Superconvergent Dispersion Relations and Vector-Meson-Baryon Couplings*

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Matrix elements of the equal-time commutators of an octet of vector current densities are considered in the infinite-momentum limit. As a consequence of locality, the Fourier transforms of these matrix elements are polynomials in the components of the momentum transfer. The resulting superconvergent sum rules are saturated with the octet and the decimet of baryons, and are evaluated at the double pole due to vector mesons. A consistent set of approximate equations for vector-meson–baryon couplings is obtained. This set has a unique nontrivial solution for the coupling constants. Together with a meson pole model for the electromagnetic form factors of baryons, these couplings give rise to relations for the form factors which are in good agreement with experiments. They are also consistent with the results obtained on the basis of collinear $U(6)$ symmetry.

1. INTRODUCTION

In two recent papers[1–3] we have given a brief description of approximate relations between vector-meson–baryon couplings which follow from the saturation of a specific set of superconvergent sum rules. These sum rules are consequences of the *locality* (microscope causality) of vector current densities and specific assumptions about the boundedness of the Fourier transforms of current commutators. *They do not depend in any way upon the validity of a current algebra.*

Our boundedness assumption is indirect. It is contained in the assumption that in those matrix elements of vector current-density commutators at equal times, which correspond to non-elastic amplitudes, we can interchange the sum over intermediate states and the infinite-momentum limit. The resulting set of sum rules is then saturated by the octet and the decimet of baryons and evaluated at the vector-meson pole.[4]

In this paper, we give a detailed derivation of our set of equations, and we discuss the solution. We obtain unique results for vector-meson–baryon couplings. Within the framework of a meson pole–dominance model, these couplings give rise to expressions for the electromagnetic form factors of baryons which are in good agreement with experiment, and which are consistent with the results obtained on the basis of collinear $U(6)$.[5]

2. SUPERCONVERGENT SUM RULES

Let us consider a commutator of vector current densities like

$$[V_{i\alpha}(x),V_{j\beta}(x')]_{x_0=x_0'}, \tag{1}$$

where i and j are isospin or $SU(3)$ indices. As a consequence of microscopic causality (locality), this equal-time commutator is given by a polynomial in the derivatives of $\delta(\mathbf{x}-\mathbf{x}')$ with q-number coefficients. Hence it follows that a matrix element like

$$E_{\alpha\beta}{}^{ij}(p,p';\mathbf{q})=\int d^4x\, e^{-iq\cdot x}\delta(x_0) \times\langle p'|[V_{i\alpha}(x/2),V_{j\beta}(-x/2)]|p\rangle \tag{2}$$

is a polynomial in the components of $\mathbf{q}$ with coefficients depending upon p, p', and spin matrices.[6] Of course, this polynomial is such that it reflects the covariance of $E_{\alpha\beta}$. In Eq. (2), and in the following equations, we generally suppress the spin indices.

Besides the matrix elements (2), we consider also the "absorptive" amplitude

$$A_{\alpha\beta}{}^{ij}(p,p';q)=\int d^4x\, e^{-iq\cdot x} \times\langle p'|[V_{i\alpha}(x/2),\, V_{j\beta}(-x/2)]|p\rangle. \tag{3}$$

We can introduce a complete set of intermediate states

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[1] R. Oehme, Phys. Letters **21**, 567 (1966).

[2] R. Oehme, Phys. Letters **22**, 206 (1966).

[3] Superconvergent sum rules have also been discussed independently by other authors, mainly by considering explicitly the high-energy behavior of scattering amplitudes. V. de Alfaro, S. Fubini, G. Furlan, and G. Rossetti, Phys. Letters **21**, 576 (1966); I. G. Aznayryan and L. D. Soloviev, Dubna Report No. E-2544, 1966 (unpublished).

