Peripheral Model for Ξ^- Associated Productions*

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A single-particle exchange model with absorptive corrections is developed for application to meson-baryon inelastic scattering processes which are dominated by baryon exchange in the u channel. The model is applied to the process $K^-+p\to\Xi^-+K^+$, which appears to be a particularly clean example of a process in which baryon exchange plays a dominant role. The complex initial- and final-state elastic amplitudes required by the absorption model are determined from the Regge model of K^-p scattering proposed by Rarita and Phillips. The f/d ratio for baryon-pseudoscalar coupling, the only parameter in the model which is not determined externally, is found to be $f/d=0.555$ if the vertex functions involved are assumed to be compatible with unbroken SU_3. The most interesting features of the Ξ^- production process are reproduced by the model as a result of the interference between the spin-$\frac{1}{2}$ Λ and Σ exchanges and the spin-$\frac{3}{2}$ Y_1^* (1385) exchange. Although many features of the process studied are qualitatively reproduced by the model, it is concluded that this formulation of the peripheral model is not an adequate representation of a baryon exchange process.

I. INTRODUCTION

THE principal assumption of a peripheral model is that singularities in the scattering amplitude which correspond to long-range potentials or low-mass intermediate states are responsible for strongly peaked differential cross sections. The long-range forces associated with poles close to the physical region scatter many partial waves; the sharp peaks observed in many differential cross sections require the constructive interference of a large number of partial-wave amplitudes. A pole in the t channel produces scattered partial-wave amplitudes which are in phase with each other; this type of singularity thus produces a forward peak ($\cos\theta=1$), since at this angle the in-phase amplitudes interfere constructively. Similarly, a u-channel pole produces partial-wave amplitudes which alternate in phase; since $P_l(\pi)=(-)^l$, these amplitudes will interfere constructively at $\cos\theta=-1$, producing a backward peak.

The simplest peripheral model is that in which the single-particle exchange terms are treated in Born approximation. This approximation fails badly in the majority of situations to which it is applied; the cross sections predicted by the Born approximation are often orders of magnitude too large, and the angular distributions are insufficiently sharply peaked. The partial-wave amplitudes corresponding to small values of the angular-momentum quantum number are overestimated by this model; the requirements of unitarity are often violated. This excess of low partial waves accounts for the failure of this approximation. One might attempt to account for this by falling back on higher order singularities or by the insertion of momentum-dependent form factors at the vertices. However, an interesting alternative, known as the absorption model, has been proposed by several authors.[1]

This model attempts to account for unitarity violations through the inclusion of initial- and final-state interactions as corrections to the Born amplitude. If the cross section for the inelastic process being considered is small, the absorption of flux due to competing inelastic channels is found to be well represented by the elastic S-matrix elements for the initial- and final-state scatterings. In the case where the range of the transition-potential term is less than the range of the forces in the entrance and exit channels, the modified amplitude for the peripheral exchange is

$$A_{\mathrm{Born}}{}^J\to A^J=(S_I{}^J)^{1/2}A_{\mathrm{Born}}{}^J(S_F{}^J)^{1/2};\qquad(1)$$

this result is independent of the initial- and final-state interactions being strongly absorptive.[2]

The absorption model has been applied to several reactions with mixed success. Gottfried and Jackson studied the reaction $\pi^-+p\to\rho^-+p$ with a driving term consisting of a single-pion exchange. The success of the model in the case of this process is outstanding. In the case of pion-nucleon charge exchange, Barger and Ebel[3] found the model to be extremely poor as a representation for a process dominated by vector-meson exchange. The majority of previous work with the absorption model has dealt with processes dominated by poles in the t channel.

The purpose of this paper is to study the consequences of the inclusion of initial- and final-state absorptive corrections in a process which is dominated by poles in the u channel. In order to choose a process for study, we have established as a criterion the absence of single-particle states in the t channel; such a process has the desirable feature that the interference between t- and u-channel effects should be minimized. In par-

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[1] L. Durand, III, and Y. T. Chiu, Phys. Rev. **139**, B646 (1965); K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).
[2] L. Durand, III, and Y. T. Chiu, Ref. 1.
[3] V. Barger and M. E. Ebel, Phys. Rev. **138**, B1148 (1965).

ticular, we have chosen the associated production process $K^- + p \rightarrow \Xi^- + K^+$ to be the cleanest example of a two-body production process which has a stable final state and which is dominated by u-channel singularities. The quantum numbers of the t channel in Ξ^- production are $I=1$, $S=+2$, and $Q=+2$. The most recent experimental evidence indicates that there is no resonance in the K^+K^+ system and that the K^+K^+ cross section is small in the region that was studied.[4] The absence of t-channel terms is further confirmed by the very noticeable lack of a forward peak in the Ξ^- production cross section.

