Calculation of the $\pi^+\pi^-$ Mass Distribution in the Reaction $\pi^-p \to \pi^-\pi^-N^{*++}$ †

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A one-pion-exchange (OPE) calculation is made of the $\pi^+\pi^-$ mass distribution from the $\pi^-p \to \pi^-\pi^-N^{*++}$ reaction in order to determine whether the σ enhancement at 400 MeV can be understood as a kinematical effect of the OPE production mechanism for N^{*++}. The results of the calculations tend to indicate that any structure in the $\pi^+\pi^-$ mass spectrum near 400 MeV cannot be explained in this way.

INTRODUCTION

In this paper we shall consider the scattering of a spin-$\frac{1}{2}$ baryon by a spinless boson into an unstable spin-$\frac{3}{2}$ baryon and a pair of spinless bosons assuming that the one-pion-exchange diagram shown in Fig. 1 dominates. We shall obtain the mass spectrum for the boson pair in which one of the particles comes from the decaying baryon and the other comes from the pion-pion scattering vertex. Our calculation includes the spin dependence of the spin-$\frac{3}{2}$ baryon as well as the additional Δ^2 dependence induced by the assumption that the unstable particle is characterized by a Rarita-Schwinger field.[1]

This calculation has been motivated by the following experimental observations: (a) There appears to be a statistically significant structure near 400 MeV in the $\pi^+\pi^-$ mass distribution in the final state of $\pi^-p \to \pi^-\pi^-N^{*++}$ (1238) at 6 BeV/c. (b) In the reaction $\pi^-p \to \pi^-\pi^-\pi^+p$ a very large fraction of the events having a π^+p mass in the N^{*++} region also have a $\pi^-\pi^+$ mass in the ρ^0 (765) region. These two effects are shown in the 6-BeV/c π^-p data of the Brookhaven National Laboratory—City College of New York (BNL-CCNY) collaboration in Fig. 2, where the 400-MeV $\pi^+\pi^-$ enhancement[2] is seen to be primarily associated with events in which the π^+p mass also falls in the N^{*++} band (1110–1390 MeV). It is possible for a structure simulating a resonance peak to appear in one mass channel because of the presence of a strong resonance production in another channel. The A_1-like structure, for instance, has been examined in terms of this type of approach.[3]

The results of our considerations show an over-all enhancement of the lower end of the $\pi^+\pi^-$ effective-mass spectrum, but strongly imply that the observed structure at 400 MeV in the $\pi^+\pi^-$ mass spectrum is not simply a kinematical effect of peripheral production of the N^{*++}. In addition, our calculations imply that both the magnitude and the shape of the $\pi^+\pi^-$ mass distribution near 400 MeV is quite independent of the spin dependence of the $N^*N\pi$ vertices. Furthermore, one can obtain from our results an estimate of the number of "ρ^0 events" which are to be expected in a sample of

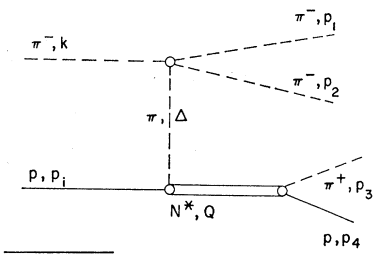

FIG. 1. One-pion-exchange diagram for $\pi^-p \to \pi^-\pi^-\pi^+p$.

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[1] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (L) (1941). The Δ^2 dependence thus induced differs from the Δ^2 dependence obtained from an analytic continuation in Δ^2 of the dispersion relations for the partial-wave amplitude near the N^{*++}(1238) resonance. See E. Ferrari and F. Selleri, Nuovo Cimento **21**, 1028 (1961). To lowest order in $\Delta^2+\mu^2$ the results of the Rarita-Schwinger model agree with those of Ferrari and Selleri. Our results for the shape of the $\pi^+\pi^-$ mass spectrum near the σ region are insensitive to this difference. J. D. Jackson and H. Pilkuhn [Nuovo Cimento **33**, 906 (1964)] have used a Rarita-Schwinger formalism and have generated this additional Δ^2 dependence in an OPE calculation of $KN \to N^*K$ and $KN \to N^*K^*$.

