Divergence Conditions and the Equal-Time Current-Current Commutators*

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The equal-time current-current commutation relations are deduced from equations for the divergence of the currents including electromagnetic and weak interactions, and canonical assumptions for equal-time commutators of electromagnetic and of weak boson fields.

IT has been shown in a previous paper[1] and, independently, by Adler and Dothan[2] that the longitudinal component of the off-mass-shell pseudoscalar-meson electroproduction amplitude can be determined from the equal-time commutator of the vector and the axial vector currents. Furthermore, it has also been pointed out that this gauge condition can be obtained without explicit use of commutators if we include to first order a weak interaction which gives rise to the meson decay. In this case, the neutral vector current of the hadrons is no longer conserved, and one finds the surprising result that the divergence of this current to first order in weak interactions is given by the equal-time vector and axial vector current-current commutator. Recently, Veltman[3] has introduced weak-interaction contributions to the divergence of the axial current as well as to the vector current and obtained many of the results which had previously been derived from the equal-time current-current commutators. The purpose of this paper is to show the fundamental connection between the equal-time current-current commutators and the assumed divergence equations for the currents including first-order electromagnetic and weak interactions.

We consider first the electromagnetic contributions to the divergence[2] of the charged components of the vector current $j_\mu{}^{V,\alpha}$ and the axial vector current $j_\mu{}^{A,\alpha}$:

$$\partial^\mu j_\mu{}^{V,+} = -ieA^\mu j_\mu{}^{V,+}, \tag{1}$$

$$\partial^\mu j_\mu{}^{A,+} = c\phi^+ - ieA^\mu j_\mu{}^{A,+}, \tag{2}$$

where ϕ^+ is the charged meson field, c is a constant, and A_μ is the electromagnetic potential satisfying the source equation

$$\Box^2 A_\mu = e\left(j_\mu{}^{V,3} + \frac{1}{\sqrt{3}} j_\mu{}^{V,8} + j_\mu{}^{\text{lept}}\right). \tag{3}$$

We assume that the charged current $j_\mu{}^+$ (which stands for either $j_\mu{}^{V,+}$ or $j_\mu{}^{A,+}$) and the electromagnetic potential A_μ commute at equal times, so that the order of the product of $j_\mu{}^+$ and A_μ in the divergence Eqs. (1) and (2) is immaterial i.e.,

$$[A_\mu(x), j_\nu{}^+(y)] = 0, \tag{4}$$

where $x_0 = y_0$. Then

$$[(\partial A_\mu/\partial x_0)(x), j_0{}^+(y)] + [A_\mu(x), \partial^\nu j_\nu{}^+(y)] = 0,$$

and substituting for the divergence of $j_\mu{}^+$ Eqs. (1) or (2), we obtain

$$[(\partial A_\mu/\partial x_0)(x), j_0{}^+(y)] = 0, \tag{5}$$

where we assumed the canonical equal-time commutation relations

$$[A_\mu(x), A_\nu(y)] = 0, \quad [A_\mu(x), \phi(y)] = 0.$$

For the space components $j_i{}^+$ we need to assume only that its equal-time commutator with $\partial A_\mu/\partial x_0$ is local.

Taking the following linear combination of partial derivatives of Eqs. (4) and (5):

$$\frac{\partial}{\partial x_0}\left[\frac{\partial}{\partial x_0} A_\mu(x), j_0{}^+(y)\right] - \nabla_x{}^2 [A_\mu(x), j_0{}^+(y)]$$

and substituting the electromagnetic source Eq. (3), we obtain

$$e\left[j_\mu{}^{V,3}(x) + \frac{1}{\sqrt{3}} j_\mu{}^{V,8}(x),\ j_0{}^+(y)\right] + \left[\frac{\partial A_\mu}{\partial x_0}(x), \partial^\nu j_\nu{}^+(y)\right] = \sum_{i=1}^{3} \frac{\partial}{\partial y_i}\left[\frac{\partial A_\mu}{\partial x_0}(x), j_i{}^+(y)\right]. \tag{6}$$

Finally, substituting Eqs. (1) and (2) into Eq. (6) for the divergence of the vector and axial vector current, respectively, and using the canonical commutation relations for the electromagnetic potential,

$$[(\partial A_\mu/\partial x_0)(x), A_\nu(y)] = -g_{\mu\nu} i\delta^3(x-y)$$

and

$$[(\partial A_\mu/\partial x_0)(x), \varphi(y)] = 0, \tag{7}$$

we arrive at the familiar current-current commutator

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[1] M. Nauenberg, Phys. Letters **22**, 201 (1966).
[2] S. L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966).
[3] M. Veltman, Phys. Rev. Letters **17**, 553 (1966).

