$.

From Table I, we find that the estimates based on the symmetry breaking due to a $W=1$ spurion belonging to only an $SU(3)$ singlet are in sharp disagreement

TABLE I. The predicted values of $spin\frac{1}{2}^+$ baryon and pseudoscalar-meson coupling constants when $SU(6)_W$ is broken with a $W=1$ spurion of the $SU(3)$ singlet. Input parameters are $g_{NN\pi}^2/4\pi = 15$ and $g_{N\Delta K}^2/4\pi = 4.8$.

Vertices	G (BeV) ⁻¹ (predicted)	$g^2/4\pi$ (predicted)
$\Sigma^0\Sigma^+\pi^-$	-13.8	60
$\Xi^ \Lambda K^-$	12.2	48
ppn	12.2	48
$\Xi^-\Xi^-\eta$	-12	48
$\Xi^-\Xi^-\pi^0$	-6.9	\sim 16
$\phi \Sigma^0 K^+$	~ 6.9	\sim 16
$\Xi^- \Sigma^0 K^+$	6.85	15
$\Sigma^0 \Sigma^0 n$	-0.06	0.001
$\Sigma^0 \Lambda \pi^0$	-0.06	0.001
ΛΛη	-0.06	0.001

⁴ M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters 21, 229 (1966). See also J. Dufour, Nuovo Cimento

34, 645 (1964) who give $(g_{p\text{A}K}^2/4\pi)$ between 5 and 6. ⁶ I. G. Aznauryan and L. D. Soloviev, Dubna Report No. **E-2544, 1966** (unpublished).

TABLE II. The predicted values of spin- $\frac{1}{2}$ ⁺ baryon and pseudoscalar-meson couplings when $SU(6)$ _{*w*} is broken with a $W=1$, $SU(3)$ octet spurion.

Vertices	G (BeV) ⁻¹ (predicted)	$g^2/4\pi$ (predicted)
$\frac{p\mathbf{\Sigma}^0 K^+}{\mathbf{\Xi}^-\mathbf{\Xi}^-\pi^0}$	5.87	11.2
$\Xi^- \Lambda K^-$	-4.51 -1.75	~ 6.5 0.9
$\Sigma^0\Sigma^+\pi^-$ $\Xi^{-}\Sigma^{0}K^{-}$	-1.51 1.45	~ 0.7 0.6
$\Sigma^0 \Lambda \pi^0$		0

(4.1) TABLE III. Predicted values of spin- $\frac{1}{2}$ ⁺ baryon and pseudoscalar-
meson couplings when $SU(6)_W$ is broken by $W=1$, $SU(3)$ octet
and singlet spurions which are in the same 35 representation.

with the present knowledge about these vertices. From Table II, we see that the pure $W=1$, $SU(3)$ octet-type breaking predicts $G(\Sigma \Lambda \pi) = 0$ and $g_{p\Sigma^0 K^{+2}}/4\pi \simeq 11.2$, which seem to disagree with experiment. Similarly, $g_{\Sigma\Sigma\pi^2}/4\pi=0.7$ is too low. On the other hand, the breaking due to a $W=1$ spurion with $I=0=Y$ that is a member of the $SU(3)$ octet and singlet and belonging to the same 35 representation seems to give better agreement, δ as is seen from Table III. [Note added in proof. It is interesting to compare the predictions of thin paper with those of I where the symmetry breaking is assumed due to a W -spin scalar spurion. Using the input value 4–8 for $(g^2 N\Lambda K/4\pi)$ in I we find that the predictions based on the W -spin scalar spurion breaking the $SU(6)_W$ symmetry are in better agreement with experiment and the important conclusion which emerges is that if at all the W -spin symmetry is good, then the nature prefers the symmetry breaking due to a W -spin scalar spurion rather than the breaking due to a W -spin $=1$ spurion.

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⁶ If we take $(g_{NAK}^2/4\pi)$ \approx 1, as determined by M. Moravscik
[Phys. Rev. Letters 2, 352 (1959)] and G. Morpurgo [Ann. Rev.
Nucl. Sci. 11, 41 (1961)] from photoproduction amplitudes, our general conclusions remain unchanged, although numerical estimates are different.