[2] V. E. Barnes, W. B. Fowler, K. W. Lai, S. Orenstein, D. Radojicic, M. S. Webster, A. H. Bachman, P. Baumel, and R. M. Lea, Phys. Rev. Letters **16**, 41 (1966). The effects were observed earlier in multibody final states (more than four) produced in 4.65 BeV/c π^-p interactions. N. P. Samios, A. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shepard, Phys. Rev. Letters **9**, 139 (1962). There is some evidence for the σ ($\pi^+\pi^-$ at 400 MeV) in other reactions. For example, R. Del Fabbro, M. De Pretis, R. Jones, G. Marini, A. Odian, G. Stoppini, and L. Tau [Phys. Rev. **139**, B701 (1965)] see some structure in $\gamma+p \to p+\pi^++\pi^-$. G. Goldhaber [in *Second Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, New York, 1965), p. 42] reports seeing this enhancement in $\pi^+p \to p\pi^+\pi^+\pi^-$ at 3.65 BeV/c. He further shows that most of these "σ" events are deleted by removing the N^{*++} band, and therefore conjectures that the σ enhancement is a kinematical effect associated with N^{*++} ρ^0 production.

[3] R. T. Deck [Phys. Rev. Letters **13**, 169 (1964)] has simulated the A_1 through the use of the diffraction scattering at the $\pi N\pi N$ vertex in a peripheral model of ρ production. N. P. Chang [Phys. Rev. Letters **14**, 806 (1965)] has obtained an A_1-like structure through the inclusion of Bose symmetrization and the "experimental" selection of events, while K. W. Lai and M. S. Webster [Brookhaven National Laboratory Internal Report No. L-40(BC) (unpublished)] have obtained an A_1-like effect in the reaction $\pi p \to \rho N^*$ because of the strongly peaked angular distribution of the decay pion from the N^*.

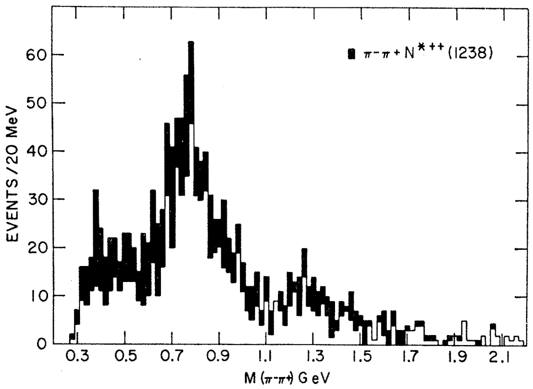

FIG. 2. Effective $\pi^+\pi^-$ mass distribution from the reaction $\pi^-p \to \pi^-\pi^-\pi^+p$ at 6 BeV/c π^- momentum. Shaded events also have the π^+p effective mass in the N^{*++} band (1110–1390 MeV).

N^{*++} events without any true ρ^0's. Our calculations may also be extended to other reactions of the same type, such as $\pi^+p \to \pi^+\pi^+N^{*0}$, $\pi^+n \to \pi^+\pi^+N^{*-}$, or $K^-p \to \pi^+\pi^-Y_1^{*0}$. In the following we shall use the terminology appropriate to the nonstrange-particle reaction and call the bosons pions and the baryons nucleons and N^*, respectively.

DERIVATION OF THE FORMULA

In this section we shall consider the reaction $\pi^-p \to N^{*++}\pi^-\pi^-$ via one-pion exchange. We shall include the Δ^2 dependence of the $N^*\pi N$ vertex function by using a Rarita-Schwinger formalism for the spin-$\frac{3}{2}$ N^*. We shall obtain formulas for the differential cross section from which one can obtain the effective-mass distribution of the $\pi^+\pi^-$ pairs. While numerical results will be obtained only for S-wave $\pi^-\pi^-$ scattering, the $\pi^+\pi^-$ mass distribution does not appear to be sensitive to this assumption.

