partial-wave amplitude[11], or a Schrödinger-equation model.[1] With the inverse propagator, the obvious direct EM self-energy diagram (nucleon plus photon) has a Coulomb part which tends to make δN negative, and a magnetic part which tends to make it positive. The pionic self-energy (nucleon plus pion) contains a feedback term due to the splitting of the intermediate nucleons; in this context it is not implausible that such feedback can change the sign predicted[9] for δN, relative to the Coulomb effect. However, approaching the problem from a different angle, we have argued in I that this mechanism for changing the sign is unlikely to operate if the nucleons are elementary, though not if they are composite. If this argument is accepted, it illustrates just how differently, from physically different viewpoints, one tends to judge the plausibility even of a given mathematical approximation scheme.

By contrast, if one regards nucleons as pion-nucleon composites from the outset, then it is tempting to use the partial-wave amplitudes or a Schrödinger-equation model, and the natural choice for the leading direct EM energy is photon exchange between the constituents. But now, both Coulomb and magnetic energies tend to make δN negative.[11] Moreover, the natural choice of leading feedback terms (namely the mass shift of the constituent nucleons) does not now lead to a sign change of δN. On this picture, such a sign change could result only from the short-range charge-dependent corrections to the strong forces (called "short-range feedback" in I), to which, in this context, no credible plausibility arguments can be applied with presently available techniques.

[11] G. Barton, Phys. Rev. 146, 1149 (1966).

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Assignment Mixing in $SU(6)$ and Radiative Meson Decays*

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It has recently been emphasized by Capps, Belinfante, and Cutkosky (CBC) that under certain conditions we may classify the 0^- (P) and 1^- (V) mesons—in $SU(6)$—by their total angular momenta rather than their spins. It was noted, however, that this gives rise to an ambiguity in filling the 35-dimensional representation and that, *a priori*, we may consider a linear combination of the available $SU(3)$ multiplets, the coefficients to be determined by experiment. Here we explore the possibility that the radiative meson decays can help determine some of these coefficients ("assignment mixing" angles). In particular, from an analysis of decays involving the η and X^0 mesons we conclude that it is consistent to include the pseudoscalar octet and the pseudoscalar singlet in the same **35**. Furthermore, the best over-all fit to the data is obtained for $\cos\theta_1/\cos\theta_8 \approx 1$, where the cosines represent the respective mixing coefficients for P_1 and P_8. This value has been used by CBC and is also required by $SU(6)_W$. Our treatment makes use of the vector pole model to separate the couplings into strong and electromagnetic vertices.

INTRODUCTION

RECENT work by Capps[1] and by Belinfante and Cutkosky[2] (referred to together as CBC) has emphasized an ambiguity in assigning the 0^- and 1^- mesons to the 35-dimensional representation of SU(6). In their static-baryon bootstrap model of meson-baryon interactions, it seems natural to combine the unit orbital angular momentum with the spin of the participating meson and to consider the resultant total angular momentum as an intrinsic property of this meson. Consequently, the 0^- nonet members (P_8 and P_1) behave as effective axial mesons and the 1^- nonet members (V_8 and V_1) can couple their spins with the orbital angular momentum and act as effective axial or effective scalar particles. It is thus possible to have two nonets of axial mesons and one nonet of scalar ones: $P(8,3)$, $P(1,3)$; $V(8,3)$, $V(1,3)$; $V(8,1)$, $V(1,1)$—in the usual $[SU(3),SU(2)]$ notation.[3] The particular combination that should make up the 35-dimensional multiplet is not determined *a priori* and we can write generally[3,4]

$$35_L = P(8,3)\cos\theta_8 + V(8,3)\sin\theta_8 + P(1,3)\cos\theta_1 + V(1,3)\sin\theta_1 + V(8,1). \quad (1)$$

The "assignment mixing" (AM)[4] angles θ_8 and θ_1 can be fixed by experiment, or else by appeal to theoretical

