Regge-Pole Formulas for Differential Cross Sections of Quasi-Two-Body πN and NN Interactions*

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Regge-pole formulas for differential cross sections of some quasi-two-body πN and NN interactions, which can have one or two meson trajectories exchanged, are given in this paper. The reactions treated are: $\pi N\to\pi\Delta(1236)$, $\pi N\to\omega\Delta(1236)$, $\pi N\to A_2N$, $\pi N\to\eta N$, $\pi N\to\eta\Delta(1236)$, $\pi N\to\rho N$, $\pi N\to\rho\Delta(1236)$, $NN\to N\Delta(1236)$, $NN\to NN^*(1500,\frac{3}{2}^-)$, $NN\to NN^*(1480,\frac{1}{2}^+)$.

I. INTRODUCTION

THE Regge-pole model has given quite good fits[1] and interesting predictions[2,3] for certain simple high-energy two-body to two-body interactions which can have single Regge-pole exchange. There remain many interesting quasi-two-body interactions, involving complications of high spins and unequal-mass kinematics, which have not been treated theoretically. With the fast accumulation of high-energy experimental data, it is of interest to know the theoretical formulas which are suitable for phenomenological fits. In this paper we shall give the Regge-pole formulas for the differential cross sections of some quasi-two-body πN and NN interactions which can have one or two meson trajectories exchanged.

According to the established method of reggeization,[4] including the deduction of residue functions free from kinematic singularities,[5] the following results will be obtained in a routine fashion.

II. πN INTERACTIONS

A. $\pi N\to\pi\Delta(1236)$

Because of the conservation of isotopic spin and G parity, only ρ can be exchanged among the experimentally established trajectories.[6] The differential cross section without polarization is

$$\begin{aligned}\frac{d\sigma}{dt}=&\frac{1}{4\pi s p_{\pi N}^2}\frac{1}{(2\times\frac{1}{2}+1)}\times2\times\tfrac{1}{4}\{|\sin\theta_t[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{3}{2}\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2\\&+|(\sin\theta_t)^2[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{3}{2}-\frac{1}{2}}(t)E_{02}{}^{\alpha,+}(\cos\theta_t)|^2+|[1-\exp(-i\pi\alpha)]\\&\times[(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{1}{2}\frac{1}{2}}(t)E_{00}{}^{\alpha,+}(\cos\theta_t)|^2+|\sin\theta_t[1-\exp(-i\pi\alpha)]\\&\times[(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{1}{2}-\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2\},\end{aligned}\tag{1}$$

where $P_{\pi N}$ is the c.m. momentum of the πN system, s is the total energy squared, θ_t is the t reaction angle,[7] $\alpha(t)$ is the ρ trajectory, and the β's are the residue functions of the partial-wave helicity amplitudes which have definite parity. The kinematic factors of the residue functions β are[8,9]

$$\beta_{00;\frac{3}{2}\frac{1}{2}}(t)\propto[\alpha(\alpha+1)]^{1/2}[t-4m_\pi^2]^{1/2}[t-(m_\Delta-m_N)^2]^{-1/2}(p_{\pi\pi}p_{\bar{N}\Delta})^{\alpha-1},\tag{2a}$$

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[1] A. V. Stirling, P. Sondereger, J. Kirz, P. Falkvairant, O. Guillaud, C. Caverzasio, and B. Amblard, Phys. Rev. Letters **14**, 763 (1965); I. Mannelli, A. Bigi, R. Carrara, M. Wahlig, and L. Sodickson, *ibid.* **14**, 408 (1965); R. K. Logan, *ibid.* **14**, 414 (1965); R. J. H. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965); F. Arbab and C. B. Chiu, *ibid.* **147**, 1045 (1966); G. Hohler, J. Baacke, N. Schlaile, and P. Sonderegger, Phys. Letters **20**, 79 (1966); W. R. Frisken, A. L. Read, H. Ruderman, A. D. Krisch, J. Orear, R. Rubin, stein, B. D. Scarl, and D. H. White, Phys. Rev. Letters **15**, 313 (1965); H. Brody, R. Lanza, R. Marshall, J. Niederer, W. Selove, M. Shochet, and R. Van Berg, *ibid.* **18**, 828 (1966); C. B. Chiu and J. D. Stack, Phys. Rev. **153**, 1575 (1967).

[2] G. F. Chew and J. D. Stack, University of California, Lawrence Radiation Laboratory Report No. UCRL-16293, 1965 (unpublished); J. D. Stack, Phys. Rev. Letters **16**, 286 (1966).

[3] L.-I., Wang, Phys. Rev. Letters **16**, 756 (1966). See also Appendix II.

[4] M. Gell-Mann, M. Goldberger, F. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

[5] L.-L. C. Wang, Phys. Rev. **142**, 1187 (1966); See also Appendix I. The procedure of obtaining the differential cross section is given very clearly in Ref. 3.

[6] Thews has studied this interaction [R. Thews, Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, 1966 (to be published in Phys. Rev.)]. He mainly investigated the energy dependence of $d\sigma/dt$ and does not concern himself with the detailed kinematic structure in $d\sigma/dt$.

[7] $\sin\theta_t=2[t\phi(s,t)]^{1/2}/\mathcal{T}_{ac}\mathcal{T}_{bd}$, where

$$\phi(s,t)\equiv st(\sum m_i^2-s-t)-s(m_b^2-m_d^2)(m_a^2-m_c^2)-t(m_a^2-m_b^2)(m_c^2-m_d^2)-(m_a^2m_d^2-m_c^2m_b^2)(m_a^2+m_d^2-m_c^2-m_b^2),$$

$\mathcal{T}_{ac}^2\equiv4tp_{ac}^2$, and $\mathcal{T}_{bd}^2=4tp_{bd}$, for $a+b\to c+d$ being the s channel and $D+b\to c+A$ being the t channel.