We shall first discuss πN scattering through an N^*. The field N_μ^* which describes the spin-$\frac{3}{2}$ N^* satisfies the Dirac equation as well as a subsidiary condition

$$(\gamma_\mu\partial_\mu+M_{33})N_\nu^*=0;\quad \gamma_\mu N_\mu^*=0. \tag{1}$$

Its propagator[4,5] is given by

$$\frac{[M_{33}-i\gamma_\nu Q_\nu]}{Q_\mu Q_\mu+M_{33}^2}\left[\delta_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu+\frac{i}{3M_{33}}(\gamma_\mu Q_\nu-\gamma_\nu Q_\mu)+\frac{2}{3M_{33}^2}Q_\mu Q_\nu\right], \tag{2}$$

where we may phenomenologically describe the instability of the N^* by allowing the mass which occurs in the denominator to have a negative imaginary part: $M_{33}=M^*-\frac{1}{2}i\Gamma$. The interaction Hamiltonian density for $N^*\pi N$ is assumed to be

$$H=(G/M)(\bar{N}\overleftrightarrow{\partial}_\mu\pi)N_\mu^*+\text{Hermitian adjoint},$$

where the field operators π, N destroy π^- and N, respectively. G is a dimensionless coupling constant and M is the nucleon mass. For pion-nucleon scattering through an N^*, the differential cross section is

$$\frac{d\sigma(\pi^+p\to\pi^+p)}{d\Omega}=\frac{G^4M^{*2}p_f'(E_f'+M)(E_i'+M)p_f'^2p_i'^2[1+3\cos^2\theta_{fi}']}{9\pi^2M^4\lambda p_i'[(\lambda-M^{*2})^2+\Gamma^2M^{*2}]}, \tag{3}$$

where λ is the square of the πN energy measured in the N^* rest frame. We shall find it convenient to denote variables measured in the N^* center-of-mass (c.m.) system by primes, so that p_i', E_i', p_f', E_f' are the 3-momenta and energy of the initial (target) and final protons measured in the N^* c.m. system. The quantity θ_{fi}' is the angle between the initial and final protons; again, measured in the N^* c.m. system.

In Eq. (3) terms of order $(\lambda-M^{*2})/M^{*2}$ have been neglected in the numerator; however, *the equation is valid for any pion mass* including (formally at least) the unphysical value of $i\Delta$ which enters in Fig. 1. The quantity Γ is seen to be the full width at half-maximum of the N_{33}^{*++} (1238 MeV) resonance. By comparing Eq. (3) with the experimental π^+p cross section, one finds $G^2/4\pi\approx 4.1$. Upon integrating Eq. (3) we arrive at an expression for $\sigma_{\pi N}(\lambda,\Delta^2)$; $\Delta^2=(Q-p_i)^2$:

$$\sigma_{\pi N}(\lambda,\Delta^2)=\frac{8G^4M^{*2}[E_i'+M][E_f'+M]p_f'^3p_i'}{9\pi\lambda M^4[(\lambda-M^{*2})^2+\Gamma^2M^{*2}]}. \tag{4}$$

The Δ^2 dependence occurs only through the variables p_i', E_i' in a known fashion.

The $\pi^-\pi^-$ scattering vertex may be described by the invariant Møller's matrix element $\mathfrak{M}_{\pi\pi}$. Denoting by s the square of the $\pi^+\pi^-$ invariant mass, we can express the differential cross section for the reaction $\pi^-p \to \pi^-\pi^-p\pi^+$ in terms of an integral of $\sigma_{\pi N}(\lambda,\Delta^2)$ and $|\mathfrak{M}_{\pi\pi}|^2$ over the three-body phase space of the N^{*++} and the two π^-.

$$\frac{\partial^2\sigma(\pi^-p\to\pi^-\pi^-N^{*++})}{\partial s\partial\lambda}=\frac{1}{p_iW}\int\frac{d^3p_1d^3p_2d^3Q\delta^4(Q+p_1+p_2-k-p_i)|\mathfrak{M}_{\pi\pi}|^2F(s,\lambda,\Delta^2)}{\omega_1\omega_2E_Q(\Delta^2+\mu^2)^2}, \tag{5}$$

[4] H. Umezawa and Y. Takahashi, Progr. Theoret. Phys. (Kyoto) **9**, 14 (1953); M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963).