* Work supported by the U. S. Atomic Energy Commission.
[1] R. H. Capps, Phys. Rev. Letters 4, 31 (1965).
[2] J. G. Belinfante and R. E. Cutkosky, Phys. Rev. Letters 14, 33 (1965).
[3] If we admit the existence of a scalar octet, $S(8,1)$—of which the $\kappa(725)$ may be the $I=\frac{1}{2}$ member—then the most general linear combination would involve a further mixing between $V(8,1)$ and $S(8,1)$. I am indebted to Dr. George Renninger for emphasizing this point.
[4] J. G. Belinfante and G. Renninger, Phys. Rev. 148, 1573 (1966).

models. The CBC assignment corresponds to taking $\theta_8=\theta_1=0$, while the $SU(6)_W$ assignment of mesons[5] is seen to be given for $\tan\theta_8=\tan\theta_1=\sqrt{2}$. The subscript "$L$" is included in (1) to denote the special character of this "light" multiplet in contradistinction to the "heavy" baryons.

In meson-meson-meson vertices there is an analogous situation in that two of the mesons interact in a relative P state. In principle, it is then possible to describe also such vertices in terms of one "light" meson—in the sense of Eq. (1)—in interaction with two "heavy" mesons, if the masses involved make such a division meaningful. Only in the PVV case can this condition be met to good approximation and we may expect sensible results for the decay $V_i \to V_f+P_k$, especially since we are free to work in the V_i rest frame.[6]

In this paper we wish to examine those decays where the PVV vertex occurs. This will include radiative decays of bosons, where the strong vertex is followed by a $V\to\gamma$ junction.[7] Appeal to experiment will then determine admissible values for the AM angles, in particular for $\cos\theta_1/\cos\theta_8$. We summarize our assumptions as follows:

(1) In processes $V_i\to V_f+P_k$, the vector mesons are assigned to the usual Gürsey-Radicati[8] **35** and the pseudoscalar mesons to the 35_L of Eq. (1).

(2) Both the on-mass-shell and the off-mass-shell coupling coefficients are given by $35_H \leftarrow 35_L\times 35_H$, or, equivalently, by the values of the matrix elements

$$G_{fi}{}^k=\langle 35_H{}^f|35_L{}^k|35_H{}^i\rangle\,, \tag{2}$$

where k, f, i refer to the particles involved in the vertex. In general, the $G_{fi}{}^k$ will depend on the assignment mixing angles θ_8 and θ_1.

(3) Since the complete matrix element for $V_i\to V_f+P_k$ is characterized by

$$\mathfrak{M}_{fi}{}^k=gG_{fi}{}^k\epsilon_{\alpha\beta\gamma\delta}P_\alpha{}^ie_\beta{}^ie_\gamma{}^fP_\delta{}^k \tag{3}$$

[the scale factor g has dimensions of mass^{-1} and is common to all PVV vertices], we reason from Lorentz invariance of $|\mathfrak{M}_{fi}{}^k|^2$ that the same coupling coefficient $G_{fi}{}^k$ should be used for the crossed process $P_k\to V_i+V_f$, since the interaction vertex is the same.[9] This crossed case is of interest, for example, in the pole model[7] for decays $\eta\to\pi\pi\gamma$, etc., examined below.

STRONG VERTICES

As it stands, Eq. (2) yields the three-meson couplings in those cases where no particle mixing enters. In practice, however, we are confronted with octet-singlet mixing and we define

$$\begin{aligned}|\omega\rangle&=a|\omega_8\rangle+b|\omega_1\rangle\,,\\ |\varphi\rangle&=-b|\omega_8\rangle+a|\omega_1\rangle\,,\end{aligned} \tag{4}$$

$$\begin{aligned}|\eta\rangle&=\alpha|\eta_8\rangle+\beta|\eta_1\rangle\,,\\ |X^0\rangle&=-\beta|\eta_8\rangle+\alpha|\eta_1\rangle\,,\end{aligned} \tag{5}$$

for the observed 1^- and 0^- isoscalar particles. The mixing angles of 40° and 10° will be used for the $\omega-\varphi$ and $\eta-X^0$ systems,[10] respectively: $a=\sin 40°$, $b=\cos 40°$; $\alpha=\cos 10°$, $\beta=\sin 10°$. The various couplings of interest can now be obtained[11] and these are shown in Table I