The Ξ^--associated production process is amenable to the absorption model. The Ξ^- production cross section is at its largest only approximately 1/50 of the total K^-p inelastic cross section. The Ξ^-K^+ threshold is at a relatively high energy; there are numerous inelastic channels open at the Ξ^-K^+ threshold, and the bulk of the strong low-energy resonances lie well below the threshold. The range of a single-particle exchange in the u channel of an inelastic process involving scattering of particles of unequal mass depends upon the energy and the external masses. At high energies where, as we shall presently show, the exchange of the $Y_1^*(1385)$ is the dominant term in our model, the range in the inelastic channel is simply the Compton wavelength of the Y_1^*, 0.72 BeV^{-1}. At lower energies the range is more complicated, and is found to be roughly 1 BeV^{-1}. Since the ranges in the elastic channels are on the order of the Compton wavelength of a ρ, 1.3 BeV^{-1}, it may be seen that the assumption concerning ranges is roughly satisfied for this process.

In addition to large amounts of differential and integrated cross section data which exist for K^- beam momenta below 3 BeV/c, the polarization of the Ξ^- has been measured at several energies between threshold and 2.5 BeV/c. Since for a short-range transition potential one need not assume strongly absorptive initial- and final-state interactions, we have used complex K^-p amplitudes in this calculation. In order to compute the necessary partial-wave amplitudes, we have used the Regge parametrization of K^-p elastic scattering which was proposed by Rarita and Phillips.[5] Although the forward K^-p amplitude is predicted to be nearly pure imaginary, the partial-wave amplitudes computed from this model display phases, as shown in Fig. 1. Present evidence suggests that the predictions of the Rarita-Phillips model are valid down to lab momenta of 2 BeV/c and below.[6] We therefore feel that the phase shifts computed from this model are as reliable as those which could be obtained in other ways. We have used Rarita and Phillips' solution 3 in this work.

Although we could have determined both the helicity-flip and helicity-nonflip elastic amplitudes from the work of Rarita and Phillips, we have assumed for

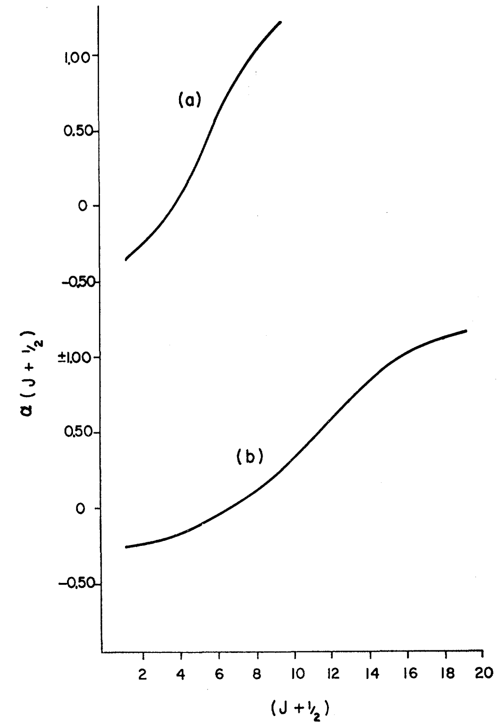

FIG. 1. The values of $\text{Re}f^j{}_{K^-p}/\text{Im}f^j{}_{K^-p}$ which are computed from the Rarita-Phillips model are plotted as a function of $(j+\frac{1}{2})$ for lab momenta of (a) 2 BeV/c and (b) 6 BeV/c.

calculational simplification that the helicity-nonflip amplitude is the dominant term and have ignored the flip contribution. We have also assumed that the S-matrix elements for Ξ^-K^+ scattering may be effectively equated to those for K^-p scattering. This again is a simplifying assumption and is nonessential, since presumably one could determine the Ξ^-K^+ terms from the Rarita-Phillips model through the use of SU_3 invariance. We do not think that these assumptions greatly influence the nature of our results.

The amplitudes produced by the single-particle exchange terms are discussed in Sec. II. Section III is devoted to a discussion of the polarization of the Ξ^- in the final state of the reaction. In addition, Sec. III deals with the f/d ratio for baryon-pseudoscalar scattering, the one free parameter to be determined by the theory. The results of the calculations are compared to experiment in Sec. IV, and the model is analyzed with respect to its predictions and to future modifications suggested by the results.

II. AMPLITUDES OF SINGLE-PARTICLE EXCHANGE TERMS

The invariant matrix element, denoted by $\mathfrak{M}$, is the quantity which is derived directly from the single-particle exchange approximation and the Feynman rules. This matrix element is simply related to the

[4] A. R. Erwin *et al.*, Phys. Rev. Letters **16**, 1063 (1966).
[5] R. J. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).
[6] V. Barger (private communication).

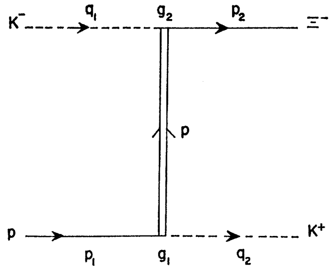

FIG. 2. The conventions adopted for particle momenta are defined in this diagram for the process $K^-+p\to\Xi^-+K^+$. Subscripts 1 (2) denote quantities in the initial (final) state; the symbol $\bar{m}$ denotes the mean of the initial and final state baryon masses, $\frac{1}{2}(m_1+m_2)$. In the text, m refers to the mass of an external baryon, μ to the mass of an external meson, and M to the mass of an exchanged particle.

transition matrix T,

$$\mathfrak{M}=\bar{u}(p_2)Tu(p_1)\,, \tag{2}$$

which is itself related to the S matrix via the relation

$$S=1-(2\pi)^4 i\delta^4(p-p')T\,. \tag{3}$$

The conventions used in labeling particle masses and momenta are illustrated in Fig. 2. The T matrix for this type of scattering process may be reduced to the sum of two terms which transform as scalars with respect to space time and to spin;

$$T=-A-Q\cdot\gamma B\,. \tag{4}$$

The quantities A and B are the invariant amplitudes; Q is defined to be $\frac{1}{2}(q_1+q_2)$.

