Effect of 2^- B Exchange in the $\pi\omega$ Channel on the Parameters of the ρ*

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We have considered the question of whether the inclusion of the attractive force in the coupled $\pi\omega$ channel due to the exchange of a 2^- B particle, for which there is some theoretical and experimental evidence, might lessen the discrepancy between the calculated and experimental values of the width of the ρ meson. We follow the procedure of Fulco, Shaw, and Wong in which the matrix ND^{-1} equations are solved with a cutoff adjusted to produce the ρ at its experimental mass. We find that the resulting width is almost totally insensitive to the inclusion of the B-exchange force. The insensitivity is due to the fact that, while the Born amplitude for B exchange is positive and of appreciable size, it decreases with energy in such a way that its contribution to the kernel of the integral equation for the N matrix is negative, and the change in the N matrix produced by B exchange is therefore suppressed. The conclusions are independent of the mass and width of the exchanged 2^- particle.

A NUMBER of authors have studied the problem of whether the ρ resonance in $\pi\pi$ scattering could be accounted for on the basis of a "bootstrap" model in which the principle force giving rise to the ρ is ρ exchange in the crossed channels; examples of such calculations are cited in Ref. 1. A common result of such studies has been that while it seems possible to understand the position of the ρ, the calculated width is much wider than the observed width. A particularly careful study of this has been made by Fulco, Shaw, and Wong (FSW).[2] They studied the width of the ρ obtained in a multichannel calculation in which the coupled $\pi\omega$ and $K\bar{K}$ channels were included in the hope that the presence of these closed inelastic channels might produce an appreciable narrowing of the theoretical width. Because of the spin of 1 of the exchanged ρ, a cutoff is required in their calculation, and this is set to reproduce the experimentally observed position of the ρ. FSW find that, while the inclusion of the inelastic channels does indeed produce appreciable narrowing of the resonance, the theoretical width is still larger than the experimental width by about a factor of 4.

An experimental enhancement in the $\pi\omega$ invariant-mass spectrum representing a possible $\pi\omega$ resonance, termed the B particle, has been reported.[3] Considerable doubt concerning its resonant nature exists as a result of further experimental study.[4,5] It is the purpose of this paper, however, to examine the effect of the B on the theoretical parameters of the ρ on the assumption that it is indeed a resonance. The experimental quantum numbers of the B are not known. Theoretically, on the basis of examination of the sign and magnitude of the force due to ρ exchange in the various partial waves in the $\pi\omega$ channel, one is led to expect a possible 2^- $\pi\omega$ resonance, and thus to assign these quantum numbers to the B.[6–10] Since 2^- exchange is attractive in the 1^- state in which the ρ occurs,[8–10] one might hope that B exchange would tend to make the ρ look more like a bound state in the closed $\pi\omega$ channel and hence narrow its width.

We follow the procedure of FSW. Since there was comparatively little effect in their calculations from the inclusion of the $K\bar{K}$ channel, we will confine ourselves to a two-channel problem for simplicity. In what follows, the indices 1 and 2 refer to the $\pi\pi$ and $\pi\omega$ channels, respectively. We define the T matrix by

$$T_{ij}(s)=(\rho_1\rho_2)^{-1/2}t_{ij}(s)\,, \tag{1}$$

where

$$t_{11}=e^{i\delta_{11}}\sin\delta_{11},\quad \rho_1=q^3/\sqrt{s},\quad \text{and}\quad \rho_2=p^3\sqrt{s}\,.$$

Here s, as usual, is the square of the total energy in the center-of-mass system, and q (p) is the center-of-mass 3-momentum in the $\pi\pi$ ($\pi\omega$) channel. We are considering, of course, the $I=l=1$ partial wave, and we will use units with $\hbar=c=\mu=1$, where μ is the pion mass. Using the ND^{-1} method we put

$$T=ND^{-1}\,, \tag{2}$$

where the N matrix is obtained by solving the following

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set of coupled integral equations

$$N_{ij}=B_{ij}+\sum_{k=1}^{2}\frac{1}{\pi}\int_{s_i}^{\Lambda}ds'K_{ik}(s,s')N_{kj}(s'). \quad (3)$$

In Eq. (3), s_i is the threshold in channel i, and Λ is the cutoff, adjusted, as previously explained, to produce the proper position for the ρ resonance.