TABLE I. Strong couplings for PVV vertices, including octet-singlet mixing defined by Eqs. (4) and (5). $r=\cos\theta_1/\cos\theta_8$.

i	f	k	$G_{fi}{}^k$
ω	ρ^0	π^0	$-(a/\sqrt{2}+b)\cos\theta_8/3$
φ	ρ^0	π^0	$+(b/\sqrt{2}-a)\cos\theta_8/3$
ρ^0	ρ^0	η	$-(\alpha/\sqrt{2}+\beta r)\cos\theta_8/3$
ρ^0	ρ^0	X^0	$+(\beta/\sqrt{2}-\alpha r)\cos\theta_8/3$
ω	ω	η	$+[(a^2/\sqrt{2}-2ab)\alpha-\beta r]\cos\theta_8/3$
ω	ω	X^0	$-[(a^2/\sqrt{2}-2ab)\beta+\alpha r]\cos\theta_8/3$
ω	φ	η	$-(ab/\sqrt{2}+a^2-b^2)\alpha\cos\theta_8/3$
φ	φ	η	$+[(b^2/\sqrt{2}+2ab)\alpha-\beta r]\cos\theta_8/3$
φ	φ	X^0	$-[(b^2/\sqrt{2}+2ab)\beta-\alpha r]\cos\theta_8/3$
ω	φ	X^0	$+(ab/\sqrt{2}+a^2-b^2)\beta\cos\theta_8/3$
K^{*+}	ρ^0	K^+	$-\cos\theta_8/2\sqrt{6}$
K^{*0}	ρ^0	K^0	$+\cos\theta_8/2\sqrt{6}$
K^*	ω	K	$+(a/2\sqrt{2}-b)\cos\theta_8/3$
K^*	φ	K	$-(b/2\sqrt{2}+a)\cos\theta_8/3$

for arbitrary particle mixing. The factor r is defined by

$$r=\cos\theta_1/\cos\theta_8\,. \tag{6}$$

It is evident that the decays involving η and X^0 mesons are of greatest interest, since they determine the AM ratio r. For purposes of orientation we also note that

[5] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965).

[6] Capps has treated three-meson vertices ($M_1\to M_2+M_3$, for example) by explicitly making two of the mesons "static" (setting, for example, $E_1=E_2=M_1$, where M_1 is mass of the heavier meson). This procedure makes the situation completely analogous to the meson-baryon static model and permits the third meson (M_3) to play the same role as the "meson" in meson-baryon situation. [See R. H. Capps, Phys. Rev. **144**, 1182 (1966).] Here, however, we wish to guarantee that the kinematics reflect the physical situation exactly and assume, instead, that Eq. (2) below still gives the correct coupling. This limits our application to PVV vertices, where the P acts as the "light" meson in each instance.

[7] M. Gell-Mann, D. H. Sharp, and W. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[8] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964).

[9] To put it another way: We choose a channel where $SU(6)$ is expected to hold and write a covariant matrix element for the process. This very covariance, however, tells us that the interaction strength stays the same for a time-crossed channel.

[10] These values are obtained by diagonalizing the relevant mass matrix and using squared masses in the Gell-Mann–Okubo formula. The $SU(6)$-derived $\omega-\varphi$ mixing of $a=1/\sqrt{3}$ prohibits $\varphi\to\rho\pi\to 3\pi$, whereas in reality this mode accounts for over 30% of the φ width.

[11] The $SU(3)$ coefficients are taken from P. McNamee and F. Chilton [Rev. Mod. Phys. **36**, 1005 (1964)] and the $SU(6)$ isoscalar factors are from C. L. Cook and G. Murtaza [Nuovo Cimento **39**, 531 (1965)].