[8] Notice that the residue functions have the usually assumed threshold behavior at $t=4m_\pi^2$ and $t=(m_\Delta+m_N)^2$; i.e., $\beta(t)\propto[t-(m_\Delta+m_N)]^L[t-4m_\pi^2]^{L'}$, where L, L' is the lowest orbital angular momentum for a fixed J_t of the $\bar{N}\Delta$ and $\pi\pi$ systems, respectively. We find that this is true for all the interactions considered in this paper.

[9] At integral values of α (and their symmetric points about $\alpha=-\frac{1}{2}$) nonsense-sense residue functions vanish in a square-root fashion. For example, if $\alpha=0$ is such a point, $\beta_{\text{sn}}\propto[\alpha(\alpha+1)]^{1/2}$, where s stands for sense and n for nonsense. The factorizability of the residue

$$\beta_{00;\frac{3}{2}-\frac{1}{2}}(t) \propto [\alpha(\alpha+1)(\alpha-1)(\alpha+2)]^{1/2}[t-(m_\Delta+m_N)^2]^{1/2}(t-4m_\pi^2)(p_{\pi\pi}p_{\Delta\bar{N}})^{\alpha-2}, \tag{2b}$$

$$\beta_{00;\frac{1}{2}\frac{1}{2}}(t) \propto [t-(m_\Delta+m_N)^2]^{-1/2}[t-(m_\Delta-m_N)^2]^{-1}(p_{\pi\pi}p_{\bar{N}\Delta})^{\alpha}, \tag{2c}$$

$$\beta_{00;\frac{1}{2}-\frac{1}{2}}(t) \propto [\alpha(\alpha+1)]^{1/2}[t-(m_\Delta-m_N)^2]^{-1/2}(t-4m_\pi^2)^{1/2}(p_{\pi\pi}p_{\bar{N}\Delta})^{\alpha-1}. \tag{2d}$$

Notice that the kinematic factors of $\beta_{00;\frac{3}{2}\frac{1}{2}}(t)$ and $\beta_{00;\frac{1}{2}-\frac{1}{2}}(t)$ are the same. For fitting we can put them together in Eq. (1). Using the asymptotic form of the E functions[10] and after some rearrangements, we obtain a formula which is suitable for phenomenological fitting :

$$\begin{aligned}\frac{d\sigma}{dt}=&\frac{1}{\mathcal{S}_{\pi N}{}^2}|1-\exp(-i\pi\alpha)|^2\left(\frac{1}{\Gamma(\alpha+1)\sin\pi\alpha}\right)^2\{|\sin\theta_t|^2\alpha^2|(4m_\pi^2-t)^{1/2}[(m_\Delta-m_N)^2-t]^{-1/2}|^2\\&\times[|\gamma_{00;\frac{3}{2}\frac{1}{2}}(t)|^2+|\gamma_{00;\frac{1}{2}-\frac{1}{2}}(t)|^2](s/s_0)^{2\alpha-2}+|(\sin\theta_t)^2|^2\alpha^2(\alpha-1)^2|(t-4m_\pi^2)[t-(m_\Delta+m_N)^2]^{1/2}|^2\\&\times|\gamma_{00;\frac{3}{2}-\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha-4}+|[t-(m_\Delta-m_N)^2]^{-1}[t-(m_\Delta+m_N)^2]^{-1/2}|^2|\gamma_{00;\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha}\},\end{aligned} \tag{3}$$

where $\mathcal{S}_{\pi N}{}^2=4sp_{\pi N}{}^2=[s-(m_N+m_\pi)^2][s-(m_N-m_\pi)^2]$. The γ's are defined through Eqs. (1) and (3). Each is a product of the corresponding β without those kinematic singularities, $(2\alpha+1)\Gamma(\alpha+\frac{1}{2})/(\pi)^{1/2}$ of the E functions and constants. The poles of $(2\alpha+1)\Gamma(\alpha+\frac{1}{2})$ at $-\frac{3}{2}$, $-\frac{5}{2}$, etc. are canceled by the zeros of β at these points.[10,11] So the γ's are analytic functions of t for $t<0$. Notice that $1/\Gamma(\alpha+1)\sin\pi\alpha$ is finite at all negative integral values of α. Therefore the odd signature factor in Eq. (3) gives zeros at negative even integers of α. In the rest of the paper, the γ's will be similarly defined. The s_0 is an arbitrary positive scale constant. Usually s_0 is chosen so as to minimize the rate of variation of the γ's as functions of t.[12] The same notation s_0 will be used throughout the paper for all interactions, even though the values of s_0 need not be the same. Notice that $\sin\theta_t=0$ and $|\cos\theta_t|=1$ in the forward direction of the s channel. We assert that the Regge asymptotic behavior holds in the s-channel forward direction for the amplitudes which are free of kinematic singularities.[13] That is why we keep the kinematic factors of $\sin\theta_t$ as they stand, and approximate the functions $E_{\lambda\mu}{}^{\alpha,+}(\cos\theta_t)$ in Eq. (1) by their leading asymptotic terms according to Ref. 10. The same assertion is applied throughout this paper.