[5] We use Hermitian γ matrices: $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu}$ for μ, $\nu=1, 2, 3, 4$. We take $\hbar=c=1$. In our metric the invariant for a physical particle is $p^2=p_\mu p_\mu=-m^2$.

where μ is the pion mass,

$$F(s,\lambda,\Delta^2)=\frac{\lambda p_i'\sigma_{\pi N}(\lambda,\Delta^2)}{32\pi^3 p_f'}\int\frac{d^3p_3d^3p_4}{\omega_3\omega_4}$$
$$\times\delta^4(Q-p_3-p_4)\delta(s+(p_1+p_3)^2)(1+3\cos^2\theta_{fi}'),\quad(6)$$

where Q is the N^{*++} momentum; p_1, p_2, p_3, p_4 are momenta of the final π^-, π^-, π^+, and proton, respectively; p_i is the incident-proton 3-momentum in the over-all c.m. system, W is the over-all c.m. energy, and p_i', p_f' are again the proton 3-momenta measured in the N^{*++} c.m. system, where they make an angle of θ_{fi}'. $\mathfrak{M}_{\pi\pi}$ depends only on the variables p_1, p_2, and $\Delta^2=(Q-p_i)^2$. For $\Delta^2=-\mu^2$ the differential cross section for $\pi^-\pi^-\to\pi^-\pi^-$ is given by

$$\frac{d\sigma}{d\Omega}=\frac{2\pi^2}{s_{12}}|\mathfrak{M}_{\pi\pi}|^2,$$

where s_{12} is the $\pi^-\pi^-$ effective mass squared.

Since F is a Lorentz-invariant we can compute it in the N^{*++} c.m. system. Choosing the z axis along p_1' and expanding $\cos^2\theta_{fi}'=\cos^2\theta_{3i}'$ in terms of the polar coordinates of p_3' and p_i', we obtain

$$F=\frac{(\lambda)^{1/2}\sigma_{\pi N}(\lambda,\Delta^2)p_i'}{32\pi^2 p_f'p_1'}(1+3C_{31}^2C_{i1}^2+\tfrac{3}{2}S_{31}^2S_{i1}^2)\theta(S_{31}^2),\quad(7)$$

where C_{31} is the cosine of the angle between p_3' and p_1', for which $(p_1+p_3)^2=-s$. $S_{31}^2=1-C_{31}^2$, and $\theta(y)$ is the usual step function: $\theta(y)=1$ for $y>0$; $\theta(y)=0$ for $y<0$. In Eq. (7), S_{31}^2 is to be considered a function of p_1' and ω_1', the momentum and energy of a π^- in the N^* c.m. system.

$$S_{31}^2=\frac{1}{4p_1'^2p_3'^2}[4p_1'^2p_3'^2-(s-2\mu^2)^2-4\omega_1'^2\omega_3'^2$$
$$+4\omega_1'\omega_3'(s-4\mu^2)],\quad(8)$$

where p_3', ω_3' are, respectively, the magnitude of the 3-momenta and energies of the π^+ in the N^{*++} c.m. system. In an invariant notation $\omega_1'=-(Q_\mu p_{1\mu})/\sqrt{\lambda}$; thus we may express S_{31}^2 in terms of manifestly Lorentz-invariant variables. In addition, C_{i1} may also be expressed in terms of invariants and we may then proceed to integrate over $d^3p_1d^3p_2$ in Eq. (5) by transforming to the rest frame of the outgoing $\pi^-\pi^-$ pair.

However, we may simplify the calculation considerably by observing that the pion propagator $\Delta^2+\mu^2$ causes the N^{*++} angular distribution in the over-all c.m. system to be peaked sharply forward in the direction of the incident proton, and it similarly causes the angular distribution of the two π^- to be peaked backward. Therefore upon Lorentz-transforming to the N^{*++} c.m. system, the π^- angular distribution is peaked in the backward direction relative to the incident proton, and we expect that over the important region[6] of phase space C_{i1} is ~-0.5 to -1.0. Furthermore, C_{i1} does not depend explicitly on s, so that we shall make a very small error in $\partial\sigma/\partial s$ by treating C_{i1}^2 as a constant. We shall call this constant $\langle C_i^2\rangle_{\rm av}$.