without $\eta-X^0$ mixing and using $a=1/\sqrt{3}$, the $G_{fi}{}^k$ reduce to

$$G_{\omega\rho}{}^{\pi}=\sqrt{3}G_{\rho\rho}{}^{\eta}=3G_{\omega\omega}{}^{\eta}=(-\sqrt{3}/2)G_{\varphi\varphi}{}^{\eta}=2G_{K^{+*}}{}_{\rho}{}^{K+}$$
$$=-2G_{K^{*0}}{}_{\rho}{}^{K^0}=2G_{K^*}{}_{\omega}{}^{K}=\sqrt{2}G_{K^*}{}^{K}=-\cos\theta_8/\sqrt{6}\,, \quad (7)$$

$$G_{\rho\rho}{}^{X^0}=G_{\omega\omega}{}^{X^0}=G_{\varphi\varphi}{}^{X^0}=-\cos\theta_1/3=-r\cos\theta_8/3\,, \quad (8)$$

$$G_{\varphi\varphi}{}^{\pi}=G_{\omega\varphi}{}^{\eta}=G_{\omega\varphi}{}^{X^0}=0\,. \quad (9)$$

Thus knowledge of r is still necessary if we are to relate decays involving the X^0 to those involving the η meson.

DECAYS $\Omega\rightarrow 3\Pi$ AND $\Phi\rightarrow 3\Pi$

Inasmuch as the radiative decays will involve additional assumptions about the $V\rightarrow\gamma$ coupling, we first wish to consider the two processes that depend on the PVV vertex and on $f_{\rho\pi\pi}=5.51$[12]: $\omega\rightarrow\rho\pi\rightarrow 3\pi$ and $\varphi\rightarrow\rho\pi\rightarrow 3\pi$. This will also enable us to evaluate the scale factor g [see Eq. (3)] that determines the absolute rates of all processes.

Multiplying Eq. (3) by $[s-M_\rho{}^2+iM_\rho\Gamma_\rho]^{-1}$, symmetrizing over the ρ states, and inserting three-body phase space, we obtain an expression for the rate of $M_i\rightarrow 3\pi$ [evaluated in the center-of-mass frame of the decaying meson of mass M_i; $i=\omega,\ \varphi$]:

$$(M_i\rightarrow 3\pi)=(gG_{\rho i}{}^{\pi})^2 f_{\rho\pi\pi}{}^2[4(2\pi)^3(2M_i)^3]^{-1}$$
$$\times\int_{4\mu^2}^{(M-\mu_0)^2} H(s)ds\,. \quad (10)$$

Here $s=M_{\pi\pi}{}^2$, μ and μ_0 are the charged and neutral pion masses, respectively, and $H(s)$ is given in the Appendix. Integrating numerically, we find

$$\Gamma(\omega\rightarrow 3\pi)=(gG_{\rho\omega}{}^{\pi})^2(3.1\times10^4\ \text{MeV}^3)\,, \quad (11)$$

$$\Gamma(\varphi\rightarrow 3\pi)=(gG_{\rho\varphi}{}^{\pi})^2(9.7\times10^5\ \text{MeV}^3)\,. \quad (12)$$

The number[13] shown in Eq. (12) can be compared with (5.4×10^5 MeV^3) for the two-body decay $\varphi\rightarrow\rho\pi$. The latter value, frequently used in estimating the $\varphi\rightarrow 3\pi$ rate, is thus too low by a factor of about 2. With the coupling included, we have, for 40° $\omega-\varphi$ mixing,

$$\Gamma(\omega\rightarrow 3\pi)=5.1\times10^3\ \text{MeV}^3(g\cos\theta_8)^2\,, \quad (13)$$

$$\Gamma(\varphi\rightarrow 3\pi)=1.1\times10^3\ \text{MeV}^3(g\cos\theta_8)^2\,, \quad (14)$$

$$R=\Gamma(\varphi\rightarrow 3\pi)/\Gamma(\omega\rightarrow 3\pi)=0.22\,. \quad (15)$$

Note that $\omega\rightarrow 3\pi$ is quite insensitive to the $\omega-\varphi$ mixing taken, while the ratio (15) very much depends on it. In fact, as has frequently been noted in other contexts,[10] $R=0$ for $a=1/\sqrt{3}$; for 39° mixing we get $R=0.14$. Finally, using $\Gamma(\omega\rightarrow 3\pi)=10.8$ MeV, we have