In order to reduce the invariant matrix element to the more familiar spin-$\frac{1}{2}$ scattering amplitudes, it is useful to introduce an additional amplitude M, which is defined by

$$\chi_2{}^\dagger M\chi_1=-(\sqrt{m_1m_2}/4\pi W)\mathfrak{M}\,. \tag{5}$$

χ_2 and χ_1 are the two component spinors representing the final and initial spin states of the baryons. M is directly related to the amplitudes f_1 and f_2, which are the usual spin-$\frac{1}{2}$–spin-0 scattering amplitudes.

$$M=f_1+(\boldsymbol{\sigma}\cdot\hat{q}_1)(\boldsymbol{\sigma}\cdot\hat{q}_2)f_2\,. \tag{6}$$

It is therefore a matter of algebra to find the relations between the spin amplitudes f_1 and f_2 and the invariant amplitudes A and B.

$$f_1=\frac{[(E_1+m_1)(E_2+m_2)]^{1/2}}{8\pi W}[A+(W-\bar{m})B]\,, \tag{7a}$$

and

$$f_2=\frac{[(E_1-m_1)(E_2-m_2)]^{1/2}}{8\pi W}[-A+(W+\bar{m})B]\,. \tag{7b}$$

It is desirable for our purposes to express the spin dependence in our calculations in terms of the helicity amplitudes of Jacob and Wick.[6a] These amplitudes, expressed in terms of f_1, f_2, the scattering angle θ, and the azimuthal angle ϕ, are

$$\begin{aligned}&f_{1/2,1/2}=(\cos\tfrac{1}{2}\theta)(f_1+f_2)\,,\quad f_{-1/2,-1/2}=(\cos\tfrac{1}{2}\theta)(f_1+f_2)\,,\\&f_{-1/2,1/2}=-e^{i\phi}(\sin\tfrac{1}{2}\theta)(f_1-f_2)\,,\\&f_{1/2,-1/2}=-e^{i\phi}(\sin\tfrac{1}{2}\theta)(f_1-f_2)\,.\end{aligned} \tag{8}$$

The partial-wave expansion of the helicity amplitudes is given by

$$f_{\lambda_2\lambda_1}=(1/P)\sum_J(J+\tfrac{1}{2})f^J{}_{\lambda_2\lambda_1}d^J{}_{\lambda_2\lambda_1}(\theta)\,, \tag{9}$$

where the $d^J{}_{\lambda_2\lambda_1}$ functions are well known and are discussed extensively by Jacob and Wick and by textbooks such as Edmonds'.[6b]

In Fig. 3 the locations of singularities close to the physical region have been indicated. The lowest mass intermediate states in the u channel are the Λ and Σ hyperons. These states are both $\frac{1}{2}^+$ states; the interaction Lagrangian representing the space-time structure of the vertices for these exchanges is therefore equal to

$$\mathfrak{L}=(4\pi)^{1/2}g\bar{B}\gamma_5BP\,. \tag{10}$$

The invariant amplitudes for the spin-$\frac{1}{2}$ exchange terms may be computed, therefore, from the Feynman rules;

$$A_{1/2^+}=\frac{4\pi g_1g_2}{u-M^2}(M-\bar{m})\,,\quad B_{1/2^+}=\frac{4\pi g_1g_2}{u-M^2}\,. \tag{11}$$

The individual coupling constants, g_1 and g_2, which appear at the vertices are related to the internal symmetries and will be discussed later.

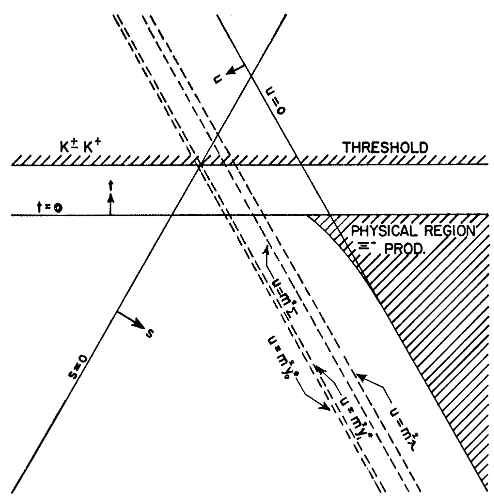

FIG. 3. The locations of important singularities of the cascade production amplitude with respect to the physical region are indicated in this Mandelstam diagram, which is drawn approximately to scale.

[6a] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).
[6b] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton, Princeton, N. J., 1957).

