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High-Energy Trident Production with Definite Helicities*

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The high-energy large-scale-angle limit of the completely differential cross section for the process $e^-+Z\rightarrow e^-+e^++e^-+Z$ (or for the process with all or some of the particles replaced by muons) is computed for arbitrary helicities of incident and final particles. This process is interesting as a test of electrodynamics and of the statistics of the muon. The formula is short and perhaps suitable for numerical integrations.

I. INTRODUCTION

TRIDENT production, i.e., electron-positron pair production by incident electrons of hydrogen (or the analogous process for muons), has been discussed in the past by different authors as a test of quantum electrodynamics at amsll distances.[1,2] For the case of the muon, this process also may provide a test of the statistics of the muon. In this article we present the differential cross section for the trident process for definite helicities of incident and outgoing leptons, in the limit of vanishing lepton mass. Although such a process is unlikely to be measured directly, the expression for the cross section is reasonably compact and amenable to machine integration and summation over the unobserved variables.

In Sec. II, the calculation is described and the result is given in Sec. III.

II. THE MATRIX ELEMENT

The eight diagrams are shown in Fig. 1. The matrix element of diagram (1) is, by the usual Feynman rule,[3]

$$\mathfrak{M}_1=-\frac{i}{(2\pi)^2}\left(\frac{m^4}{EE_1E_2E_+}\right)^{1/2}\frac{1}{q^2}\frac{1}{(p-p_1)^2}\times\left[\bar{u}(p_2,\lambda_2)\gamma^\mu\frac{1}{p_1+p_2-p-m}\gamma^0 v(p_+,\lambda_+)\right]\cdot\left[\bar{u}(p_1,\lambda_1)\gamma^\mu u(p,\lambda)\right]. \quad (1)$$

All the notations are summarized in the Appendix. The method we use is to calculate directly the amplitude for each graph, sum the amplitudes, and then square. To calculate the amplitude explicitly, we multiply and divide the first square bracket by

$$\bar{v}(p_+,\lambda_+)\gamma^0 u(p_2,\lambda_2)=|A_1|e^{i\theta_1}=A_1e^{i\theta_1} \quad (2)$$

and the second square bracket by

$$\bar{u}(p,\lambda)\gamma^0 u(p_1,\lambda_1)=|A_2|e^{i\theta_2}=A_2e^{i\theta_2}. \quad (3)$$

The numerators and denominators after multiplication can be put into invariant form by the use of spin-projection operators[4] and the usual trace method.

The invariant-spin projection operator O_λ for positive energy states, defined by $O_\lambda u(p,\lambda')=\delta_{\lambda\lambda'}u(p,\lambda')$, is reduced to $O_\lambda=\frac{1}{2}(1+\lambda\gamma^5)$ in the limit of zero mass. It is then easily verified that for diagrams of any order, so long as mass is neglected, the matrix element vanishes unless the fermion lines entering and leaving the diagram have the same chirality.[5] This asserts that only one spin projection operator is needed in summing the

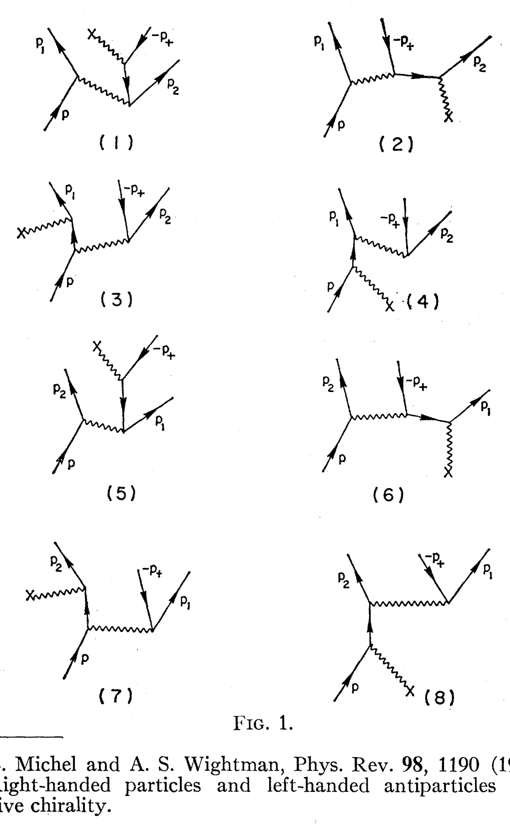

FIG. 1.