$$(g\cos\theta_8)^2=2\times10^{-3}\ \text{MeV}^{-2}\,. \quad (16)$$

RADIATIVE DECAYS

In the vector pole model[7] the decays $X^0\rightarrow\pi\pi\gamma$, $\eta\rightarrow\pi\pi\gamma$, $\rho\rightarrow\eta\gamma$, and $\omega\rightarrow\pi^0\gamma$ also involve the $\rho-\gamma$ vertex, which is given by $f_\rho=em_\rho{}^2/f_\rho$. It has been the practice to assume ρ dominance of the isovector form factor and use $f_\rho\approx f_{\rho\pi\pi}=5.51$ [for $\Gamma(\rho\rightarrow 2\pi)=124$ MeV]. However, the recently measured ratio[14]

$$\Gamma(\rho\rightarrow\mu^+\mu^-)/\Gamma(\rho\rightarrow 2\pi)=(0.33_{-0.07}^{+0.16})\times10^{-4} \quad (17)$$

provides direct access to f_ρ and the above approximation can be avoided. Using[15]

$$\Gamma(\rho\rightarrow\mu^+\mu^-)=\frac{\alpha^2}{f_\rho{}^2/4\pi}\frac{m_\rho}{3}\left(1-\frac{4m_\mu{}^2}{m_\rho{}^2}\right)^{1/2}\left(1+\frac{2m_\mu{}^2}{m_\rho{}^2}\right)$$
$$=(f_\rho{}^2/4\pi)^{-1}(1.36\times10^{-2}\ \text{MeV}) \quad (18)$$

leads to

$$f_\rho{}^2/4\pi=3.3_{-2.0}^{+2.6}\,, \quad (19)$$

where we have included the uncertainty in the ρ width. Equivalently, we can write $f_{\rho\pi\pi}/f_\rho=0.86_{-0.65}^{+0.50}$, so that the ρ may dominate but not to the exclusion of other substantial contributions. A rough consistency check on (19) can be obtained from[16] $(\omega\rightarrow\pi\gamma)/(\omega\rightarrow 3\pi)=(9.7\pm1.6)\times10^{-2}$, as follows. From the expression

$$\Gamma(\omega\rightarrow\pi^0\gamma)=(gG_{\omega\rho}{}^{\pi})^2(f_{\rho\gamma}/m_\rho{}^2)^2\left(\frac{m_\omega{}^2-\mu^2}{m_\omega}\right)^3\Big/96\pi\,, \quad (20)$$

we have, using (11),

$$\frac{\Gamma(\omega\rightarrow\pi^0\gamma)}{\Gamma(\omega\rightarrow 3\pi)}=(3.44\times10^{-1})(f_\rho/4\pi)^{-1}$$
$$=(9.7\pm1.6)\times10^{-2} \quad (20')$$

and therefore, $f_\rho{}^2/4\pi=3.6\pm0.4$. We shall use

$$f_\rho{}^2/4\pi=3.5\,.$$

Since, in $X^0(\eta)\rightarrow\pi\pi\gamma$, no $\omega-\varphi$ mixing enters and it is not necessary to assume that the photon transform as a pure octet, then these decays allow a relatively clear-cut determination of the AM ratio. In fact, inserting a Breit-Wigner form into Eq. (3) and integrating over phase space, we have

$$\Gamma(M_i\rightarrow\pi\pi\gamma)=(gG_{\rho\rho}{}^{i})^2 f_{\rho\pi\pi}{}^2(f_{\rho\gamma}/m_\rho{}^2)^2$$
$$\times[96M_i{}^3(2\pi)^3]^{-1}I\,, \quad (21)$$

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[12] Unless otherwise indicated, all experimental values are taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965): $\Gamma_\rho=124$ MeV, $M_\rho=763$ MeV.

[13] Note that this number depends also on the value of $f_{\rho\pi\pi}{}^2$. We agree with J. Yellin, Phys. Rev. **147**, 1080 (1966).

[14] J. K. dePagter *et al.*, Phys. Rev. Letters **16**, 35 (1966).

[15] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **8**, 79 (1965).