The interaction between the spin-$\frac{3}{2}^+ Y_1^*(1385)$ resonance state and the external meson and nucleon may be represented by the interaction Lagrangian

$$\mathcal{L}_{3/2^+}=(4\pi)^{1/2}g\tfrac{1}{2}\bar{D}_\mu[P\partial_\mu B-B\partial_\mu P]. \quad (12)$$

Adopting the form of the spin-$\frac{3}{2}$ propagator presented by Abers and Zemach,[7] the invariant amplitudes corresponding to the exchange of the state are found, after much algebra, to be

$$A_{3/2^+}=[-4\pi g_1g_2/(u-M^2)][(M+\bar{m})\alpha+(M-\bar{m})\beta],$$
$$B_{3/2^+}=[4\pi g_1g_2/(u-M^2)](\alpha-\beta). \quad (13)$$

The quantities α and β are related to the masses and invariant four-vector products in the following manner:

$$\alpha=(p_1\cdot p)(p_2\cdot p)/M^2-(p_1\cdot p_2), \quad (14a)$$

and

$$\beta=\frac{1}{3}\left(\frac{m_2(p_1\cdot p)+m_1(p_2\cdot p)}{M}+m_1m_2+\frac{(p_1\cdot p)(p_2\cdot p)}{M^2}\right). \quad (14b)$$

Evaluation of the quantities α and β with the Y_1^* on the mass shell affects only the s and p waves; since the lowest partial waves are strongly absorbed, we have evaluated α and β at $p^2=M^2$ and taken advantage of the resultant simplification of calculations.

The final intermediate state considered, the $Y_0^*(1405)$, is a spin-$\frac{1}{2}$ particle but of opposite parity to the Λ and Σ. The interaction Lagrangian for the vertices is

$$\mathcal{L}_{1/2^-}=(4\pi)^{1/2}g\bar{B}BP, \quad (15)$$

and the amplitudes for this exchange are

$$A_{1/2^-}=[-4\pi g_1g_2/(u-M^2)](M+\bar{m}),$$
$$B_{1/2^-}=4\pi g_1g_2/(u-M^2). \quad (16)$$

The various coupling constants required for our calculation may be determined by SU_3 invariance from experimentally determined parameters, the width of the $N^*(1238)$, the width of the $Y_0^*(1405)$, and the pion-nucleon coupling constant, and the f/d ratio for baryon-pseudoscalar coupling, which is considered to be an undetermined parameter in this model. This is the only parameter in the model which is not externally determined.

The validity of the couplings determined from SU_3 symmetry is questionable at the present time. Gautam and Ghose,[8] in their discussion of this question, indicate that the couplings of the decuplet baryon-meson resonances may deviate from the SU_3 values by fairly large amounts. Data from the above reference indicate that the particular combination of coupling constants in which we are interested, $g_{Y_1^*pK^+}g_{Y_1^*K^-\Xi^-}$, which is equal to -2.50 BeV^{-2} from SU_3 analysis, is rather well predicted by SU_3. The value of the $Y_0^*(1405)$ coupling derived from the width of the state into $\Sigma\pi$ is very small[9]; this exchange term has only minor effects and even fairly large deviations from SU_3 would not be expected to affect the calculations to a large extent.

The Λ and Σ^0 hyperons are very nearly degenerate in mass; if the masses are assumed to be equal, the net spin-$\frac{1}{2}$ coupling deduced from SU_3 is

$$g_1g_2=\tfrac{4}{3}g_{\pi N}^2(1-2f-2f^2), \quad (17)$$

where the parameter f is equal to the percentage of f-type coupling present in the Lagrangian. Lusignoli *et al.*[10] have recently presented evidence for substantial violations of SU_3 by the couplings of K mesons. It is therefore possible that one value of f will not adequately describe the entire group of baryon-pseudoscalar couplings. However, the results of our work are used to determine the effective coupling of the Λ and Σ, and this is expressed in terms of f, assuming SU_3 invariance.

III. Ξ^- POLARIZATION AND f/d RATIO

In terms of the helicity amplitudes, the polarization of the Ξ in the final state may be expressed as

$$\mathbf{P}(\theta)=\frac{2\,\mathrm{Im}f_{++}{}^*f_{-+}}{|f_{-+}|^2+|f_{++}|^2}\hat{n}_y, \quad (18)$$

where the x, z plane has been chosen to be the scattering plane and $\hat{n}_y$ is the normal to this plane. An experimentally measured quantity, the average polarization, is defined by

$$\bar{P}\sigma_T=\int d\Omega\,P(\theta)d\sigma/d\Omega. \quad (19)$$

Because of the phases introduced by the initial- and final-state interactions, the amplitudes computed with this model, unlike the Born amplitudes, are compatible with Ξ^- polarization in the final state, notwithstanding unpolarized target protons.

The most interesting feature of the experimental polarization data reviewed by Stevenson[11] is a zero of the average Ξ^- polarization which occurs at K^- beam momentum of 1.5 BeV/c. The zero is reproduced by this model; here it is found to be a result of the effects of the interference between the Y_1^* and the $\Lambda\Sigma$ ex-

[7] E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963).

[8] V. P. Gautam and P. Ghose, Phys. Rev. **145**, 1166 (1966).

[9] A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1325 (1965). However, a determination of this coupling from an extrapolation of low energy K^-p scattering data to the Y_0^* pole indicates that the value of the coupling determined from the Y_0^* width may be too small by a factor of 5. Our results will not be terribly sensitive to even a symmetry breaking of this magnitude. On the other hand, the suppression of the backward peak in Y_0^* production indicates that the coupling in question is actually very small. We wish to thank Dr. C. Goebel for discussions of this point.