B. $\pi N\to\omega\Delta(1236)$

Only ρ can be exchanged in this interaction and the ρ can couple to $\pi\omega$ only when ω is in helicity state one.[8] The differential cross section is

$$\begin{aligned}\frac{d\sigma}{dt}=&\frac{1}{4\pi sp_{\pi N}{}^2}\frac{1}{(2\times\frac{1}{2}+1)}\times2\times\tfrac{1}{4}\{2[1+(\cos\theta_t)^2]|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{10;\frac{3}{2}\frac{1}{2}}(t)E_{11}{}^{\alpha,+}(\cos\theta_t)|^2\\&+2|\sin\theta_t|^2[1+(\cos\theta_t)^2]|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{10;\frac{3}{2}-\frac{1}{2}}(t)E_{21}{}^{\alpha,+}(\cos\theta_t)|^2+2|\sin\theta_t|^2\\&\times|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{10;\frac{1}{2}\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2+2[1+(\cos\theta_t)^2]\\&\times|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{10;\frac{1}{2}-\frac{1}{2}}(t)E_{11}{}^{\alpha,+}(\cos\theta_t)|^2\}.\end{aligned} \tag{4}$$

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function says that $\beta_{\rm sn}{}^2=\beta_{\rm ss}\beta_{\rm nn}$. Therefore either $\beta_{\rm ss}$ or $\beta_{\rm nn}$ has a factor $\alpha(\alpha+1)$. Which one is chosen is a dynamical question. Throughout then $\beta_{\rm sn}\propto[\alpha(\alpha+1)]^{1/2}$, $\beta_{\rm ss}\propto1$, $\beta_{\rm nn}\propto[\alpha(\alpha+1)]$. If it couples to nonsense-nonsense channels, then $\beta_{\rm sn}\propto[\alpha(\alpha+1)]^{1/2}$, $\beta_{\rm ss}\propto\alpha(\alpha+1)$, $\beta_{\rm nn}\propto1$. For example, the $\beta_{00;\frac{1}{2}\frac{1}{2}}$ of Eq. (2c) should than have a factor $\alpha(\alpha+1)$; the $\beta_{10;\frac{1}{2}\frac{1}{2}}$ of Eq. (5a) and $\beta_{10;\frac{1}{2}-\frac{1}{2}}$ of Eq. (5b) should not have a factor $\alpha(\alpha+1)$; $\beta_{20;\frac{1}{2}-\frac{1}{2}}{}^{\rho}$ of Eq. (9c) and $\beta_{10;\frac{1}{2}-\frac{1}{2}}{}^{\rho}$ of Eq. (9f) should not have the factor $\alpha_\rho(\alpha_\rho+1)$; $\beta_{\frac{3}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}$ of Eq. (23c) should have a factor $\alpha(\alpha+1)$, and $\beta_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}$ of Eq. (23d) and $\beta_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}$ of Eq. (22) should not have the factor $\alpha(\alpha+1)$. Also the differential cross sections should be changed accordingly. Hopefully, by fitting the experimental data, we can determine which is the correct coupling.

[10] The asymptotic forms of the E functions used in this paper are:

$$E_{00}{}^{\alpha,+}(\cos\theta_t)\approx\frac{\Gamma(\alpha+\frac{1}{2})}{(\pi)^{1/2}\Gamma(\alpha+1)}\left(\frac{s}{p_tp_t'}\right)^{\alpha},\qquad E_{02}{}^{\alpha,+}(\cos\theta_t)\approx\frac{4\alpha(\alpha-1)}{[\alpha(\alpha+1)(\alpha-1)(\alpha+2)]^{1/2}}\frac{\Gamma(\alpha+\frac{1}{2})}{(\pi)^{1/2}\Gamma(\alpha+1)}\left(\frac{s}{p_tp_t'}\right)^{\alpha-2},$$

$$E_{01}{}^{\alpha,+}(\cos\theta_t)\approx\frac{2\alpha}{[\alpha(\alpha+1)]^{1/2}}\frac{\Gamma(\alpha+\frac{1}{2})}{(\pi)^{1/2}\Gamma(\alpha+1)}\left(\frac{s}{p_tp_t'}\right)^{\alpha-1},\qquad E_{11}{}^{\alpha,+}(\cos\theta_t)\approx\frac{2\alpha^2}{\alpha(\alpha+1)}\frac{\Gamma(\alpha+\frac{1}{2})}{(\pi)^{1/2}\Gamma(\alpha+1)}\left(\frac{s}{p_tp_t'}\right)^{\alpha-1},$$

$$E_{12}{}^{\alpha,+}(\cos\theta_t)\approx\frac{4\alpha^2(\alpha-1)}{\alpha(\alpha+1)[(\alpha-1)(\alpha+2)]^{1/2}}\frac{\Gamma(\alpha+\frac{1}{2})}{(\pi)^{1/2}\Gamma(\alpha+1)}\left(\frac{s}{p_tp_t'}\right)^{\alpha-2}.$$

The E functions have poles at all half-integers of α. When a physical trajectory passes a positive half-integer, the pole of the E function in the amplitude will be either canceled by its compensating trajectory passing $-\alpha-1$ or its residue function vanishes (Ref. 11). However the leading term at high $\cos\theta_t$ in the E has poles only at negative half-integers. Thus the vanishing of the residue can give a zero in the asymptotic form of the amplitude. When the leading trajectory passes a negative half-integer, there obviously can not be any compensating trajectory. Therefore it is the vanishing of the residue function that cancels the poles at negative half-integers. The author would like to thank Professor S. Mandelstam for informing her that the compensating trajectory always exists for a trajectory passing half-integers in the potential theory.

[11] S. Mandelstam, Ann. Phys. **19**, 254 (1959).

[12] See F. Arbab and C. Chiu's paper in Ref. 1.