[4] Our method is, of course, general. It can be used in many other cases. The meson matrix elements of the equal-time commutators of vector densities are being considered by G. Venturi, University of Chicago. The equal-time commutators of axial-vector densities are also of interest. However, there is no reason to expect that it is always sufficient to use only the octet and the decimet of baryons in the saturation of the superconvergent sum rules (Ref. 1). Certainly, an infinite number of particles with unlimited spin is needed for the saturation of the full set of nonforward relations. It may also be of interest to consider the application of our method to anticommutation relations of fermion current densities at equal times, and to boson-fermion commutators.

[5] R. Oehme, in *Preludes in Theoretical Physics*, edited by A. de Shalit, H. Feshbach, and L. Van Hove (North-Holland Publishing Company, Amsterdam, 1966), p. 143; R. Oehme, in *High-Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965), p. 533. These articles contain further references.

[6] R. Oehme, Phys. Rev. **100**, 1503 (1955).

$|n,p_n\rangle$ and write $A_{\alpha\beta}$ in the form

$$A_{\alpha\beta}{}^{ij}(p,p';q)=2\pi\sum_{|n\rangle}{}'\{\delta(s-m_n{}^2)[2p_{n0}N(n)\times\langle p'|V_{i\alpha}(0)|n,p_n\rangle\langle n,p_n|V_{j\beta}(0)|p\rangle]_{p_n=\frac{1}{2}(p+p')+q}-\delta(u-m_n{}^2)[2p_{n0}N(n)\langle p'|V_{j\beta}(0)|n,p_n\rangle\times\langle n,p_n|V_{i\alpha}(0)|p\rangle]_{p_n=\frac{1}{2}(p+p')-q}\}\,, \quad (4)$$

where

$$s=-[\tfrac{1}{2}(p+p')+q]^2\,,\quad u=-[\tfrac{1}{2}(p+p')-q]^2\,, \quad (5)$$

and $N(n)$ is a normalization factor which appears because we use a covariant normalization of states. The prime on the sum in Eq. (4) indicates that the total momentum $\mathbf{p}_n$ is kept fixed. Performing an analogous decomposition in Eq. (2), we see that $E_{\alpha\beta}$ can be written in a *formal* way as an integral over the absorptive amplitude $A_{\alpha\beta}$:

$$E_{\alpha\beta}{}^{ij}(p,p';\mathbf{q})=\left[\frac{1}{2\pi}\int_0^\infty ds\,\tfrac{1}{2}\{s+[\tfrac{1}{2}(\mathbf{p}+\mathbf{p}')+\mathbf{q}]^2\}^{-1/2}\times A_{\alpha\beta}{}^{ij}(p,p';q)-\frac{1}{2\pi}\int_0^\infty du\,\tfrac{1}{2}\{u+[\tfrac{1}{2}(\mathbf{p}+\mathbf{p}')-\mathbf{q}]^2\}^{-1/2}\times A_{\alpha\beta}{}^{ij}(p,p';q)\right]_{\mathbf{q}\text{ fixed}}\,, \quad (6)$$

where s and u are defined in Eq. (5).[7]

We are interested in the infinite-momentum limit[8] of $E_{\alpha\beta}$, and since it is not our purpose here to give a general discussion, we consider only the case $p=p'$, which is relevant for this paper. We write

$$p=(\mathbf{p}+\gamma\hat{e}_z,\, i[(\mathbf{p}+\gamma\hat{e}_z)^2+m^2]^{1/2})$$

and make the assumption that the limit $\gamma\to\infty$ can be interchanged with the integral in Eq. (6), *but only for those amplitudes which correspond to nonelastic processes.* This assumption implies a certain boundedness for the amplitude $A_{\alpha\beta}$, which is related to the degree of singularity of the commutator on the light cone, and hence also to the degree of the polynomial in $\mathbf{q}$. A detailed discussion of these connections is outside the scope of this paper.