[16] S. M. Flatté *et al.*, Phys. Rev. Letters **14**, 1095 (1965).

$$I=\int_{2\mu}^{M_i} dQ\frac{(M_i^2-s)^3(s-4\mu^2)^{3/2}}{(s-m_\rho^2)^2+(m_\rho\Gamma_\rho)^2}$$

$$=\begin{cases}2.7\times10^{18}\text{ MeV}^6;\ i=X^0\\ 5.2\times10^{12}\text{ MeV}^6;\ i=\eta\end{cases}$$

$$s=(\pi^+ + \pi^-)^2=Q^2.$$

With the appropriate couplings in, this yields for $+10°$ $\eta-X^0$ mixing

$$\Gamma(X^0\rightarrow\pi\pi\gamma)=(g\cos\theta_8)^2(1-8r)^2(1.7\text{ MeV}^3), \tag{22}$$

$$\Gamma(\eta\rightarrow\pi\pi\gamma)=(g\cos\theta_8)^2(1+0.25r)^2(4.4\times10^{-2}\text{ MeV}^3), \tag{23}$$

$$P\equiv\frac{\Gamma(X^0\rightarrow\pi\pi\gamma)}{\Gamma(\eta\rightarrow\pi\pi\gamma)}=38\left(\frac{1-8r}{1+0.25r}\right)^2. \tag{24}$$

These quantities are seen to be very sensitive to the choice of r. We may now use the experimental information that $\Gamma(X^0\rightarrow\pi\pi\gamma)/\Gamma(X^0\rightarrow\text{all})=0.25$ and $\Gamma(X^0\rightarrow\text{all})<4$ MeV to find, using Eq. (16),

$$\begin{aligned}\Gamma(X^0\rightarrow\pi\pi\gamma)&=30\text{ keV},\quad r=\tfrac{1}{2}\\&=167\text{ keV},\quad r=1\\&=760\text{ keV},\quad r=2.\end{aligned} \tag{25}$$

Thus, $r=2$ is an upper limit for $\Gamma(X^0\rightarrow\text{all})<4$ MeV. We should note that $0°$ $\eta-X^0$ mixing yields $\Gamma(X^0\rightarrow\pi\pi\gamma)=225r^2$ keV, and again an upper limit of about 2. For $-10°$ mixing, we just change the minus to a plus in Eq. (22) and find an upper limit of ~2. On the other hand, the choice $r=\frac{1}{2}$ predicts $\Gamma(X^0\rightarrow\eta\pi\pi)=(90\text{–}160)$ keV for the above $\eta-X^0$ mixing angles. Although this is consistent with the data, it may be on the low side for what is supposedly a strong decay mode.[17] In any case, the $SU(6)_W$ (and CBC) choice of $r=1$ is certainly in harmony with experiment. Note that the decay $M\rightarrow\pi\pi\gamma$ is collinear—and therefore qualifies for $SU(6)_W$—precisely because we are using the pole model. With $r=1$ we also have

$$\Gamma(\eta\rightarrow\pi\pi\gamma)=140\text{ eV}, \tag{26}$$

thereby predicting an η width of 2.5 keV (we have used $\eta\rightarrow\pi\pi\gamma/\eta\rightarrow\text{all}=5.5\%$). This is considerably larger than most estimates.[18]

In principle, the quantity

$$R_{\text{exp}}=\Gamma(\eta\rightarrow2\gamma)/\Gamma(\eta\rightarrow\pi\pi\gamma)=7.0\pm3$$

offers an independent determination of the AM ratio. However, in $\eta\rightarrow2\gamma$ we also have contributions from ω and φ intermediate states, and there is some question about the coupling appropriate for the $\omega(\varphi)\rightarrow\gamma$ vertices. If we assume that the photon transforms as a pure octet member of $SU(3)$ and does not couple to ω_1, then we can write [relating $f_{\omega_8\gamma}$ to $f_{\rho\gamma}$ by $S(U3)$ and using Eq. (4)],

$$\begin{aligned}f_{\omega\gamma}&=(a/\sqrt{3})f_{\rho\gamma},\\ f_{\varphi\gamma}&=(-b/\sqrt{3})f_{\rho\gamma}.\end{aligned} \tag{27}$$