The matrix K in Eq. (3) is given by

$$K_{ik}(s,s')=\left(\frac{1}{s'-s}\right)\left[B_{ik}(s')-\frac{(s-s_0)}{(s'-s_0)}B_{ik}(s)\right]\rho_k(s'), \quad (4)$$

where B is the matrix of Born amplitudes. We depart from FSW in the inclusion of a contribution to B_{22} due to the exchange of a 2^- particle. Once N has been obtained, the D matrix can be calculated by direct integration from

$$D_{ij}(s)=\delta_{ij}-\left(\frac{s-s_0}{\pi}\right)\int_{s_i}^{\Lambda}ds'\frac{\rho_i(s')N_{ij}(s')}{(s'-s_0)(s'-s)}. \quad (5)$$

In Eqs. (4) and (5), s_0 represents the subtraction point, which was taken to be 1 for most of our calculations, although it was verified that T was independent of s_0 as it should be. A common value of the cutoff was taken for both channels, because FSW report no improvement in the results when different cutoffs are allowed, and it seems clear that our general conclusions would also be unaltered by the presence of an additional cutoff parameter.

The Born amplitudes for ρ exchange are well known. (See FSW or the last paper cited in Ref. 1.) The B-exchange contribution to the $J=1$, odd-parity partial-wave amplitude, $T_{22B}(s)$, is given by

$$T_{22B}(s)=(p/16\pi\sqrt{s})M(s), \quad (6)$$

where

$$\begin{aligned}M=g_B^2&\left(-\frac{1}{3}+\frac{10}{3}x-\frac{4}{3}x^2+\frac{s+2p^2}{m_B^2}\right)I^{(0)}(B)\\&+\left(\frac{13}{3}-\frac{10}{3}x+\frac{4}{3}x^2-\frac{(s+2p^2)}{m_B^2}\right)I^{(2)}(B)\\&+[-2s-4p^2+2x(m^2-\mu^2)-2p^4/m_B^2]I^{(1)}(B)/p^2\\&+(2p^2/m_B^2)I^{(3)}(B),\end{aligned} \quad (7)$$

where m_B is the B mass, 1220 MeV, g_B is the π-ω-B coupling, $g_B^2/4\pi=1.03$ corresponding to a width of 125 MeV, m is the ω mass, and

$$x=(m^2-\mu^2)/m_B^2,$$

$$I^{(0)}(B)=\frac{1}{2p^2}\ln\frac{2p^2-C_B}{-2p^2-C_B};\quad I^{(2)}(B)=\frac{C_B}{2p^2}I^{(1)}(B);$$

$$I^{(1)}(B)=\frac{1}{p^2}+\frac{C_B}{2p^2}I^{(0)}(B);\quad I^{(3)}(B)=\frac{1}{3p^2}+\frac{C_B}{2p^2}I^{(2)}(B);$$

$$C_B=-\tfrac{1}{2}[s+2(m_B^2-m^2-\mu^2)-(m^2-\mu^2)^2/s].$$

In calculating T_{22B} we have neglected the primarily F-wave π-ω-B coupling; this neglect we justify on the basis of the ρ-exchange model for the B.[6,9,10] We have included in T_{22B} the terms coming from the $p_\mu p_\nu/m_B^2$ terms in the 2^- propagator; these terms were neglected in Ref. 9.

Equation (3) was solved by the usual process of approximating it by a set of linear algebraic equations; the latter set of equations was solved numerically by computer, following the convenient systematic procedure outlined by FSW. The final results involved the inversion of a 30×30 matrix, and each run required about 50 min on an IBM 1620 computer.

Our results are shown in Table I, where we give the value of the $\pi\pi$ cross section as a function of s, with $\Lambda=600\,\mu^2$, with and without the inclusion of the B-exchange contribution. In agreement with FSW, we find that when the peak in cross section occurs at the experimental value of the ρ mass, the peak has upper and lower half-widths of about 300 and 200 MeV, respectively, with B exchange not included. (It might be remarked that, because of the width of the resonance, there is a considerable difference between the energy at which the cross section peaks, and that at which the phase shift becomes 90°; the latter occurs around $s=44$, or almost 200 MeV above the peak in the cross section.) It will also be seen from the results in Table I that the position and width of the resonance are almost totally insensitive to the inclusion of the 2^- exchange force. It seems clear that, even if the B should be confirmed as a resonance and not merely a kinematic effect, we cannot

TABLE I. The $l=1$ contribution to the $\pi\pi$ total cross section, in units of millibarns, as a function of s, the square of the total center-of-mass energy, with and without the inclusion of the B-exchange force in the $\pi\omega$ channel. The cutoff for both cases is set at 600 pion masses squared.