Indicating $i\leftrightarrow j$ odd amplitudes by $[ij]$, we write

$$A_{\alpha\beta}{}^{[ij]}=p_\alpha p_\beta a_{[ij]}+p_\alpha p_\beta b_{[ij]}+\cdots\,, \quad (7)$$

where $a_{[ij]}$ etc. are invariant coefficients which are still matrices in spin space. Taking the limit $\gamma\to\infty$ inside the integral in Eq. (6), and keeping track of normalization factors p_0/m, we find from the leading term in γ the relations

$$\int_{-\infty}^{+\infty} d\nu\, a_{[ij]}(\nu,\mathbf{q})=\text{polynomial in }\mathbf{q}\,, \quad (8)$$

where the variable ν is given by

$$\nu\equiv-2p\cdot q=s-u\,. \quad (9)$$

We can assume $\hat{e}_z\cdot q=0$ without loss of generality.

The absorptive amplitude (3) has the symmetry

$$A_{\alpha\beta}{}^{ij}(p,q)=-A_{\beta\alpha}{}^{ji}(p,-q)\,, \quad (10)$$

and hence the integral in Eq. (8) is an even function of $\mathbf{q}$. If we expand $a_{[ij]}$ with respect to a complete set of spin matrices, then the integrals (8) of the invariant coefficients $a^{(n)}$ are functions of $\mathbf{q}^2$, for which we obtain relations of the form

$$\alpha_{[ij]}{}^{(n)}(q^2)\equiv\int_{-\infty}^{+\infty} d\nu\, a_{[ij]}{}^{(n)}(\nu,q^2)=\text{polynomial in }q^2. \quad (11)$$

Here we can reinterpret $\mathbf{q}^2$ as the square of the four-momentum transfer q^2, because $q_0\to(\hat{e}_z\cdot q)=0$ in the infinite-momentum limit $\gamma\to\infty$ with fixed ν and q^2. It is reasonable to assume that the functions $\alpha^{(n)}(q^2)$ are analytic and can be continued from real $q^2\geq 0$ to $q^2<0$. Since we also assume that the currents $V_{i\alpha}$ are coupled to the corresponding vector-meson fields, we expect poles at $q^2=-\mu^2$. For the purpose of this paper, we consider an octet of degenerate vector mesons and we are interested in the homogeneous relations for the residue of the corresponding *double poles* which follow from Eq. (11). If we truncate the sum rules (9) by using a limited set of single baryon states, then we obtain from the residue a set of relations between the vector-meson–baryon coupling constants. Of course, we can also take into account the meson mass splitting and obtain correspondingly generalized relations.

By extracting a residue like

$$\lim_{q^2\to-\mu^2}(q^2+\mu^2)^2\alpha_{[ij]}{}^{(n)}(q^2)=0\,, \quad (12)$$

we are effectively dealing with superconvergent dispersion relations for the corresponding amplitudes describing vector-meson–baryon scattering. Our use of amplitudes in the infinite-momentum limit corresponds to the selection of a specific class of these dispersion formulas. There may be other sum rules which have convergence properties corresponding to those selected by our prescription, but for the purpose of saturation with single-particle states, the use of the infinite-momentum limit seems to be preferable. Nonelastic $i\leftrightarrow j$ even amplitudes will be included in Sec. 6.

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[7] Note that we can also write the formal relation

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} dq_0\, A_{\alpha\beta}{}^{ij}(p,p';q)=E_{\alpha\beta}{}^{ij}(p,p';\mathbf{q}).$$

A priori, the integration over q_0 does *not* correspond to an integral over the total energy variable of the absorptive amplitude for fixed values of $k^2, k'^2, (p-p')^2$, where $q=\frac{1}{2}(k+k')$ and $p+k=p'+k'$. Rather, as in Eq. (6), we have considered $\mathbf{q}$ fixed. These differences are eliminated in the infinite-momentum limit.

[8] S. Fubini and G. Furlan, Physics **1**, 229 (1965); R. F. Dashen and M. Gell-Mann, Phys. Rev. Letters **17**, 340 (1966); R. Oehme, Phys. Rev. **143**, 1138 (1966).