[10] M. Lusignoli *et al.*, Phys. Letters **21**, 229 (1966).

[11] M. L. Stevenson, Lawrence Radiation Laboratory Report No. UCRL 11493, 1964 (unpublished).

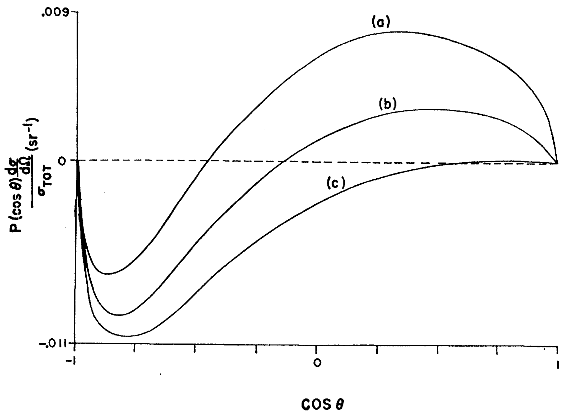

FIG. 4. The differential polarization is shown in the vicinity of the zero of the average polarization. The quantity plotted, $P(\cos\theta) \times (d\sigma/d\Omega)\sigma_{tot}^{-1}$, is the polarization weighted by the angular distribution; hence, the integral of this curve is just $\frac{1}{2}\pi P_{av}$. The curves (a), (b), and (c) refer to different values of the f/d ratio, or, alternatively of the energy; the variation from (a) to (c) may be characterized by decreasing energy or by increasing f.

change terms. In order to demonstrate the occurrence of this zero, we temporarily simplify the model by the assumption that all masses in the problem have a common value, m.

The invariant amplitudes in the equal mass limit are the following:

$$A_{1/2^+}=0, \qquad B_{1/2^+}=\frac{4\pi g_1g_2}{u-m^2}, \tag{20a}$$

$$A_{1/2^-}=\frac{-4\pi g_1g_2}{u-m^2}(2m), \qquad B_{1/2^-}=\frac{4\pi g_1g_2}{u-m^2}, \tag{20b}$$

$$A_{3/2^+}=\frac{-4\pi g_1g_2}{u-m^2}(\tfrac{3}{2}m^3-mW^2), \quad B_{3/2^+}=\frac{-4\pi g_1g_2}{u-m^2}(W^2/2). \tag{20c}$$

The helicity-nonflip amplitude f_{++} is given in this equal-mass limit by

$$f_{++}=(1/8\pi)[WB+(2m/W)(A-mB)]. \tag{21}$$

Therefore, the condition for the appearance of a zero in f_{++} is found to be

$$A=(m-W^2/2m)B. \tag{22}$$

Defining $x=W/m$, $\alpha=g_1g_2(\frac{1}{2}^+)/m^2g_1g_2(\frac{3}{2}^+)$, and $\beta=g_1g_2(\frac{1}{2}^-)/m^2g_1g_2(\frac{3}{2}^+)$, the condition of (22) reduces to a quadratic equation in x^2,

$$x^4-x^2[6+2(\alpha+\beta)]+4(\alpha+\beta)+8\beta+6=0, \tag{23}$$

which has the solution

$$x^2=3+\alpha+\beta+[(3+\alpha+\beta)^2-(4(\alpha+\beta)+8\beta+6)]^{1/2}. \tag{24}$$

The other root of the quadratic has been discarded because it does not lead to zeros of f_{++} in the physical region $W\geq 2m$.

If we assign to m the reasonable value of 1 BeV, it is found that a zero of f_{++} will appear in the physical region $x>2$ for values of α which correspond to values of f such that

$$f\geq 0.346. \tag{25}$$

The value of W for which the zero occurs increases as the value of f is increased.

Because of the equality of the exchanged masses in this limit, this zero of f_{++}^{Born} will occur also for all partial-wave helicity-nonflip amplitudes. The amplitude modified by the absorptive corrections will therefore also contain this zero; and the polarization, which is proportional to f_{++}^*, will have a zero at the same energy. In this equal-mass case, $P(\theta)$ will be identically zero as a function of angle. When the equal-mass condition is relaxed, $P(\theta)$ will not be identically zero at any energy, but the average polarization will exhibit the zero as $P(\theta)$ varies with energy. This is demonstrated by calculated polarizations in Fig. 4.

It has been determined in a previous paper that the qualitative appearance of the differential cross section for Ξ^- production may be compatible only with values of f such that[12]

$$f\geq 0.35. \tag{26}$$

Therefore, although f_{-+} may have a zero of the same origin as the one which has been discussed, this zero is not relevant, since it may appear in the physical region only for values of f less than 0.35.