[13] With the discovery of the daughter trajectories, D. Z. Freedman and J. M. Wang [Phys. Rev. **153**, 1596 (1967)] have shown that this is true. (Related references are given in the paper.)

The kinematic factors of the residue functions are

$$\beta_{10;\frac{3}{2}\frac{1}{2}}(t) \quad \text{and} \quad \beta_{10;\frac{1}{2}-\frac{1}{2}}(t) \propto \alpha(\alpha+1)[t-(m_\Delta-m_N)^2]^{-1/2}\mathcal{T}_{\pi\omega}(p_{\pi\omega}p_{\bar{N}\Delta})^{\alpha-1}, \tag{5a}$$

$$\beta_{10;\frac{3}{2}-\frac{1}{2}}(t) \propto \alpha(\alpha+1)[(\alpha-1)(\alpha+2)]^{1/2}t^{-1/2}[t-(m_\Delta+m_N)^2]^{1/2}\mathcal{T}_{\pi\omega}{}^2(p_{\pi\omega}p_{\bar{N}\Delta})^{\alpha-2}, \tag{5b}$$

$$\beta_{10;\frac{1}{2}\frac{1}{2}}(t) \propto [\alpha(\alpha+1)]^{1/2}t^{-1/2}[t-(m_\Delta-m_N)^2]^{-1/2}\mathcal{T}_{\pi\omega}(p_{\pi\omega}p_{\bar{N}\Delta})^{\alpha-1}, \tag{5c}$$

where $\mathcal{T}_{\pi\omega}{}^2 \equiv 4tp_{\pi\omega}{}^2=[t-(m_\pi+m_\omega)^2][t-(m_\pi-m_\omega)^2]$. The factor of $\alpha(\alpha+1)$ in Eqs. (5a) and (5b) arises from the assumption that the ρ trajectory couples to a sense-sense channel.[9] Notice that by factorizability, these residue functions are related to those of $\pi N \to \pi\Delta(1236)$, i.e.,

$$\frac{\beta_{10;N\Delta}(\pi\omega \leftarrow \bar{N}\Delta)}{\beta_{10;N'\Delta'}(\pi\omega \leftarrow \bar{N}\Delta)}=\frac{\beta_{00;\bar{N}\Delta}(\pi\pi \leftarrow \bar{N}\Delta)}{\beta_{00;\bar{N}'\Delta'}(\pi\pi \leftarrow \bar{N}\Delta)}. \tag{6}$$

Therefore in addition to the pure-t kinematic factors of the full helicity amplitude given by Ref. 5, *the factorizability relation of Eq. (6) can require additional kinematic factors of the residue functons.* These additional factors are always analytic. All the β kinematic factors of all the interactions considered in this paper are obtained with consideration of this additional constraint given by the factorizability of the residue functions. The final form of the differential cross section, suitable for phenomenological fitting, is

$$\begin{aligned}\frac{d\sigma}{dt}=\frac{1}{s_{\pi N}{}^2}|1-\exp(-i\pi\alpha)|^2\left(\frac{1}{\Gamma(\alpha+1)\sin\pi\alpha}\right)^2\{[1+(\cos\theta_t)^2]\alpha^4|[t-(m_\Delta-m_N)^2]^{-1/2}\mathcal{T}_{\pi\omega}|^2[|\gamma_{10;\frac{3}{2}\frac{1}{2}}(t)|^2\\ +|\gamma_{10;\frac{1}{2}-\frac{1}{2}}(t)|^2](s/s_0)^{2\alpha-2}+|t^{-1/2}\sin\theta_t|^2[1+(\cos\theta_t)^2]\alpha^4(\alpha-1)^2|\mathcal{T}_{\pi\omega}{}^2[t-(m_\Delta+m_N)^2]^{1/2}|^2|\gamma_{10;\frac{3}{2}-\frac{1}{2}}(t)|^2\\ \times(s/s_0)^{2\alpha-4}+|t^{-1/2}\sin\theta_t|^2\alpha^2|[t-(m_\Delta-m_N)^2]^{-1/2}\mathcal{T}_{\pi\omega}|^2|\gamma_{10;\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha-2}\},\end{aligned} \tag{7}$$

where the γ's are defined through Eqs. (4) and (7). Notice that a factor α appears in every term of Eq. (7). Therefore the differential cross section has a minimum where $\alpha(t)=0$.

C. $\pi N \to A_2 N$

For non-charge-exchange $\pi N \to A_2N$, the Pomeranchuk P, P' and the ρ trajectories can be exchanged. None can couple to the πA_2 system with A_2 in the helicity zero state.