3. SATURATION

In this section we consider the sum rules (11) for the octet and the decimet of baryons, assuming that we can restrict the sum over intermediate states to this same set of particles. In order to simplify the calculations, we assume here $SU(3)$ invariance, and later we will also use a common mass for all baryons.

We write the relevant vertices in the general form

$$\langle N(p')|V_\alpha|N(p)\rangle=\left(1+\frac{q^2}{4m^2}\right)^{-1}\bar{u}(p')\times\left\{\frac{P_\alpha}{2m}G_E(q^2)-\frac{ir_\alpha}{2m}G_M(q^2)\right\}u(p),\quad(13)$$

$$\langle N^*(p')|V_\alpha|N(p)\rangle=\bar{w}_\beta(p')\{\delta_{\alpha\beta}D_1{}^V(q^2)+i\gamma_\alpha q_\beta(m^*+m)^{-1}D_2{}^V(q^2)+P_\alpha q_\beta(m^*+m)^{-2}D_3{}^V(q^2)+q_\alpha q_\beta(m^*+m)^{-2}D_4{}^V(q^2)\}u(p),\quad(14)$$

and

$$\langle N^*(p')|V_\alpha|N^*(p)\rangle(1+q^2/4m^{*2})=\bar{w}_\beta(p')\left\{\left[\frac{P_\alpha}{2m^*}H_E(q^2)-\frac{ir_\alpha}{2m^*}H_M(q^2)\right]\delta_{\beta\beta'}+\left[\frac{P_\alpha}{2m^*}\bar{H}_E(q^2)-\frac{ir_\alpha}{2m^*}\bar{H}_M(q^2)\right]\frac{q_\beta q_{\beta'}}{4m^{*2}}\right\}w_{\beta'}(p),\quad(15)$$

with $P=p+p'$, $q=p-p'$, and $r_\alpha=\epsilon_{\alpha\beta\gamma\delta}P_\beta q_\gamma\gamma_\delta\gamma_5$. We note that the assumption of a conserved vector current implies

$$D_1{}^V(q^2)+D_2{}^V(q^2)+\frac{m^*-m}{m^*+m}D_3{}^V(q^2)+\frac{q^2}{(m^*+m)^2}D_4{}^V(q^2)=0.$$

Actually, in place of the complicated expression (14), we use later the generalized $M1$ form

$$\langle N^*(p')|V_\alpha|N(p)\rangle=\frac{1}{4m^2}\left(1+\frac{q^2}{4m^2}\right)^{-1}\times D_1{}^V(q^2)\epsilon_{\alpha\beta\gamma\delta}P_\beta q_\gamma\bar{w}_\delta(p')u(p),\quad(16)$$

which, for $m=m^*$, corresponds to

$$\left(1+\frac{q^2}{4m^2}\right)^{-1}D_1{}^V(q^2)=-D_2{}^V(q^2)=D_3{}^V(q^2)=-D_4{}^V(q^2),\quad(17)$$

and which turns out to be consistent with our equations, as well as with the empirical dominance of the $M1$ amplitude.[9]

At first, we take the matrix elements of the equal-time commutator (1) with respect to octet states of momentum $p=(\mathbf{p},ip_0)$ and insert the octet and the decimet as intermediate states with $p_n=(\mathbf{p}\pm\mathbf{q},\ ip_{n0})$, $\mathbf{p}\cdot\mathbf{q}=0$. Taking the limit $|\mathbf{p}|\to\infty$, we obtain then the equations[1]

$$[G_E{}^f(q^2)+G_E{}^d(q^2)]^2+q^2[G_M{}^f(q^2)+G_M{}^d(q^2)]^2-\frac{4}{9}\frac{q^2}{4m^2}D^2(q^2)=1+\cdots,\quad(18)$$