By requiring that the zero $\bar{P}_{\Xi^-}$ occur at 1.5 BeV/c lab momentum, the value of f which is determined from the model is

$$f=0.357. \tag{27}$$

This value of f is in very close agreement with values determined by other methods. If one assumes partially

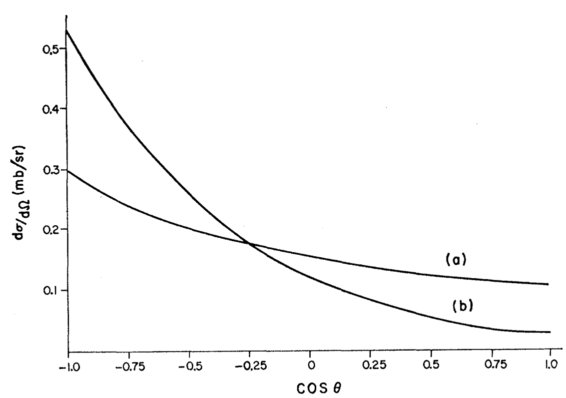

FIG. 5. The differential cross sections computed from single-particle exchange terms with no initial- and final-state absorptive corrections included are plotted as a function of center-of-mass scattering angle. The cross sections are given in mb/sr for (a) Λ and Σ exchange only and (b) Λ, Σ, Y_1^*, and Y_0^* exchanges at a lab momentum of 1.75 BeV/c.

[12] M. E. Ebel and P. B. James, Phys. Rev. Letters 15, 805 (1965).

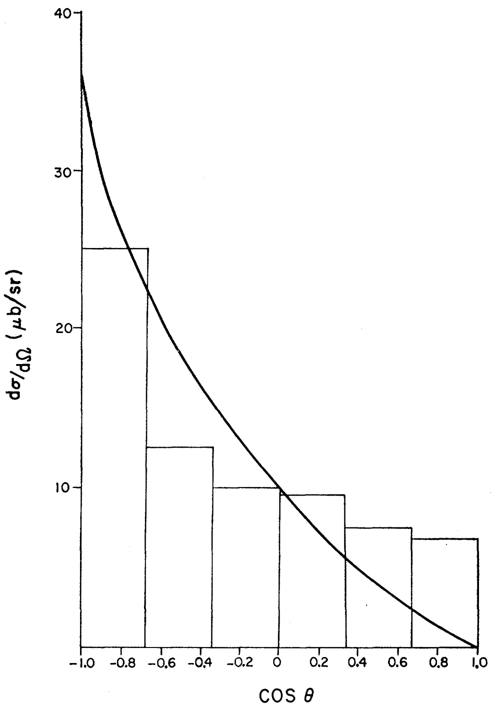

FIG. 6. The computed differential cross section (in μb/sr) is compared with the experimental data from Stevenson. The cross section in this figure corresponds to a lab momentum of 1.5 BeV/c and to a value of f equal to 0.36.

conserved axial-vector current (PCAC) in weak-interaction theory, the divergence of the axial-vector current is proportional to the pseudoscalar field. The algebra of the weak currents is, in the PCAC limit, the same as the algebra of the baryon pseudoscalar couplings. Willis *et al.*[13] have determined a value of $f=0.37$ from weak interactions, while Courant *et al.*[14] determined $f=0.37\pm0.04$. Harari has determined a value for f by use of the Gell-Mann current commutation relations; this value is $f=0.35$. Finally, Goebel's[15] strong-coupling theory predicts a value of $f=0.357$, exactly equal to ours. SU_6 theory of strong interactions predicts a value of $f=0.40$. On the other hand, Jarlskog and Pilkuhn[16] have determined that f is equal to 0.29. This value is incompatible with even the qualitative limit (26), and this discrepancy remains unexplained, although it may be a result of the SU_3 symmetry breaking mentioned previously.

IV. RESULTS

The most striking feature of the Ξ^- production process, the pronounced backward peaking of the K^+, is reproduced by the model fairly well. It will be noted from Fig. 5 that the greatest effect in the production of the sharp peak in our model is the inclusion of the spin-$\frac{3}{2}$ exchange term. As we mentioned in Sec. III and discussed in Ref. 12, for values of f greater than 0.35 the interference between the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ contributions to the amplitude is constructive in the helicity flip amplitude and destructive in the nonflip amplitude. It is this interference effect which enhances the sharpness of the peak to such a degree. The spin-$\frac{1}{2}$ exchange term alone does not produce a peak as narrow as that which should be obtained. The addition of the absorptive corrections, while of the correct size to bring the cross-section magnitudes into agreement with the data at low energies, do not seem to have a very pronounced effect on the angular distribution.

It is interesting to note that this spin-$\frac{1}{2}$–spin-$\frac{3}{2}$ interference apparently plays an important role in the reaction $K^-+p\rightarrow\Sigma^-+\pi^+$, as well as in the process which is being discussed. Neutron and N^* exchanges in this process take the place of $\Lambda\Sigma$ and Y_1^*, Y_0^* exchanges in Ξ^- production. In this case, however, the interference between the n and N^* terms is constructive in the flip amplitude for values of f greater than $\frac{1}{2}$ and destructive for values of f less than $\frac{1}{2}$. If f is assumed to be of the order of 0.37, one would expect that the backward peaking of π mesons produced in this process would be diminished at intermediate energies by the interference effects. Experimentally, the peak is observed to become less prominent at intermediate energies.[11]

Figures 6–8 display the differential cross sections obtained from the model; Figs. 6 and 7 present the