Let us now denote by q the $\pi_1^-\pi_2^-$ momentum in the over-all center-of-mass system and let $s_{12}=-q^2$; then upon inserting an additional δ function we can write

$$\frac{\partial^3\sigma}{\partial s_{12}\partial s\partial\lambda}=\frac{(\lambda)^{1/2}}{32\pi^2p_iW}$$
$$\times\int\frac{d^3Qd^3q\,p_i'\sigma_{\pi N}(\lambda,\Delta^2)J(s_{12},\lambda,s,W)\delta^4(k+p_i-Q-q)}{E_QE_qp_f'(\Delta^2+\mu^2)^2},\quad(9)$$

where

$$J(s_{12},\lambda,s,W)=\int\frac{d^3p_1d^3p_2}{2\omega_1\omega_2p_1'}\delta^4(q-p_1-p_2)|\mathfrak{M}_{\pi\pi}|^2$$
$$\times[(1+3\langle C_i^2\rangle_{\rm av})+\tfrac{3}{2}(1-3\langle C_i^2\rangle_{\rm av})S_{31}^2]\theta(S_{31}^2).\quad(10)$$

The invariant integral J may be written in the rest frame of the two π^-, for which $\mathbf{q}=0$, as

$$J(s_{12},\lambda,s,W)=\frac{1}{2Q''}\left(\frac{\lambda}{s_{12}}\right)^{1/2}\int_0^{2\pi}d\phi_1\int_{\omega_1'\min}^{\omega_1'\max}\frac{d\omega_1'}{p_1'}|\mathfrak{M}_{\pi\pi}|^2$$
$$\times[(1+3\langle C_i^2\rangle_{\rm av})+\tfrac{3}{2}(1-3\langle C_i^2\rangle_{\rm av})S_{31}^2]\theta(S_{31}^2),\quad(11)$$

where Q'' is the 3-momentum of the N^* measured in the $\pi^-\pi^-$ rest frame. Note that a value of $\langle C_i^2\rangle_{\rm av}=\frac{1}{3}$ is equivalent to ignoring the N^{*++} spin dependence.

It is worthwhile observing that the numerator of S_{31}^2 in Eq. (8) is a quadratic in ω_1' which opens downward, so that we need integrate only between the roots of the quadratic. For S-wave $\pi\pi$ scattering the integration is particularly simple since there is no ϕ dependence and we can take $|\mathfrak{M}_{\pi\pi}|^2$ outside of the integral. The result for $J(s_{12},\lambda,s,W)$ may be conveniently expressed if we define

$$\alpha=1+3\langle C_i^2\rangle_{\rm av};\quad(12a)$$

$$\beta=(3\mu^2/2p_3'^2)(1-\langle 3C_i^2\rangle_{\rm av});\quad(12b)$$

$$Z_\pm=(1/2\mu^2)[\omega_3'(s-2\mu^2)\pm p_3'(s(s-4\mu^2))^{1/2}];\quad(12c)$$

$$\omega_\pm=(1/4(\lambda)^{1/2})[(W^2-\lambda-s_{12})\pm(1-4\mu^2/s_{12})^{1/2}$$
$$\times((W^2-\lambda-s_{12})^2-4\lambda s_{12})^{1/2}];\quad(12d)$$

$$B=\text{Minimum of }(Z_+,\omega_+);\quad(12e)$$

$$A=\text{Maximum of }(Z_-,\omega_-).\quad(12f)$$

In terms of these quantities we may write

$$J(s_{12},\lambda,s,W)=\frac{\pi}{Q''}(\lambda/s_{12})^{1/2}|\mathfrak{M}_{\pi\pi}|^2J_1(s_{12},s,\lambda,W)\quad(13)$$

[6] In a preliminary sample of 542 $\pi^+\pi^-$ pairs from $N^{*++}\pi^-\pi^-$ events at 6 BeV/c π^-p, 250 have $C_{i1}<-0.5$. In the same sample $\langle C_i^2\rangle_{\rm av}\simeq0.42$. Private communication from the BNL-CCNY π^-p collaboration.