For an independent check on Eq. (27) (and on the present model, as such), we can consider the $\pi^0\rightarrow2\gamma$ decay [proceeding via $\pi^0\rightarrow\rho+\omega(\varphi)\rightarrow2\gamma$]. With $f_\rho^2/4\pi=3.5$ and using (16) [there is no dependence on r], we obtain

$$\Gamma(\pi^0\rightarrow2\gamma)=7.8\text{ eV}, \tag{28}$$

which is in excellent agreement with the measured width of 6.3 eV.[19] From (20′) we would estimate $\Gamma(\pi^0\rightarrow2\gamma)=(7.4\pm1.6)$ eV. It has been discussed in the literature (see Ref. 7, for example) that the vector pole model predicts too high a value for $\Gamma(\pi^0\rightarrow2\gamma)$. In these estimates, however, use is made of $f_\rho\simeq f_{\rho\pi\pi}$, and our result would suggest that in a treatment such as ours the difficulty may be traced to this added assumption rather than to use of the basic model for π^0 decay. Of course, it would be sufficient for our purpose to reverse the argument and consider Eq. (28) as fortuitous supporting evidence for the large f_ρ obtained in (19) and (20′). We might re-emphasize here that recent fits to the isovector form factor [see, for example, Wilson *et al.*, Phys. Rev. **141**, B1298 (1966)] cause us to question the complete dominance of the ρ contribution and would lead us to expect $f_\rho/f_{\rho\pi\pi}>1$.

For the η decays we have

$$R=\frac{\Gamma(\eta\rightarrow2\gamma)}{\Gamma(\eta\rightarrow\pi\pi\gamma)}=4.0\left(\frac{1+0.31r}{1+0.25r}\right)^2, \tag{29}$$

for $10°$ $\eta-X^0$ mixing. This gives $R=4.7$, 4.4, 4.2 for $r=2$, 1, $\frac{1}{2}$, respectively—all equally consistent with 7 ± 3 and, therefore, also rather unenlightening. In practice, then, the slow dependence on r renders R unsuitable for determining the AM ratio, although it does serve as a check on the model.

On the other hand, the ratios $\Gamma(\varphi\rightarrow\eta\gamma)/\Gamma(\omega\rightarrow\eta\gamma)$ and $\Gamma(\varphi\rightarrow\eta\gamma)/\Gamma(\varphi\rightarrow\text{all})$ can provide a sensitive test. Using expressions analogous to Eq. (20) to calculate the rates, we find for $+10°$

$$Q=\frac{\Gamma(\varphi\rightarrow\eta\gamma)}{\Gamma(\omega\rightarrow\eta\gamma)}=25\left(\frac{1-0.11r}{1+0.31r}\right)^2, \tag{30}$$

which yields $Q=17$, 12, 6 for $r=\frac{1}{2}$, 1, 2, respectively. Experimental numbers are $\Gamma(\varphi\rightarrow\eta\gamma)/\Gamma(\varphi\rightarrow\text{all})<0.12$ and $\Gamma(\omega\rightarrow\eta\gamma)/\Gamma(\omega\rightarrow\text{all})<0.17$, and this gives $r=2$ for the upper limits.

No other data are available at this time, but it would

[17] See L. M. Brown and H. Faier, Phys. Rev. Letters **13**, 73 (1964).

[18] See, however, F. A. Berends and P. Singer [Phys. Letters **19**, 249 (1965); **19**, 616 (1965)] who estimate $\Gamma(\eta\rightarrow\text{all})=2.2$ keV.

[19] G. von Dardel *et al.*, Phys. Letters **4**, 51 (1963).

be of interest to list some additional predictions for future verification. In particular we have

$$\Gamma(\rho \to \eta\gamma) = 37(1+0.25r)^2 \text{ keV}, \quad (31)$$

$$\frac{\Gamma(X^0 \to \omega\gamma)}{\Gamma(X^0 \to \pi\pi\gamma)} = 0.05\left(\frac{1-10r}{1-8r}\right)^2, \quad (32)$$

$$\frac{\Gamma(X^0 \to 2\gamma)}{\Gamma(\eta \to 2\gamma)} = 0.25\left(\frac{1-10r}{1+0.31r}\right)^2. \quad (33)$$

The last ratio is particularly sensitive to the value of r and equals 4, 12, 35 fo $r=\frac{1}{2}$, 1, 2, respectively. For $(-10°)$ mixing just reverse the signs preceding r in the above expressions. To get $\Gamma(\rho \to \eta\gamma)$, we have used Eq. (16) again.