$$G_E{}^{f2}(q^2)+\tfrac{1}{3}G_E{}^{d2}(q^2)+q^2[G_M{}^{f2}(q^2)+\tfrac{1}{3}G_M{}^{d2}(q^2)]+\frac{1}{18}\frac{q^2}{4m^2}D^2(q^2)=1+\cdots,\quad(19)$$

$$[G_E{}^f(q^2)-G_E{}^d(q^2)]^2+q^2[G_M{}^f(q^2)-G_M{}^d(q^2)]^2+\frac{2}{9}\frac{q^2}{4m^2}D^2(q^2)=1+\cdots,\quad(20)$$

where the dots indicate a homogeneous polynomial in q^2. With the $M1$-type amplitude (16) and $m=m^*$, the function $D(q^2)$ is given by

$$D(q^2)=D_1{}^V(q^2).$$

We note here that the right-hand sides of Eqs. (18)–(20) would be given by $1+q^2/4m^2$ if we used the $SU(3)$ density algebra

$$[V_{i0}(x),V_{j0}(x')]_{x_0=x_0'}=if_{ijk}V_{k0}(x)\delta(\mathbf{x}-\mathbf{x}'),\quad(21)$$

instead of only the general causality condition.

In order to have a saturation of our set of sum rules (11), we must also consider the decimet-octet and decimet-decimet matrix elements of the commutators (1) which are *odd* in the $SU(3)$ indices i, j. Again we have an octet and a decimet as intermediate states. Since we want to have simple equations, we omit terms which are of higher order in $q^2/4m^2$, and consequently also the quadrupole form factors appearing in the vertex (15). With these approximations, we find from the **10-8** matrix elements the relations[2]

$$q^2D_1{}^V(q^2)[G_M{}^f(q^2)+G_M{}^d(q^2)-(5/3)H_M(q^2)]=0+\cdots,$$
$$q^2D_1{}^V(q^2)[G_M{}^f(q^2)-G_M{}^d(q^2)+\tfrac{1}{3}H_M(q^2)]=0+\cdots,\quad(22)$$

whree the dots again indicate a homogeneous polynomial in q^2. Correspondingly, the **10-10** matrix elements give rise to the formulas

$$H_E{}^2(q^2)+\tfrac{1}{3}q^2H_M{}^2(q^2)+\frac{1}{6}\frac{q^2}{4m^2}[D_1{}^V(q^2)]^2=1+\cdots,$$
$$H_E{}^2(q^2)+\frac{7}{9}q^2H_M{}^2(q^2)+\frac{1}{18}\frac{q^2}{4m^2}[D_1{}^V(q^2)]^2=1+\cdots,\quad(23)$$

and

$$q^2H_M{}^2(q^2)-\frac{1}{4}\frac{q^2}{4m^2}[D_1{}^V(q^2)]^2=0+\cdots.\quad(24)$$

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[9] See, for example, R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).

The last equation (24) corresponds to a $|\Delta J_z|=2$ matrix element, whereas all other relations are $\Delta J_z=0$. Because of our restriction to $i\leftrightarrow j$-odd amplitudes, we find no matrix elements with $|\Delta J_z|=1$.

4. VECTOR-MESON–BARYON COUPLINGS

As discussed in Sec. 2, we now evaluate our set of equations between form factors at the double pole $q^2=-\mu^2$ corresponding to the vector-meson octet.[1,2] We may write

$$G_{E,M}{}^{f,d}(q^2)=\frac{g_{E,M}{}^{f,d}\chi\mu^2}{q^2+\mu^2}+\Gamma_{E,M}{}^{f,d}(q^2)\,,$$
$$D_1{}^V(q^2)=\frac{d_1\chi\mu^2}{q^2+\mu^2}+\Delta(q^2)\,, \tag{25}$$
$$H_{E,M}(q^2)=\frac{h_{E,M}\chi\mu^2}{q^2+\mu^2}+\Theta(q^2)\,,$$