and

$$J_1(s_{12},s,\lambda,W)=\theta(B-A)\left[(\alpha-\beta)\ln(\omega_i'+p_i')+\frac{\beta}{p_i'}\times\left\{\omega_i'\left(1+\frac{Z_+Z_-}{\mu^2}\right)-Z_+-Z_-\right\}\right]_{\omega_i'=A}^{\omega_i'=B}. \quad (14)$$

We may now go to the over-all rest frame and, after integrating over the energy-momentum δ function, arrive at

$$\frac{\partial^4\sigma}{\partial\Delta^2\partial s_{12}\partial s\partial\lambda}=\frac{s_{12}\lambda\sigma_{\pi\pi}(s_{12},\Delta^2)(p_i'/p_f')\sigma_{\pi N}(\lambda,\Delta^2)J_1(s_{12},s,\lambda,W)}{2(4\pi)^3p_i^2W^2(\Delta^2+\mu^2)^2[(W^2-\lambda-s_{12})^2-4\lambda s_{12}]^{1/2}}, \quad (15)$$

where we have written $\sigma_{\pi\pi}(s_{12},\Delta^2)$ for

$$8\pi^3|\mathfrak{M}_{\pi\pi}(s_{12},\Delta^2)|^2/s_{12}.$$

In the absence of further knowledge about the Δ^2 dependence of the $\pi\pi$ scattering amplitude, we shall assume that it varies little with Δ^2. We shall, however, make full use of the Δ^2 dependence of the πN cross section[7] which arises from our previous assumption that the N^{*++} is characterized by a Rarita-Schwinger field:

$$(p_i'/p_f')\sigma_{\pi N}(\lambda,\Delta^2)=\sigma_{\pi N}(\lambda,-\mu^2)\times\left(1+\frac{\Delta^2+\mu^2}{(M+\sqrt{\lambda})^2-\mu^2}\right)\left(\frac{p_i'^2}{p_f'^2}\right), \quad (16)$$

where

$$p_i'^2=(1/4\lambda)(\lambda+M^2+\Delta^2)^2-M^2;$$
$$p_f'^2=(1/4\lambda)(\lambda+M^2-\mu^2)-M^2.$$

We may now integrate over Δ^2 (the limits of integration of course depend upon the square of the $\pi^-\pi^-$ effective mass s_{12}) and arrive at an expression for $\partial^3\sigma/(\partial s_{12}\partial s\partial\lambda)$:

$$\frac{\partial^3\sigma}{\partial s_{12}\partial s\partial\lambda}=\frac{s_{12}\lambda\sigma_{\pi\pi}(s_{12})\sigma_{\pi N}(\lambda,-\mu^2)J_1(s_{12},s,\lambda,W)I(s_{12},\lambda,W)}{2(4\pi)^3p_i^2W^2[(W^2-\lambda-s_{12})^2-4\lambda s_{12}]^{1/2}}, \quad (17)$$

where

$$I(s_{12},\lambda,W)=\int_{\Delta_-^2}^{\Delta_+^2}d\Delta^2\left[\frac{p_i'}{p_f'(\Delta^2+\mu^2)}\right]^2\left[1+\frac{\Delta^2+\mu^2}{(M+\sqrt{\lambda})^2-\mu^2}\right];$$

$$\Delta_\pm^2=\frac{E_i}{W}(W^2+\lambda-s_{12})-\lambda-M^2\pm\frac{p_i}{W}[(W^2+\lambda-s_{12})^2-4\lambda W]^{1/2}. \quad (18)$$

Let us review the assumptions which we have made in the derivation of the formulas (15) and (17). We have approximated the square of the cosine of the angle between an outgoing π^- and the incoming proton by a constant, which we may now consider to be a parameter of the model. Before integrating, this angle depends in a complicated way upon the kinematical invariants, however, it is completely independent of the invariant of interest s. Hence we may well expect that this is a good approximation for the calculation of $\partial\sigma/\partial s$. We have assumed that the $\pi^-\pi^-$ scattering amplitude is predominantly S wave.