(to zero order in e),[4]

$$[j_\mu{}^{V,3}(x)+\tfrac{1}{3}\sqrt{3}j_\mu{}^{V,8}(x),\ j_0{}^+(y)]$$
$$=j_\mu{}^+(x)\delta^3(x-y)+\frac{1}{e}\sum_{i=1}^{3}\frac{\partial}{\partial y_i}\left[\frac{\partial A_\mu}{\partial x_0}(x),j_i{}^+(y)\right]. \quad (8)$$

In an entirely similar manner we obtain corresponding commutation relations between two axial vector currents if we introduce, in direct analogy with the electromagnetic potential A_μ, a weak boson field W_μ which has the weak interaction current as a source.[3] It should be clear that we can not assume in general that the equal-time commutator of $\partial A_\mu/\partial x_0$ with the space component of $j_\mu{}^+$ vanishes, since this commutator accounts for the presence of gradients of $\delta^3(x-y)$ terms[5] in the current-current commutator, Eq. (8).

This result can also be obtained readily if we consider, for example, the matrix elements of the divergence equations between a hadron state (a) and another hadron state (b,γ) containing a single photon. Applying the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula to the photon and keeping only first-order terms in e, we find Eq. (8), showing explicitly how the compensation of the gradient of $\delta^3(x-y)$ terms occurs in the divergence equations.[2]

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[4] We have left out of Eq. (8) a term

$$-i\sum_{i=1}^{3}A_i(y)\left[\frac{\partial A_\mu}{\partial x_0}(x),j_i{}^+(y)\right],$$

which is first order in e.

[5] The presence of a gradient of $\delta^3(x-y)$ term in the vacuum expectation value of the equal-time commutator of the time component with the space component of the electromagnetic current appears to have been first noticed by T. Gotô and T. Imanura, Progr. Theoret. Phys. (Kyoto) 14, 396 (1955). I am indebted to Professor G. Källén for calling my attention to this reference. See also J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

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Baryon-Meson Couplings in Broken $SU(6)_W$. II

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The baryon-meson vertex is studied within the framework of broken $SU(6)_W$ symmetry. The breaking is provided by a $W=1$ spurion transforming like the $I=0=Y$ member of the adjoint **35** representation of $SU(6)_W$ and which belongs to an $SU(3)$ octet and/or $SU(3)$ singlet. The spin kinematic factor in this case is different from the $W=0$ spurion breaking. The present type of breaking forbids all decuplet decays of the type $D\rightarrow B+P$, and in this respect the present situation resembles that of the static $SU(6)_S$ in exact symmetry. The BBP couplings are related, in general, by 5 parameters which, under certain simplifying assumptions, reduce to a smaller number. Sum rules are listed for all the possibilities that might arise, and it is found that the predictions for the symmetry broken by a $W=1$ spurion belonging to an $SU(3)$ octet plus an $SU(3)$ singlet—both in the same **35** representation—are consistent with the present knowledge of the couplings.

I. INTRODUCTION

In an earlier paper[1] (hereafter referred to as I) we have considered the baryon–pseudoscalar-meson vertex and the decuplet decays within the framework of exact and broken $SU(6)_W$ symmetry. In I we attributed the breaking of the $SU(6)_W$ symmetry to a W-spin scalar spurion having $I=0=Y$ and belonging to the adjoint 35-dimensional representation of $SU(6)_W$. However, this is not the only way in which the symmetry might be broken. For example, the symmetry may also be broken by a $W=1$ spurion which may belong to an $SU(3)$ octet and/or an $SU(3)$ singlet having $I=0=Y$ and belonging to the 35-dimensional adjoint representation of $SU(6)_W$. This type of breaking introduces a spin kinematic factor different from that for the $W=0$ spurion. Thus the two cases have to be considered quite independently. This has provided the motivation for the present paper, and we investigate in an exhaustive way the consequences following from the W-spin 1 spurion breaking the symmetry. In Sec. 2 we write down the interaction Lagrangian and outline the various interesting possibilities. In Sec. 3 the sum rules are listed for various cases, followed by discussions in Sec. 4.

II. BARYON-MESON VERTEX

The $SU(3)$ octet and singlet spurions with $W=1$ which transform like the $I=0=Y$ member and belong to the adjoint 35-dimensional representation may be

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[1] S. N. Gupta, Phys. Rev. 151, 1235 (1966). All other references are listed in this paper.