$$\begin{aligned}\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{\pi N}{}^2}\frac{1}{(2\times\frac{1}{2}+1)}\times 2\times\tfrac{1}{4}\{2|(\sin\theta_t)^2|^2|[1+\exp(-i\pi\alpha_P)][(2\alpha_P+1)/\sin\pi\alpha_P]\beta_{20;\frac{1}{2}\frac{1}{2}}{}^P(t)E_{02}{}^{\alpha_P,+}(\cos\theta_t)\\ +[1-\exp(-i\pi\alpha_\rho)][(2\alpha_\rho+1)/\sin\pi\alpha_\rho]\beta_{20;\frac{1}{2}\frac{1}{2}}{}^\rho(t)E_{02}{}^{\alpha_\rho,+}(\cos\theta_t)|^2+2|\sin\theta_t|^2[1+(\cos\theta_t)^2]\\ \times|[1+\exp(-i\pi\alpha_\rho)][(2\alpha_\rho+1)/\sin\pi\alpha_\rho]\beta_{20;\frac{1}{2}-\frac{1}{2}}{}^P(t)E_{1,2}{}^{\alpha_\rho,+}(\cos\theta_t)+[1-\exp(-i\pi\alpha_\rho)][(2\alpha_\rho+1)/\sin\pi\alpha_\rho]\\ \times\beta_{20;\frac{1}{2}-\frac{1}{2}}{}^\rho(t)E_{1,2}{}^{\alpha_\rho,+}(\cos\theta_t)|^2+2|\sin\theta_t|^2|[1+\exp(-i\pi\alpha_\rho)][(2\alpha_\rho+1)/\sin\pi\alpha_\rho]\beta_{10;\frac{1}{2}\frac{1}{2}}{}^P(t)E_{01}{}^{\alpha_\rho,+}(\cos\theta_t)+\cdots|^2\\ +2[1+(\cos\theta_t)^2]|[1+\exp(-i\pi\alpha_\rho)][(2\alpha_\rho+1)/\sin\pi\alpha_\rho]\beta_{10;\frac{1}{2}-\frac{1}{2}}{}^P(t)E_{11}{}^{\alpha_\rho,+}(\cos\theta_t)\\ +[1-\exp(-i\pi\alpha_\rho)][(2\alpha_\rho+1)/\sin\pi\alpha_\rho]\beta_{10;\frac{1}{2}-\frac{1}{2}}{}^\rho(t)E_{11}{}^{\alpha_\rho,+}(\cos\theta_t)|^2\}.\end{aligned} \tag{8}$$

The contribution from the P' trajectory gives exactly the same form as that from the P trajectory. The kinematic factors of the residue functions are[14]

$$\beta_{20;\frac{1}{2}\frac{1}{2}}{}^P(t) \quad \text{and} \quad \beta_{20;\frac{1}{2}\frac{1}{2}}{}^\rho(t) \propto [\alpha(\alpha+1)(\alpha-1)(\alpha+2)]^{1/2}\mathcal{T}_{\pi A}(t-4m_N{}^2)^{1/2}(p_{\pi A}p_{N\bar{N}})^{\alpha-2}, \tag{9a}$$

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[14] In this paper all the formulas involving P, P', and A_2 are written according to the mechanism proposed by Gell-Mann to cancel the pole at $\alpha=0$ in the amplitudes: these trajectories all couple to nonsense-nonsense channels at $\alpha=0$ and the pole in the nonsense-nonsense channels is canceled by another trajectory passing $\alpha=-1$, {i.e., $\beta_{\text{sn}} \propto [\alpha(\alpha+1)]^{1/2}$, $\beta_{\text{ss}} \propto \alpha(\alpha+1)$, $\beta_{\text{nn}} \propto 1$}. An alternative mechanism is to have the pole in the nonsense-nonsense channels annihilated by a zero of the residue functions, [i.e., $\beta_{\text{sn}} \propto \alpha[\alpha(\alpha+1)]^{1/2}$, $\beta_{\text{ss}} \propto \alpha^2(\alpha+1)$, $\beta_{\text{nn}} \propto \alpha$].

G. F. Chew has proposed a different mechanism [G. F. Chew, Phys. Rev. Letters **16**, 60 (1966)], in which the trajectories couple to the sense-sense channel at $\alpha=0$, but β_{ss} vanishes there, i.e., $\beta_{\text{sn}} \propto \alpha[\alpha(\alpha+1)]^{1/2}$, $\beta_{\text{ss}} \propto \alpha$, $\beta_{\text{nn}} \propto \alpha^2(\alpha+1)$. J. Finkelstein and Boris Kayser pointed out that in potential theory the coupled sense-sense residue function cannot change sign for $t<$ threshold. Thus if β_{ss} vanishes, it must have a double zero, [i.e., $\beta_{\text{sn}} \propto \alpha[\alpha(\alpha+1)]^{1/2}$, $\beta_{\text{ss}} \propto \alpha^2$, $\beta_{\text{nn}} \propto \alpha(\alpha+1)$]. However, for $t<0$ nonrelativistic arguments are of dubious value. In the relativistic case, the kinematic singularity free residue functions are real analytic and have right-hand cuts. This does not rule out the possibility of their changing sign for $t<0$. Actually there are experimental indications that the residue functions need to change sign to explain certain aspects of πN and NN scattering [R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965); E. Leader and R. C. Slansky, *ibid.* **148**, 1491 (1966)]. The author would like to thank Professor Chew and Dr. Finkelstein for discussions.

where α is α_P(or α_ρ) for β^P (or β^ρ);

$$\beta_{20;\frac{1}{2}-\frac{1}{2}}{}^P(t) \propto [(\alpha_\rho-1)(\alpha_\rho+2)]^{1/2} t^{1/2} \mathcal{T}_{\pi A}(t-4m_N^2)^{1/2}(p_{\pi A}p_{N\bar{N}})^{\alpha_\rho-2}, \tag{9b}$$

$$\beta_{20;\frac{1}{2}-\frac{1}{2}}{}^\rho(t) \propto \alpha_\rho(\alpha_\rho+1)[(\alpha_\rho-1)(\alpha_\rho+2)]^{1/2} t^{1/2} \mathcal{T}_{\pi A}(t-4m_N^2)^{1/2}(p_{\pi A}p_{N\bar{N}})^{\alpha_\rho-2}, \tag{9c}$$

$$\beta_{10;\frac{1}{2}\frac{1}{2}}{}^P(t) \quad \text{and} \quad \beta_{10;\frac{1}{2}\frac{1}{2}}{}^\rho(t) \propto [\alpha(\alpha+1)]^{1/2}(p_{\pi A}p_{N\bar{N}})^{\alpha-1}, \tag{9d}$$

$$\beta_{10;\frac{1}{2}-\frac{1}{2}}{}^P(t) \propto t^{1/2}(p_{\pi A}p_{N\bar{N}})^{\alpha_\rho-1}, \tag{9e}$$

$$\beta_{10;\frac{1}{2}-\frac{1}{2}}{}^\rho(t) \propto \alpha_\rho(\alpha_\rho+1) t^{1/2}(p_{\pi A}p_{N\bar{N}})^{\alpha_\rho-1}. \tag{9f}$$