The results of this calculation shows that $\partial\sigma/\partial s$ is rather insensitive to the spin of the N^{*++} vertex,[8] hence we infer, *a posteriori*, that the spin dependence of the other vertex will be of minor importance in determining the shape of $\partial\sigma/\partial s$.

DISCUSSION

The inclusion of the energy dependence of the $\pi^-\pi^-$ scattering cross section $\sigma_{\pi^-\pi^-}(s_{12})$ produces little change in the shape of $\partial\sigma/\partial s$ (the $\pi^+\pi^-$ mass distribution). We have verified this by calculating $\partial\sigma/\partial s$ for a fictitious resonance in $\sigma_{\pi^-\pi^-}$. We have therefore assumed for simplicity a constant $\sigma_{\pi^-\pi^-}$ of $1/2\mu^2$.

We have integrated Eq. (17) over s_{12} and over an "experimental" N^{*++} region of (1236±140) MeV to get the $\pi^+\pi^-$ mass distribution. In Fig. 3 the results are presented for several values of the parameter $\langle C_i^2\rangle_{\rm av}$ and for an incident π^- energy of 6 BeV/c. The resulting curve for $\langle C_i^2\rangle_{\rm av}=0.42$, which is the experimentally observed value at 6 BeV/c, agrees reasonably well with the result of a Monte Carlo integration over the four-body phase space of the exact matrix element without the $\langle C_i^2\rangle_{\rm av}$ approximation and with D-wave $\pi^-\pi^-$ scattering. In Fig. 4 the calculated $\pi^+\pi^-$ mass distribution for $\langle C_i^2\rangle_{\rm av}=0.42$ is compared with the experimental results[9] at 6 BeV/c.

[7] The results of Ferrari and Selleri [Nuovo Cimento **21**, 1028 (1961)] replaces the term $1+(\Delta^2+\mu^2)/[(\lambda^{1/2}+M)^2-\mu^2]$ by $1+(\Delta^2+\mu^2)/(4M^2)$, which may be attributed to the simplifying approximations of Ferrari and Selleri (Ref. 1); see their Eqs. 48, 49. In addition, they obtain an over-all factor of $[1+3\eta(\Delta^2+\mu^2)]^2\times[1+\eta(\Delta^2+\mu^2)]^{-6}$ because of their approximate solutions of the off-the-mass-shell dispersion relation. Note that the term of this additional factor which is linear in $(\Delta^2+\mu^2)$ vanishes identically. Hence to the lowest order in $(\Delta^2+\mu^2)$ our results agree with theirs. Of course for large Δ^2 our results do not agree, but insofar as the present investigation of the dipion mass distribution is concerned, it appears that our results are not particularly sensitive to this difference.

[8] Recall that the value $\langle C_i^2\rangle_{\rm av}=\frac{1}{3}$ is equivalent to ignoring the spin dependence of the $N^*N\pi$ vertices. From Fig. 3 one sees that changes in $\partial\sigma/\partial E$ due to changes in $\langle C_i^2\rangle_{\rm av}$ are quite small over ranges of E of the order of a few hundred MeV.

[9] V. E. Barnes, W. B. Fowler, K. W. Lai, S. Orenstein, D. Radojicic, M. S. Webster, A. H. Bachman, P. Baumel, and R. M. Lea (Ref. 2). It should be note that the experimental results presented in Fig. 4 contain only ρ^0 events which also are N^{*++} events.