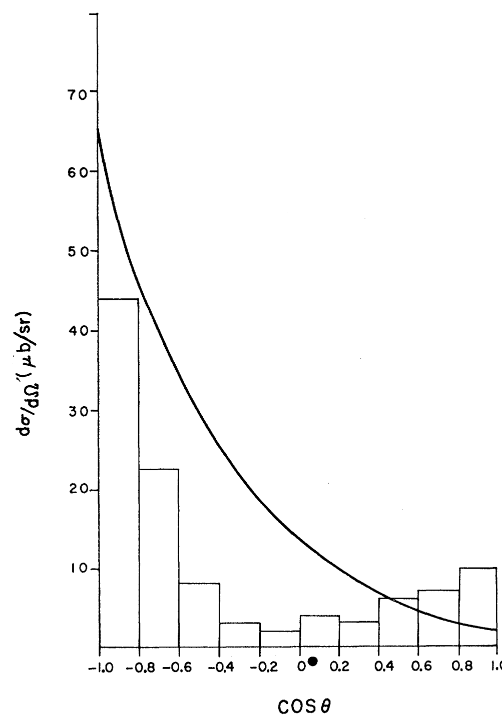

FIG. 7. The computed differential cross section (in μb/sr) is compared with the experimental data of Stevenson. The lab momentum corresponding to this figure is 1.75 BeV/c, and the value of f is 0.36.

[13] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964).
[14] H. Courant *et al.*, Phys. Rev. **136**, B1791 (1964).
[15] C. Goebel, Phys. Rev. Letters **16**, 1130 (1966).
[16] C. Jarlskog and H. Pilkuhn, Phys. Letters **20**, 428 (1966).

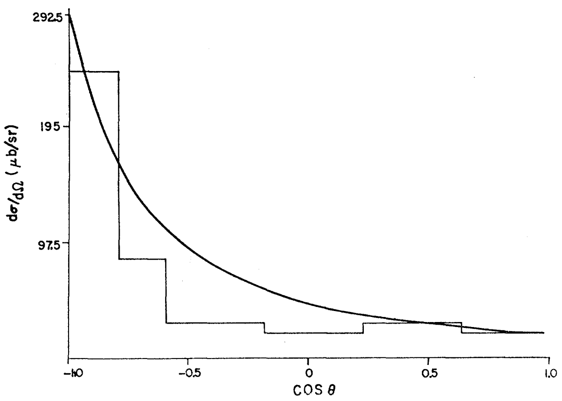

FIG. 8. The differential cross section computed from the model at lab momentum of 3.0 BeV/c is compared with the experimental angular distribution of Badier. Since these data consisted of a small number of events and were not converted to μb/sr, his data have been normalized to fit the model curve.

differential cross sections at K^- beam momenta of 1.5 and 1.75 BeV/c, respectively; these are compared with the experimental data at those energies which were summarized by Stevenson.[11] Figure 8 compares the experimental distribution of Badier[17] with that of the model at 3 BeV/c; in this case, the data of Badier have been normalized to the theoretical curve.

The divergences which appear in the magnitude of the cross section are most evident in the integrated cross section displayed in Fig. 9. The very serious divergence apparent in curves b and d is due to the energy dependence of the Y_1^* exchange term. The extra momentum factors present in the vertex functions of a spin-$\frac{3}{2}$ particle, as well as the momentum dependence of the projection operator appearing in the spin-$\frac{3}{2}$ propagator, account for this divergence. This is the same difficulty which precludes renormalization of spin-$\frac{3}{2}$ interactions in field theory.

The failure of the theory for energies above 1.5 BeV/c may be attributed to a breakdown of the single-particle exchange approximation in the form imposed by this model. It is possible that vertex form factors, which are functions of invariant u in this case, are needed in order to correctly describe the exchange of such a state. Because we must project partial-wave amplitudes from the total amplitude, contributions from increasingly negative values of u are included in our corrected amplitude at high energies. Vertex form factors could therefore be expected to modify the amplitudes substantially at high energies. As the energy increases, the individual partial-wave amplitudes increase and, even with the absorptive corrections, violate unitarity at high energy. Therefore multiple exchanges may be necessary to cancel the divergence at high energies.

The cross sections computed in the absence of the spin-$\frac{3}{2}$ exchange term (a and c) indicate that total suppression of the Y_1^* term will not suffice to bring the model into agreement with experiment. The energy dependence of the spin-$\frac{1}{2}$ contribution to the amplitude fails, although not as badly as that of the $\frac{3}{2}$ contribution, to agree with experiment.

Another possibility for removing the divergence is Reggeization of the various exchange contributions. The high-energy behavior of the integrated Ξ^- production cross section is roughly proportional to p_{lab}^{-3}. Gribov,[18] in his analysis of u-channel trajectories, found that an energy dependence of $d\sigma/d\Omega \propto s^{-(n+3)}$ is possible, where $n=1, 2, \cdots$. It is possible that a Regge theory for this process would retain the desirable low-energy features of this model while circumventing the disagreeable energy dependence.