The difference of a factor of $\alpha(\alpha+1)$ between Eqs. (9b) and (9c), and between Eqs. (9e) and (9f), arises from the assumption that the P trajectory couples to nonsense-nonsense channels[14] [the amplitudes do not have a pole at $\alpha_P(t)=0$], while the ρ trajectory couples to sense-sense amplitudes[9] at $\alpha=0$. After some rearrangement we obtain

$$\begin{aligned}
\frac{d\sigma}{dt}=\frac{1}{s_{\pi N}{}^2}\Bigg\{ &|(\sin\theta_t)^2|^2|\mathcal{T}_{\pi A}(t-4m_N^2)^{1/2}|^2 \left|[1+\exp(-i\pi\alpha_\rho)]\frac{\alpha_\rho(\alpha_\rho-1)}{\Gamma(\alpha_\rho+1)\sin\pi\alpha_\rho}\gamma_{20;\frac{1}{2}\frac{1}{2}}{}^P(t)\left(\frac{s}{s_0}\right)^{\alpha_\rho-2}\right. \\
&\left.+[1-\exp(-i\pi\alpha_\rho)]\frac{\alpha_\rho(\alpha_\rho-1)}{\Gamma(\alpha_\rho+1)\sin\pi\alpha_\rho}\gamma_{20;\frac{1}{2}\frac{1}{2}}{}^\rho(t)\left(\frac{s}{s_0}\right)^{\alpha_\rho-2}\right|^2+|\sin\theta_t|^2[1+(\cos\theta_t)^2]|t^{1/2}\mathcal{T}_{\pi A}(t-4m_N^2)^{1/2}|^2 \\
&\times\left|[1+\exp(-i\pi\alpha_\rho)]\frac{\alpha_\rho(\alpha_\rho-1)}{(\alpha_\rho+1)\Gamma(\alpha_\rho+1)\sin\pi\alpha_\rho}\gamma_{20;\frac{1}{2}-\frac{1}{2}}{}^P(t)\left(\frac{s}{s_0}\right)^{\alpha_\rho-2}+[1-\exp(-i\pi\alpha_\rho)]\frac{\alpha_\rho{}^2(\alpha_\rho-1)}{\Gamma(\alpha_\rho+1)\sin\pi\alpha_\rho}\right. \\
&\left.\times\gamma_{20;\frac{1}{2}-\frac{1}{2}}{}^\rho(t)\left(\frac{s}{s_0}\right)^{\alpha_\rho-2}\right|^2+|\sin\theta_t|^2\left|[1+\exp(-i\pi\alpha_\rho)]\frac{\alpha_\rho}{\Gamma(\alpha_\rho+1)\sin\pi\alpha_\rho}\gamma_{10;\frac{1}{2}\frac{1}{2}}{}^P(t)\left(\frac{s}{s_0}\right)^{\alpha_\rho-1}+[1-\exp(-i\pi\alpha_\rho)]\right. \\
&\left.\times\frac{\alpha_\rho}{\Gamma(\alpha_\rho+1)\sin\pi\alpha_\rho}\gamma_{10;\frac{1}{2}\frac{1}{2}}{}^\rho(t)\left(\frac{s}{s_0}\right)^{\alpha_\rho-1}\right|^2+[1+(\cos\theta_t)^2]|t|\left|[1+\exp(-i\pi\alpha_\rho)]\frac{\alpha_\rho}{(\alpha_\rho+1)\Gamma(\alpha_\rho+1)\sin\pi\alpha_\rho}\right. \\
&\left.\times\gamma_{10;\frac{1}{2}-\frac{1}{2}}{}^P(t)\left(\frac{s}{s_0}\right)^{\alpha_\rho-1}+[1-\exp(-i\pi\alpha_\rho)]\frac{\alpha_\rho{}^2}{\Gamma(\alpha_\rho+1)\sin\pi\gamma_\rho}\gamma_{10;\frac{1}{2}-\frac{1}{2}}{}^\rho(t)\left(\frac{s}{s_0}\right)^{\alpha_\rho-1}\right|^2\Bigg\},
\end{aligned} \tag{10}$$

where the γ's are defined through Eqs. (8) and (10). The γ's are related to those of $\pi N\to\pi N$ scattering by[15]

$$\frac{\beta_{A0;\frac{1}{2}\frac{1}{2}}(\pi A_2\leftarrow N\bar{N})}{\beta_{A0;\frac{1}{2}-\frac{1}{2}}(\pi A_2\leftarrow N\bar{N})}=\frac{\beta_{00;\frac{1}{2}\frac{1}{2}}(\pi\pi\leftarrow N\bar{N})}{\beta_{00;\frac{1}{2}-\frac{1}{2}}(\pi\pi\leftarrow N\bar{N})}.$$

The contribution of the P' trajectory is exactly the same as that from the P. One can see how hopeless it is to do fitting with non-charge-exchange $\pi N\to A_2N$ interaction. However there is one interesting point. Notice that in the first and the second term of Eq. (10) there is a factor (α_P-1). Since $\alpha_P(0)=1$, their contribution in the s-channel forward direction will be very small. Also notice that every term in Eq. (10) has kinematic factors of sines and cosines of θ_t which do not increase with energy s (i.e., $\sin\theta_t=0$ and $|\cos\theta_t|=1$) in the forward direction of s channel; therefore the Regge trajectory, especially the P, cannot contribute with full strength in the forward direction of s channel and thus the production of A_2 in very high energies will be more limited than if P could contribute with full strength.[16] For the charge-exchange $\pi N\to A_2N$ interaction, only ρ contributes. Since every term in Eq. (10) has a factor α_ρ, the differential cross section has a minimum at $\alpha_\rho=0$.