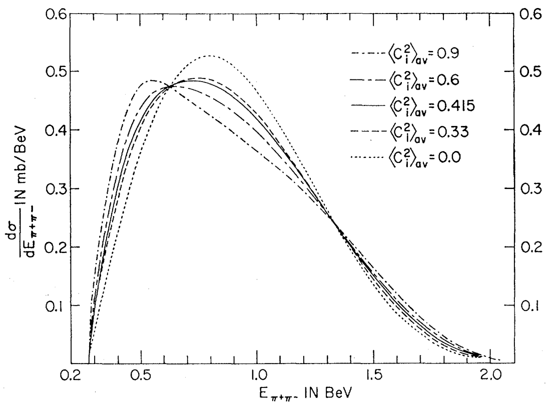

FIG. 3. Differential cross section $\partial\sigma/\partial E_{\pi^+\pi^-}$ in mb/BeV versus the $\pi^+\pi^-$ invariant mass $E_{\pi^+\pi^-}$ in BeV for different values of the phenomenological parameter $\langle C_i^2\rangle_{\text{av}}$ for 6-BeV/c incident π^- lab momentum. The parameter $\langle C_i^2\rangle_{\text{av}}$ is some mean value of the cosine of the angle between the incident proton and an outgoing π^- measured in the N^{*++} center-of-mass system. The plotted curves have been normalized to the experiment of Fig. 4, as indicated in the text.

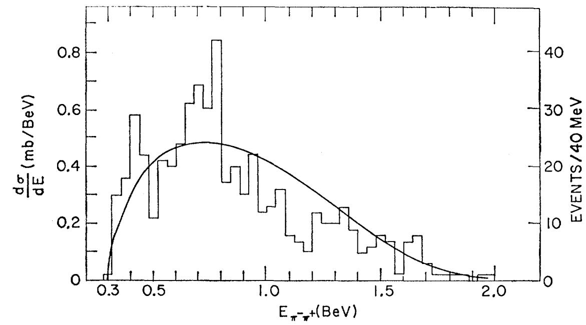

FIG. 4. Comparison of the experimental results with the calculated differential cross section $\partial\sigma/\partial E_{\pi^+\pi^-}$ as a function of the invariant $\pi^+\pi^-$ mass $E_{\pi^+\pi^-}$. The theoretical curve is calculated with $\langle C_i^2\rangle_{\text{av}}=0.42$, which is the experimental value taken from the same sample of events. The experimental results are taken from the 6-BeV/c BNL-CCNY collaboration. Both the experimental and theoretical curves contain all $\pi^+p\pi^-\pi^-$ events for which the effective mass of the $p\pi^+$ system is in the N^* region, (1238 ± 140) MeV. Thus, the experimental results include ρ^0 events which are also N^{*++} events. The smooth curve has been normalized to the experiment, as indicated in the text.

The comparison of the shape of the calculated curve with the experimental result seems satisfactory; however, in order to obtain agreement with the total cross section it is necessary to divide the calculated curve by a factor of 32. This may be attributed to the behavior of $\partial\sigma/\partial\Delta^2$ for large Δ^2. The correction factor of 1/32 has been used in Fig. 4.

In an effort to determine the sensitivity of the $\pi^+\pi^-$ mass distribution to the additional Δ^2 dependence of the $N^*\pi N$ vertex we have also calculated $\partial\sigma/\partial s$ without the additional Δ^2 dependence of the $N^*\pi N$ vertex. We found that the shape of the curve changed very little, but the total integrated cross section was reduced by an order of magnitude. Hence, we would infer that the use of form factors of the type used by Ferrari and Selleri or by Jackson and Pilkuhn, which decrease rapidly for large Δ^2, would not yield any significant change in the structure of $\partial\sigma/\partial s$ near 400 MeV, although such form factors would improve the agreement with the integrated cross section.

It thus appears that the $\pi^+\pi^-$ enhancement at 400 MeV may not be understood as a strict kinematic effect of the OPE production of the N^{*++}.

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Erratum

Dynamics of a Singlet and an Octet of $D_{3/2}$ Meson-Baryon Composite States, PAUL DU T. VAN DER MERWE [Phys. Rev. **145**, 1257 (1966)]. There are two misprints in this paper:

1. Equation (3.7) should read

$$u_\lambda(p_i)=\left(\frac{E_i+M_i}{2M_i}\right)^{1/2}\begin{pmatrix}\varphi_\lambda\\ [\boldsymbol{\sigma}\cdot\mathbf{p}_i/(E_i+M_i)]\varphi_\lambda\end{pmatrix}, \tag{3.7}$$

2. In the paragraph following Eq. (4.12) the line: "In particular, for $d=0.40$ and 0.33, the ratio . . ." should read: "In particular, for $d=0.33$ and 0.40, the ratio . . .".