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† A. P. Sloan Fellow, 1962–64.
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[2] M. C. Chen, Phys. Rev. **127**, 1844 (1962).
[3] We use $\hbar=c=1$, $\gamma_5^2=1$. Our notation, metric, etc., are those of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1964).
[4] L. Michel and A. S. Wightman, Phys. Rev. **98**, 1190 (1955).
[5] Right-handed particles and left-handed antiparticles have positive chirality.

spins of the spinors. Thus the first bracket may be written

$$\frac{1}{A_1e^{i\theta_1}}\sum_{\lambda_2'=\pm1}\bar{u}(p_2,\lambda_2')\gamma^\mu\frac{1}{\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}-m}\times\gamma^0\sum_{\lambda_+'=\pm1}v(p_+,\lambda_+)\bar{v}(p_+,\lambda_+)\gamma^0O_{\lambda_2}u(p_2,\lambda_2').$$

This will be simplified by the usual energy-state projection operator and trace method, and then becomes

$$\frac{1}{A_1e^{i\theta_1}}\mathrm{Tr}\left[\frac{\boldsymbol{p}_2}{2m}\gamma^\mu\frac{1}{\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}-m}\gamma^0\frac{\boldsymbol{p}_+}{2m}\gamma^0\tfrac{1}{2}(1+\lambda_2\gamma^5)\right]\epsilon_\mu$$
$$=\frac{1}{A_1e^{i\theta_1}}\frac{1}{2m^2}\frac{1}{(p-p_1-p_2)^2-m^2}\times\{\tilde{p}_+\cdot\epsilon p_2\cdot(p-p_1)+\epsilon\cdot(p_1+p_2-p)p_2\cdot\tilde{p}_+$$
$$-\tilde{p}_+\cdot(p-p_1-p_2)p_2\cdot\epsilon+i\lambda_2(p-p_1)\cdot\epsilon p_2\tilde{p}_+\}\,,\quad(4)$$

where $|ABCD|$ is a short notation for the determinant formed by the components of the four vectors:

$$|ABCD|=\begin{vmatrix}A^0 & B^0 & C^0 & D^0\\ A^1 & B^1 & C^1 & D^1\\ A^2 & B^2 & C^2 & D^2\\ A^3 & B^3 & C^3 & D^3\end{vmatrix}$$

and $\tilde{p}_+=(E,\,-\mathbf{p}_+)$.

The quantity A_1 may be computed explicitly by multiplying by the complex conjugate and then using spin and energy projection operators as before; thus

$$A_1^2=\sum_{\lambda_+'=\pm1}\bar{v}(p_+,\lambda_+')\gamma^0O_{\lambda_2}\times\sum_{\lambda_2'=\pm1}u(p_2,\lambda_2')\bar{u}(p_2,\lambda_2')\gamma^0v=\frac{1}{2m^2}p_+\cdot\tilde{p}_2.\quad(5)$$

The second bracket of (1) may be treated in the same manner, and found to be

$$\bar{u}(p_1,\lambda_1)\gamma^\mu u(p,\lambda)\epsilon_\mu=\frac{1}{2m^2}\frac{1}{A_2e^{i\theta_2}}\times\{E_1p\cdot\epsilon+Ep_1\cdot\epsilon-\epsilon_0p\cdot p_1+i\lambda_1|p_1p\eta\epsilon|\}\,,$$
$$A_2^2=\frac{1}{2m^2}p_1\cdot\tilde{p}.\quad(6)$$

It is then only a matter of putting (4), (5), and (6) into (1) to obtain the full matrix element. The matrix elements of the remaining seven diagrams may be obtained from m_1 by interchange of parameters. The results are summarized in the next section.

We note that the phase factor θ_1 is the same for diagrams (1) to (4) and θ_2 the same for diagrams (5) to (8). If $\lambda=\lambda_1\neq\lambda_2$, only diagrams (1) to (4) contribute, and the phase factors have no effect to the cross section. If, on the other hand, $\lambda=\lambda_2\neq\lambda_1$, only diagrams (5) to (8) contribute, and the phase factors have no effect either. The phase factors are important only if $\lambda=\lambda_1=\lambda_2$. In this case all eight diagrams contribute, but only the relative phase θ need to be computed. From the definitions of (2) and (3), θ can again be evaluated by means of trace techniques. The result is given in the next section.

III. THE CROSS SECTION

We may write down the differential cross section

$$d\sigma=\frac{\alpha^4}{4\pi^4}\frac{|\mathbf{p}_1||\mathbf{p}_2||\mathbf{p}_+|}{E}\frac{1}{q^4}\left|\delta_{\lambda\lambda_1}\frac{F_1+F_2+F_3+F_4}{(p_+\cdot\tilde{p}_2p_1\cdot\tilde{p})^{1/2}}+\delta_P\delta_{\lambda\lambda_2}\frac{F_5+F_6+F_7+F_8}{(p_+\cdot\tilde{p}_1p_2\cdot\tilde{p})^{\frac{1}{2}}}e^{i\theta}\right|^2dE_1dE_2d\Omega_1d\Omega_2d\Omega_+.\quad(7)$$