[15] See Ref. 11 of Ref. 3.

[16] See Appendix III.

D. $\pi N\to\eta N$

Of the known trajectories, only that of A_2 can be exchanged here.[17]

$$\begin{aligned}
\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{\pi N}{}^2}\tfrac{1}{2}\times 2\times\tfrac{1}{4}\{ &|[1+\exp(-i\pi\alpha)] \\
&\times[(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{1}{2}\frac{1}{2}}(t)E_{00}{}^{\alpha,+}(\cos\theta_t)|^2 \\
&+|\sin\theta_t[1+\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha] \\
&\times\beta_{00;\frac{1}{2}-\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2\}.
\end{aligned} \tag{11}$$

The kinematic factors of the residue functions are[14]

$$\beta_{00;\frac{1}{2}\frac{1}{2}}(t)\propto\alpha(\alpha+1)(t-4m_N^2)^{-1/2}(p_{\pi\eta}p_{N\bar{N}})^\alpha, \tag{12a}$$

$$\beta_{00;\frac{1}{2}-\frac{1}{2}}(t)\propto[\alpha(\alpha+1)]^{1/2}\mathcal{T}_{\pi\eta}(p_{\pi\eta}p_{\bar{N}N})^{\alpha-1}. \tag{12b}$$

[17] R. J. N. Phillips and W. Rarita studied this interaction [Phys. Rev. Letters 15, 807 (1965); 15, 938(E) 1965)]. They applied the kinematics of $\pi N\to\pi N$ to this case; i.e., instead of $|\mathcal{T}_{\pi\eta}\sin\theta_t(\pi\eta\leftarrow N\bar{N})|^2$ in the second term of Eq. (13), they used $|\mathcal{T}_{\pi\pi}\sin\theta_t(\pi\pi\leftarrow N\bar{N})|^2$. The author found that numerically the two can differ by a factor of 2 for small $|t|$.

The final form of the differential cross section is

$$\frac{d\sigma}{dt}=\frac{1}{\mathcal{S}_{\pi N}{}^2}|1+\exp(-i\pi\alpha)|^2\left[\frac{\alpha}{\Gamma(\alpha+1)\,\sin\pi\alpha}\right]^2 \times\{(\alpha+1)^2|(t-4m_N{}^2)^{-1/2}|^2|\gamma_{00;\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha} +|\sin\theta_t|^2|\mathcal{T}_{\pi\eta}|^2|\gamma_{00;\frac{1}{2}\frac{1}{2}-}(t)|^2(s/s_0)^{2\alpha-2}\}, \quad (13)$$

where the γ's are defined through Eqs. (11) and (13). They are smoothly varying functions of t for $t<0$. Since $\alpha/\Gamma(\alpha+1)\sin\pi\alpha$ is finite at negative integers of α, the vanishing of the signature factor $|1+\exp(-i\pi\alpha)|^2$ at negative odd integers of α gives minimums in the differential cross sections at these points.

E. $\pi N\rightarrow\eta\Delta(1236)$

Here only the A_2 trajectory can be exchanged.

$$\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{\pi N}{}^2}\tfrac{1}{2}\times 2\times\tfrac{1}{4}\{|\sin\theta_t[1+\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{3}{2}\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2+|(\sin\theta_t)^2[1+\exp(-i\pi\alpha)] \times[(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{3}{2}-\frac{1}{2}}(t)E_{02}{}^{\alpha,+}(\cos\theta_t)|^2+|[1+\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{1}{2}\frac{1}{2}}(t)E_{00}{}^{\alpha,+}(\cos\theta_t)|^2 +|\sin\theta_t[1+\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{00;\frac{1}{2}-\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2\}. \quad (14)$$

The kinematic factors of the residue functions are

$$\beta_{00;\frac{3}{2}\frac{1}{2}}(t)\quad\text{and}\quad\beta_{00;\frac{1}{2}-\frac{1}{2}}(t)\propto[\alpha(\alpha+1)]^{1/2}t^{-1/2}[t-(m_\Delta-m_N)^2]^{1/2}\mathcal{T}_{\pi\eta}(p_{\pi\eta}p_{\bar{N}\Delta})^{\alpha-1}, \quad (15a)$$

$$\beta_{00;\frac{3}{2}-\frac{1}{2}}(t)\propto[\alpha(\alpha+1)(\alpha-1)(\alpha+2)]^{1/2}t^{-1}[t-(m_\Delta-m_N)^2][t-(m_\Delta+m_N)^2]^{1/2}\mathcal{T}_{\pi\eta}{}^2(p_{\pi\eta}p_{\bar{N}\Delta})^{\alpha-2}, \quad (15b)$$

and

$$\beta_{00;\frac{1}{2}\frac{1}{2}}(t)\propto\alpha(\alpha+1)[t-(m_\Delta-m_N)^2]^{-1/2}(p_{\pi\eta}p_{N\Delta})^{\alpha}. \quad (15c)$$