where the functions Γ, Δ, and Θ are regular at $q^2=-\mu^2$, and where χ is a common factor. Introducing the ratios

$$\xi_E=g_E{}^d/g_E{}^f\,,\qquad \xi_M=g_M{}^d/g_M{}^f\,,$$
$$\delta=\frac{\mu}{2m}d_1/g_E{}^f\,,\qquad a=\tfrac{3}{2}\mu g_M{}^f/g_E{}^f\,, \tag{26}$$

and

$$\eta_E=h_E/g_E{}^f\,,\quad \eta_M=\mu h_M/g_E{}^f\,,$$

we can write the relations for the residue in the form

$$(1+\xi_E)^2-(4/9)a^2(1+\xi_M)^2+(4/9)\delta^2=0\,,$$
$$(1+\tfrac{1}{3}\xi_E{}^2)-(4/9)a^2(1+\tfrac{1}{3}\xi_M{}^2)-(1/18)\delta^2=0\,, \tag{27}$$
$$(1-\xi_E)^2-(4/9)a^2(1-\xi_M)^2-(2/9)\delta^2=0\,.$$

These are the expressions obtained from Eqs. (18)–(20). Furthermore we find

$$\delta[2a(1+\xi_M)-5\eta_M]=0\,,$$
$$\delta[2a(1-\xi_M)+\eta_M]=0\,, \tag{28}$$

and

$$\eta_E{}^2-\tfrac{1}{3}\delta_M{}^2-\tfrac{1}{6}\delta^2=0\,,$$
$$\eta_E{}^2-(7/9)\eta_M{}^2-(1/18)\delta^2=0\,, \tag{29}$$
$$\eta_M{}^2-\tfrac{1}{4}\delta^2=0\,,$$

from Eqs. (22) and (23)–(24), respectively.

We are interested only in nontrivial solutions of these equations for which $\delta\neq0$. From Eqs. (27) alone we can then express a^2, ξ_M, and δ in terms of ξ_E, and find, for example

$$\xi_M=3\frac{3-2\xi_E}{6+5\xi_E}\,,\quad\text{or}\quad \xi_E=3\frac{3-2\xi_M}{6+5\xi_M}\,, \tag{30}$$

as a relation between the electric and the magnetic d/f ratios. The additional requirement that the two equations (28) be compatible with each other implies, however, that $\xi_M=\frac{3}{2}$, and hence $\xi_E=0$, $\delta=4$, $a^2=1$. The relations (28) and (29) are consistent with this solution if $\eta_E=1$ and $\eta_M=1$. Hence we find for the *vector-meson–baryon couplings*

$$g_E{}^d=0\,,\quad g_M{}^d/g_M{}^f=\tfrac{3}{2}\,, \tag{31}$$

the important relation

$$g_M{}^d=(1/\mu)g_E{}^f\,, \tag{32}$$

and

$$d_1=4mg_M{}^d. \tag{33}$$

We also have

$$h_E=g_E{}^f\,,\quad h_M=g_M{}^d. \tag{34}$$

The expressions (31)–(33) are consistent with the results obtained for the vector-meson–baryon couplings from collinear $U(6)$ symmetry or from the "dissociated quark model," which describes hadrons as if they were loosely bound systems of quarks.

5. ELECTROMAGNETIC FORM FACTORS

Of special interest is the use of the results (31)–(34) in connection with the assumption of *meson pole dominance* for the form factors. We obtain then the familiar relations[10]

$$G_E{}^d(q^2)=0\,,\quad G_M{}^d(q^2)/G_M{}^f(q^2)=\tfrac{3}{2}\,, \tag{35}$$

which imply

$$G_E{}^n(q^2)=0\,,\quad G_M{}^p(q^2)/G_M{}^n(q^2)=-\tfrac{3}{2}\,,\quad\text{etc.} \tag{36}$$