It has thus been shown that our analysis of the above decays yields results consistent with present observations. Indeed, the assignment mixing approach relates in a natural way the P_1VV vertex to the P_8VV coupling strength, and we have seen that a value for this pseudoscalar mixing of $\cos\theta_1/\cos\theta_8 = 1$ is in over-all agreement with the data. These conclusions would be much strengthened if the η and X^0 widths were known well and, also, if the sign and magnitude of the $\eta - X^0$ mixing were unambiguously determined.[20]

We might emphasize that for single-photon decays such as $\omega \to \pi^0\gamma$—which do not depend on $\eta - X^0$ mixing—our treatment gives essentially the same results as obtained previously[21] by using Gell-Mann's postulate that the magnetic dipole operator transforms like a tensor component of the **35** representation of $SU(6)$.[22] Our more specific reliance on the pole model is offset perhaps by the unification achieved in the treatment of all decays involving a PVV vertex, while we also avoid the reference to quarks implicit in the above postulate.

[20] A. J. MacFarlane and R. H. Socolow [Phys. Rev. **144**, 1194 (1966)] have proposed several tests to determine the $\eta - X^0$ mixing angle.

[21] See, for example, S. Badier and C. Bouchiat, Phys. Rev. Letters **15**, 96 (1965).

[22] M. Gell-Mann, Phys. Rev. Letters **14**, 77 (1965).

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APPENDIX

The integrand in Eq. (10) is given by

$$H(s) = [\tfrac{4}{3}M_i^3[\varphi(s)]^3 s - 2F(s)][(s-m_\rho^2)^2 + (m_\rho\Gamma_\rho)^2]^{-1},$$

where

$$\varphi(s) = \left[\left(\frac{M_i^2+\mu_0^2-s}{2M_i}\right)^2 - \mu_0^2\right]^{1/2}\left[\frac{s-4\mu^2}{s}\right]^{1/2},$$

$$\begin{aligned} F(s) = {} & [(M_iX + 4M_i m_\rho^2)A' + 2M_iB']\varphi(s) \\ & + [(3m_\rho^4 - B_\rho^2)A'/2 + m_\rho^2B' + C'/2]\ln(T_{\max}/T_{\min}) \\ & + [m_\rho^2(m_\rho^4 - 3B_\rho^2)A' + (m_\rho^4 - B_\rho^2)B' + m_\rho^2C' + D'] \\ & \times\{\tan^{-1}[B_\rho(t_{\max} - m_\rho^2)^{-1}] \\ & \qquad - \tan^{-1}[B_\rho(t_{\min} - m_\rho^2)^{-1}]\}/B_\rho. \end{aligned}$$

Here

$$\begin{aligned} B_\rho &= m_\rho\Gamma_\rho, \\ X &= M_i^2 + 2\mu^2 + \mu_0^2 - s, \\ A' &= s(s - m_\rho^2), \\ B' &= [B_\rho^2 - m_\rho^2(s - m_\rho^2) - (s - m_\rho^2)X]s, \\ C' &= [(M_i^2 - \mu^2)(\mu_0^2 - \mu^2)s + (M_i^2 - \mu_0^2)\mu^2](s - m_\rho^2) \\ &\quad - [B_\rho^2 - m_\rho^2(s - m_\rho^2)]sX, \\ D' &= [B_\rho^2 - m_\rho^2(s - m_\rho^2)] \\ &\quad \times[(M_i^2 - \mu_0^2)(\mu_0^2 - \mu^2)s + (M_i^2 - \mu_0^2)^2\mu^2], \end{aligned}$$

$$\begin{aligned} T_{\max,\min} &= (t_{\max,\min} - m_\rho^2)^2 + B_\rho^2, \\ t_{\max} &= \tfrac{1}{2}X + M_i\varphi(s), \\ t_{\min} &= \tfrac{1}{2}X - M_i\varphi(s). \end{aligned}$$

After setting up the matrix element, we did not keep track of the different pions so that $H(s)$ does *not* represent the invariant mass spectrum of a particular pair.