s	$\sigma_{\pi\pi}$ (no B)	$\sigma_{\pi\pi}$ (B included)
4.2	0.023	0.022
11.0	17.2	16.6
17.8	70.8	70.0
24.6	102.0	103.0
31.4	100.2	101.4
38.2	86.8	87.3
45.0	73.3	73.6
51.8	61.2	61.5
58.6	49.3	49.5

TABLE II. The ρ- and B-exchange contributions, B_ρ and B_B, to the Born amplitudes for $\pi\omega$ scattering as a function of s. The amplitudes are normalized as defined in the text and are given in units with $\hbar=c=\mu=1$, and have been multiplied by 10^3.

s	B_ρ	B_B
48.7	1.700	0.569
188.1	0.316	0.273
326.9	0.135	0.097
465.9	0.081	0.066
600.0	0.056	0.050

look to it to explain any of the discrepancy between the experimental and theoretical width of the ρ.

It is perhaps of interest to ask why the effect of B exchange is so small. In Table II, we give several values of the ρ- and B-exchange contributions to the Born amplitudes for $\pi\omega$ scattering; it will be seen that they are of comparable magnitude so that, *a priori*, one might expect the B exchange to have at least an appreciable effect. The reason it does not is that the 22 matrix elements of the kernel K in the integral equation (3) depend on the Born amplitude B_{22} and on its variation with energy. For example, if one made a linear approximation to the Born matrix element, one would obtain from Eq. (4) the expression

$$K_{ik}(s,s')=B_{ik}(s)/(s'-s)+B_{ik}'(s)\,, \tag{8}$$

where the prime on the second term indicates differentiation. Thus if the Born amplitude is positive but decreases sufficiently rapidly with energy one might expect to find the kernel becoming negative. This is, in fact, what happens with the B-exchange contribution to B_{22}; although it is positive, it decreases with energy, as may be seen from Table II, and the resulting contribution to the kernel turns out to be negative. The result is that the change in the numerator function when B exchange is included is only of the order of magnitude of the Born term; in fact, the second term in Eq. (3) actually causes the change in N to be smaller than the change in B. In contrast, for example, the kernel due to ρ exchange in the $\pi\pi$ channel is positive, so that N is larger than the Born term. Consequently, the inclusion of B exchange has little effect on the N matrix and hence on the amplitude. Another example of this same effect is seen in the fact that if one calculates $\pi\omega$ scattering due to B exchange as a single-channel problem, then one finds that the scattering is quite weak; the denominator function is only down to about 0.9 at threshold, and remains above 0.6 throughout the whole range of energies less than Λ. The same thing is, in fact, true of ρ exchange in the $\pi\omega$ channel; it also leads to a negative K_{22} and hence a small contribution to N_{22}. However, ρ exchange of course contributes to the reaction $\pi+\pi\rightarrow\pi+\omega$ also, unlike B exchange which contributes only to the 22 matrix elements, and it is for this reason that the inclusion of the $\pi\omega$ channel has an appreciable effect on the parameters of the ρ when ρ exchange is taken as the dominant force.

In view of the uncertainty as to whether the effect at 1220 MeV reported in Ref. 3 actually is a 2^- resonance, and the theoretical suggestions that such a resonance might be expected to exist,[6–10] one might ask whether the exchange of a 2^- particle with a different mass in the $\pi\omega$ channel might have a greater effect on the width of the ρ. That is, it could be that the different energy dependence in the kernel of the integral equation resulting from a different B mass would avoid the effect discussed in the preceding paragraph. In fact, however, reasonable changes in the B mass do not alter our conclusion that the effect of the exchange of a 2^- particle on the width of the ρ is negligible. We repeated our calculations for B masses of 7 and 11 pion masses (as opposed to 8.7 pion masses for the particle reported in Ref. 3) and the results were essentially the same as those in the second column of Table I; the $\pi\pi$ cross sections at the given energies changed by, typically, around 5% with these variations in the exchanged mass. These small changes in the individual cross-section values left the width and shape of the resonance peak for all practical purposes unchanged. As one would expect, changing the width of the B has even less effect on the results, since this does nothing to alter the energy dependence of the kernel. Increasing the B width by a factor of 4 produced changes only of the order of 1% for the cross sections from those given in Table I. Hence we can apparently not look to a 2^- $\pi\omega$ resonance, discovered or undiscovered, to account for the discrepancy in the calculated width of the ρ.

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