The agreement in magnitude between the cross section predicted by the model and that deduced from experiment is good at lab momenta less than 1.5 BeV/c. It is encouraging that this good agreement occurs for a value of f equal to that deduced from the zero of $\bar{P}_{\Xi^-}$ (Sec. III). Although the discrepancy between the model and experiment with respect to the integrated cross section would seem to discredit a determination of f by this means, one would expect the energy range below

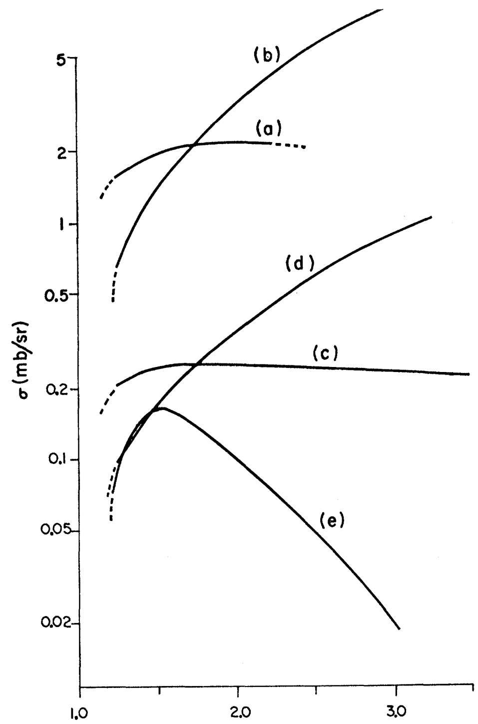

FIG. 9. The integrated cascade production cross section (in mb) is plotted for the following cases: (a) Λ and Σ exchanges with no absorption, $f=0.40$; (b) Λ, Σ, Y_1^*, Y_0^* exchanges with no absorption; (c) Λ and Σ exchanges with absorptive corrections; (d) Λ, Σ, Y_1^*, and Y_0^* exchanges with absorption; and (e) the experimental cross section from Stevenson.

[17] J. Badier, Proceedings of Argonne Accelerator User's Group Meeting, 1965 (unpublished).

[18] V. N. Gribov, Proceedings of the 1962 CERN Conference, 1962, p. 437 (unpublished).

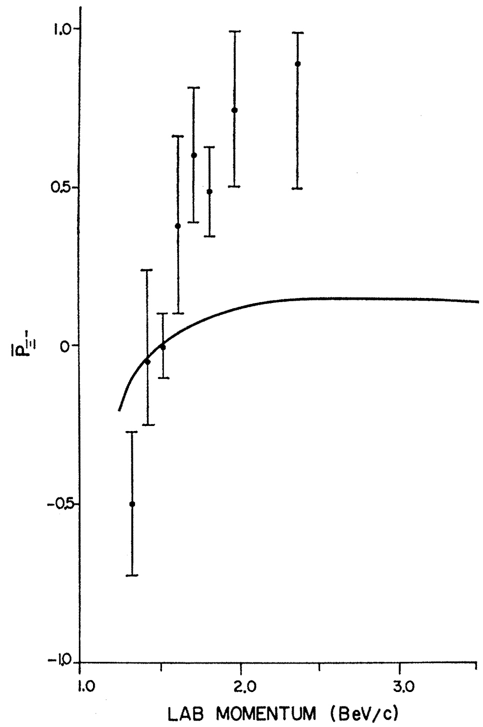

FIG. 10. The computed average polarization of the Ξ hyperons produced in $K^-p \to \Xi^- K^+$ is plotted as a function of beam momentum for a value of $f=0.357$. The experimental points shown are from Stevenson. The value of the average polarization predicted by the model in the region around $P_{\text{lab}}=2$ BeV/c is 16%.

1.5 BeV/c to offer the most sensitive test of the value of f, since it is at these energies that the spin-$\frac{1}{2}$ contribution is most important.

Qualitatively, the energy dependence of the polarization predicted by the model agrees with that observed experimentally (Fig. 10). This energy dependence is well reproduced by the model for a value of $f=0.357$, which is reasonable when compared to other determinations. However, the polarizations predicted by the model are smaller roughly by a factor of 3 than those determined experimentally. This discrepancy could be attributed to the omission of resonance states in the s channel of the production process. We have briefly investigated the effects of the spin-$\frac{7}{2}^+$ recurrence of the Y_1^* at lab momentum of 1.6 BeV/c; the results of this calculation were negative in that this resonance led to cross sections and polarizations which did not agree with the data as well as did those of our model. It is also possible that the use of elastic amplitudes computed from the Rarita-Phillips model led to the error in the magnitude of the polarization. Resonances in the elastic channels will presumably modify certain partial-wave amplitudes and lead to a modified polarization through the change in the absorption parameters. It is interesting to note the qualitative agreement of the behavior of $P(\theta)$ predicted by the model with the behavior recently determined by Berge *et al.*[19]

The qualitative predictions of this model indicate that the peripheral model is at least partially applicable to effects in the backward direction in production processes. The important contributions of the Y_1^* exchange term indicate that the spin-$\frac{3}{2}$ amplitudes should not be neglected in favor of spin-$\frac{1}{2}$ contributions when dealing with processes of this type. However, the quantitative failure of the model at high energies with or without the spin-$\frac{3}{2}$ exchange term leads to the conclusion that the model, as formulated here, is an insufficient description of a system of the type studied. The quantitative agreement obtained at low energies and the qualitative correctness of polarization and angular distribution indicate that absorption is an important effect in this process.

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[19] J. P. Berge *et al.*, Phys. Rev. **147**, 945 (1966).