It follows from the assumption of pole dominance that

$$G_M{}^p(q^2)/G_E{}^p(q^2)=\mu_p/2m\,, \tag{37}$$

and from Eq. (32) we find the formulas

$$\mu_p=2m/\mu \tag{38}$$

and

$$\tfrac{1}{6}\langle r_p{}^2\rangle=(\mu_p/2m)^2. \tag{39}$$

For the form factor of the $(M-1)$-type decimet-octet vertex, we have

$$D_1{}^V(q^2)=4mG_M{}^p(q^2)\,. \tag{40}$$

This relation also corresponds to the collinear $U(6)$ result.[11] At $q^2=0$, we have $D_1{}^V(0)=2\mu_p$. In view of the crudeness of our model, the results (35)–(40) are in good agreement with experiments. Finally, we note the approximate equalities

$$H_E(q^2)=G_E{}^p(q^2)\,,\quad H_M(q^2)=G_M{}^p(q^2) \tag{41}$$

for the form factors of N^{+*}. They follow from Eq. (34) and the pole-dominance assumption.

We note especially the result (39) which, as we have pointed out in Ref. 1, is quite different from what we would obtain on the basis of the $SU(3)$ density algebra. With the density algebra, we would have relations

[10] P. G. O. Freund and R. Oehme, Phys. Rev. Letters **14**, 1085 (1965); K. J. Barnes, P. Carruthers, and F. von Hippel, *ibid.* **14**, 82 (1965).

[11] R. Oehme, Phys. Letters **19**, 518 (1965).

corresponding to Eqs. (18)–(24), but with specified right-hand sides. For the electromagnetic form factors of the nucleons, we would obtain the relation

$$[G_E{}^p(q^2)]^2+q^2[G_M{}^p(q^2)]^2=1+\frac{q^2}{4m^2}. \qquad (42)$$

Evaluating this formula near $q^2=0$, we find

$$\tfrac{1}{6}\langle r_p{}^2\rangle=\tfrac{1}{2}(\mu_p/2m)^2 \qquad (43)$$

instead of the more satisfactory equation (39).[12] The latter relation follows from Eq. (42) if it is evaluated at the vector-meson pole and if we use the pole model for the extrapolations to $q^2=0$.

The factor $\frac{1}{2}$ appearing in Eq. (43) is, of course, related to the corresponding factor in the Cabibbo Radicati[13] sum rule

$$-\tfrac{1}{6}(\langle r_p{}^2\rangle-\langle r_n{}^2\rangle)+\frac{1}{2}\left(\frac{\mu_p-\mu_n}{2m}\right)^2 +\frac{1}{4\pi^2\alpha}\int_{(m+m_\pi)^2}^{\infty} ds\,\frac{2\sigma_{1/2}(s)-\sigma_{3/2}(s)}{s-m^2}=0. \qquad (44)$$

This sum rule is obtained from the $SU(2)$ density algebra. If truncated, it corresponds to our Eq. (18), with the term $1+q^2/4m^2$ on the right-hand side, and the whole equation being evaluated near $q^2=0$. It is then of the form

$$-\tfrac{1}{6}(\langle r_p{}^2\rangle-\langle r_n{}^2\rangle)+\frac{1}{2}\left(\frac{\mu_p-\mu_n}{2m}\right)^2+\left(\frac{\mu^*}{2m}\right)^2=0, \qquad (45)$$

where $\mu^*=(\sqrt{2}/3)D_1{}^V(0)$ is the transition moment. The relation (45) is rather unsatisfactory, which may indicate that contributions from higher resonances are not negligible. It could also mean that the density commutation relation (21) is not valid. For instance, there may be gradient terms which imply a term proportional to q^2 on the right-hand side of Eq. (18).