In the above expression, δ_P is the signature of identical particles in the final state, and in our case $\delta_P=-1$ if the fermions obey Fermi-Dirac statistics and $\delta_P=+1$ were they to obey Bose-Einstein statistics. The function F_1 is given by

$$F_1(p,p_1,p_2,p_+,\lambda_1,\lambda_2)=\frac{1}{(p-p_1)^2[(p-p_1-p_2)^2-m^2]}\Big[2p_+\cdot\tilde{p}_2(-E_2p\cdot p_1+E_1p\cdot p_2+Ep_1\cdot p_2)$$
$$-\tilde{p}_+\cdot p\{(E+E_1)p_1\cdot p_2-E_2p\cdot p_1\}+\tilde{p}_+\cdot p_1\{(E+E_1)p\cdot p_2E_2p\cdot p_1\}+E_+p\cdot p_1(p_1\cdot p_2-p\cdot p_2)$$
$$-i\lambda_1\{\tilde{p}_+\cdot(p-p_1-2p_2)|p_1p\eta p_2|+p_2\cdot(p-p_1)|p_1p\eta\tilde{p}_+|\}+i\lambda_2\{(E+E_1)|pp_1p_2\tilde{p}_+|+p\cdot p_1|(p-p_1)\eta p_2\tilde{p}_+|\}$$
$$-\lambda_1\lambda_2\begin{vmatrix}E_+ & p_1\cdot\tilde{p}_+ & p\cdot\tilde{p}_+\\ E_2 & p_1\cdot p_2 & p\cdot p_2\\ E-E_1 & p\cdot p_1 & -p\cdot p_1\end{vmatrix}\Big]\quad(8)$$

The other functions F_2 to F_8 are obtained from $F_1(p,p_1,p_2,p_+,\lambda_1,\lambda_2)$ by the following exchange:

$$F_2=-F_1(p,\,p_1,\,p_+,\,p_2,\,\lambda_1,\,-\lambda_2)\,,$$
$$F_3=-F_1(-p_+,\,p_2,\,-p,p_1,\,\lambda_2,\,+\lambda_1)\,,$$
$$F_4=F_1(-p_+,\,p_2,\,p_1,\,-p,\,\lambda_2,\,-\lambda_1)\,,$$

and F_5, F_6, F_7, and F_8 are obtained from F_1, F_2, F_3, and F_4, respectively, by the interchange $p_1\leftrightarrow p_2$, $\lambda_1\leftrightarrow\lambda_2$. The relative phase θ is given by

$$\theta=\tan^{-1}\left(\frac{\lambda|p_1\tilde{p}_+p_2\tilde{p}|}{p_1\cdot\tilde{p}_+p_2\cdot\tilde{p}+p_1\cdot\tilde{p}p_2\cdot\tilde{p}_+-p_1\cdot p_2\tilde{p}\cdot\tilde{p}_+}\right).\quad(9)$$

Our calculation assumes a static Coulomb field with $Z=1$. Practical considerations will probably force the use of targets of $Z>1$, e.g., carbon. For the elastic trident production (no nuclear excitation) Eq. (7) need by modified only be inclusion of an extra factor $[ZF(q^2)]^2$, with $F(q^2)$ the charge form factor measured in elastic electron scattering. Higher orders in Z are of course neglected in this Born approximation calculation.

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APPENDIX

p, p_1, p_2, and p_+ are four-momenta of incoming μ^- (μ^+), two outgoing μ^- (μ^+) and one μ^+ (μ^-). E and p are energy and three-momentum, etc. λ, λ_1, λ_2, and λ_+ are the chiralities of these particles, and take the value $+1$ when the particle is right handed or antiparticle left handed. A four-vector with a tilde is related to its original vector by changing the sign of the space momentum, e.g., if $p=(E,\mathbf{p})$, then $\tilde{p}=(E,-\mathbf{p})$. The vector η^μ is a unit four-vector with zeroth component only. $p=\mathbf{p}$ = magnitude of space momentum. $d\Omega_1$, $d\Omega_2$, and $d\Omega_+$ are solid angles of the corresponding particles. $\alpha=1/137$; $q=p-p_1-p_2-p_+$.

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Compton Scattering and Detailed Balance*

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Possible detection of the effects due to C and T noninvariance in proton Compton scattering is discussed. It is shown that the measurement of the recoil-proton polarization together with the measurement of the differential cross section for a polarized target can provide information about the breakdown of detailed balance. The region of the second πN resonance (1520 MeV) is shown to be most suitable for such an experiment.

IT has been conjectured[1] that the electromagnetic interactions of strongly interacting particles do not conserve C and T, in connection with the fact[2] that the observed amplitude of the decay process $K_L{}^0\to 2\pi$ is smaller by the factor $\sim\alpha/\pi$ than that of the process $K_S{}^0\to 2\pi$, where α is the fine-structure constant. Recently it was reported that the energy spectrum of the positive pion is slightly different from that of the negative pion in the process $\eta\to\pi^++\pi^-+\pi^0$.[3] The observed C violation in the η process would be of electromagnetic origin. In order to confirm unambiguously the validity of the conjecture, it is necessary to observe the effects due to C and T violation in the processes which involve real photons, such as photoproduction of pions and proton Compton scattering; there is no real photon involved in the processes $K_L{}^0\to 2\pi$ and $\eta\to\pi^++\pi^-+\pi^0$.

In this paper we would like to stress the importance of detailed-balance experiments in proton Compton scattering, and to point out that the effect of T violation in the process could be quite large and observable with present experimental techniques, at least in the second resonance region $E_\gamma=700$ MeV and at $\theta=90°$, where θ is the scattering angle in the center-of-mass system.

Since parity is conserved with high accuracy in electromagnetic interactions, we are interested in a process which involves at least one real photon and further gives

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