On the other hand, if we follow the rules described in this paper, evaluate Eq. (18) at the quadratic meson pole, and use a pole model in order to extrapolate to $q^2=0$, then we find instead of Eq. (45) the relation

$$-\tfrac{1}{6}(\langle r_p{}^2\rangle-\langle r_n{}^2\rangle)+\left(\frac{\mu_p-\mu_n}{2m}\right)^2+2\left(\frac{\mu^*}{2m}\right)^2=0, \qquad (46)$$

which is a special case of our Eqs. (35)–(39),[14] and which is more satisfactory than Eq. (45). Furthermore, it does not depend upon the validity of a current algebra.

[12] The relation $\frac{1}{6}\langle r_p{}^2\rangle=(\mu_p/2m)^2$ can also be obtained from commutation relations for the densities of the space components V_{ia}, $a=1, 2, 3$, provided Schwinger gradient terms are ignored in these relations [R. F. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965); B. W. Lee, Phys. Rev. Letters 14, 676 (1965)]. If, in the quark model, the Schwinger terms are eliminated by proper symmetrization, one finds again a relation corresponding to Eq. (43) [C. Bouchiat and Ph. Meyer, Orsay Report No. TH/143 (to be published)].

[13] N. Cabibbo and L. A. Radicati, Phys. Letters 19, 697 (1966).

[14] A relation similar to Eq. (46) has been obtained by S. Fubini and G. Segrè, Nuovo Cimento (to be published).

6. DISCUSSIONS AND CONCLUSIONS

Granting the approximations we have made, especially in neglecting terms of higher order in $q^2/4m^2$ in Eqs. (22)–(24),[15] we see that there emerges a consistent set of equations between the couplings of the vector mesons to the octet and the decimet of baryons. Essentially, these equations have a unique nontrivial solution. They have been obtained by the saturation of those sum rules which are obtained in the infinite-momentum limit, and which are *odd* under the interchange of the $SU(3)$ indices. These sum rules are actually integrals over the absorptive parts of nonelastic amplitudes for vector-meson–baryon scattering. We cannot expect similar superconvergence properties for $i\leftrightarrow j$ *even* amplitudes as a group, since this group contains also elastic amplitudes. If we nevertheless consider the $i\leftrightarrow j$ *even* sum rules in our particular case, we find that for most of the amplitudes we have formally

$$\int_{-\infty}^{+\infty} d\nu\, a_{\{ij\}}(\nu,\mathbf{q})=0, \qquad (47)$$

simply because $a_{\{ij\}}(\nu,\mathbf{q})$ is odd under crossing $(\nu\to-\nu)$. Here we indicate $i\leftrightarrow j$ even amplitudes by $\{ij\}$. Since

$$a_{\{ij\}}(\nu,\mathbf{q})=-a_{\{ij\}}(-\nu,-\mathbf{q}), \qquad (48)$$

only terms which are odd in $\mathbf{q}$ can give rise to a sum rule. We have two such sum rules resulting from the **8-10** matrix elements of the commutator with $|\Delta J_z|=1$. Omitting, as before, terms of higher order in $q^2/4m^2$, these relations are

$$\begin{aligned} &q^2D_1{}^V(q^2)[G_E{}^f(q^2)+G_E{}^d(q^2)-H_E(q^2)]=0+\cdots,\\ &q^2D_1{}^V(q^2)[G_E{}^f(q^2)-G_E{}^d(q^2)-H_E(q^2)]=0+\cdots, \end{aligned} \qquad (49)$$

where the dots indicate again a homogeneous polynomial. Equations (49) imply

$$\delta[(1+\xi_E)-\eta_E]=0, \quad \text{and} \quad \delta[(1-\xi_E)-\eta_E]=0 \qquad (50)$$

for the residue of the vector-meson double pole. Hence we find that also the $i\leftrightarrow j$ *even* sum rules are consistent with our solutions as given in Eqs. (31)–(34).

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[15] More detailed calculations are being performed by G. Venturi, University of Chicago. They show that our results remain unchanged if all $q^2/4m^2$ terms are included.