After some rearrangements we obtain

$$\frac{d\sigma}{dt}=\frac{1}{\mathcal{S}_{\pi N}{}^2}|1+\exp(-i\pi\alpha)|^2\left[\frac{\alpha}{\Gamma(\alpha+1)\,\sin\pi\alpha}\right]^2\{|t^{-1/2}\sin\theta_t|^2[t-(m_\Delta-m_N)^2]^{1/2}\mathcal{T}_{\pi\eta}|^2[|\gamma_{00;\frac{3}{2}\frac{1}{2}}(t)|^2+|\gamma_{00;\frac{1}{2}-\frac{1}{2}}(t)|^2] \times(s/s_0)^{2\alpha-2}+|t^{-1}(\sin\theta_t)^2|^2(\alpha-1)^2|[t-(m_\Delta-m_N)^2][t-(m_\Delta+m_N)^2]^{1/2}\mathcal{T}_{\pi\eta}{}^2|^2|\gamma_{00;\frac{3}{2}-\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha-4} +(\alpha+1)^2|[t-(m_\Delta+m_N)^2]^{-1/2}|^2|\gamma_{00;\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha}\}. \quad (16)$$

The γ's are defined through Eqs. (14) and (16). Notice that the differential cross section has minimums at negative odd integers of α.

F. $\pi N\rightarrow\rho N$

For charge-exchange $\pi N\rightarrow\rho N$ interaction, A_2 and π can be exchanged. The A_2 can couple to $\pi\rho$ only when ρ is in helicity state one. Because of conservation of G parity, the π can couple to the $N\bar{N}$ system only when they are in the same helicity states. It turns out that there is no interference between the contributions from the two trajectories.

$$\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{\pi N}{}^2}\tfrac{1}{2}\times 2\times\tfrac{1}{4}\{|[1+\exp(-i\pi\alpha_\pi)][(2\alpha_\pi+1)/\sin\pi\alpha_\pi]\beta_{00;\frac{1}{2}\frac{1}{2}}{}^{\pi}(t)E_{00}{}^{\alpha_\pi,+}(\cos\theta_t)|^2+2|\sin\theta_t|^2|[1+\exp(-i\pi\alpha_\pi)] \times[(2\alpha_\pi+1)/\sin\pi\alpha_\pi]\beta_{10;\frac{1}{2}\frac{1}{2}}{}^{\pi}(t)E_{01}{}^{\alpha_\pi,+}(\cos\theta_t)|^2+2|\sin\theta_t|^2|[1+\exp(-i\pi\alpha_A)][(2\alpha_A+1)/\sin\pi\alpha_A] \times\beta_{10;\frac{1}{2}\frac{1}{2}}{}^{A}(t)E_{01}{}^{\alpha_A,+}(\cos\theta_t)|^2+2[1+(\cos\theta_t)^2]|[1+\exp(-i\pi\alpha_A)][(2\alpha_A+1)/\sin\pi\alpha_A] \times\beta_{10;\frac{1}{2}-\frac{1}{2}}{}^{A}(t)E_{11}{}^{\alpha_A,+}(\cos\theta_t)|^2\}. \quad (17)$$

The kinematic factors of the residue functions are[14]

$$\beta_{00;\frac{1}{2}\frac{1}{2}}{}^{\pi}(t)\propto t^{1/2}\mathcal{T}_{\pi\rho}{}^{-1}(p_{\pi\rho}p_{N\bar{N}})^{\alpha_\pi}, \quad (18a)$$

$$\beta_{10;\frac{1}{2}\frac{1}{2}}{}^{\pi}(t)\propto[\alpha_\pi(\alpha_\pi+1)]^{1/2}t^{1/2}(t-4m_N{}^2)^{1/2}(p_{\pi\rho}p_{N\bar{N}})^{\alpha_\pi-1}, \quad (18b)$$

$$\beta_{10;\frac{1}{2}\frac{1}{2}}{}^{A}(t)\propto[\alpha_A(\alpha_A+1)]^{1/2}\mathcal{T}_{\pi\rho}(p_{N\bar{N}})^{\alpha_A-1}, \quad (18c)$$

$$\beta_{10;\frac{1}{2}-\frac{1}{2}}{}^{A}(t)\propto t^{1/2}\mathcal{T}_{\pi\rho}(p_{\pi\rho}p_{N\bar{N}})^{\alpha_\pi-1}. \quad (18d)$$

Here we assume that the π trajectory couples to the sense-sense channels at $\alpha_\pi=0$.

$$\frac{d\sigma}{dt}=\frac{1}{\mathcal{S}_{\pi N}{}^2}\Bigg\{|1+\exp(-i\pi\alpha_\pi)|^2\left(\frac{1}{\Gamma(\alpha+1)\sin\pi\alpha_\pi}\right)^2\Big[|t^{1/2}\mathcal{T}_{\pi\rho}{}^{-1}|^2|\gamma_{00;\frac{1}{2}\frac{1}{2}}{}^\pi(t)|^2(s/s_0)^{2\alpha_\pi}+|\sin\theta_t|^2\alpha_\pi{}^2|t^{1/2}(t-4m_N{}^2)^{1/2}|^2$$
$$\times|\gamma_{10;\frac{1}{2}\frac{1}{2}}{}^\pi(t)|^2(s/s_0)^{2\alpha_\pi-2}\Big]+|1+\exp(-i\pi\alpha_A)|^2\left[\frac{\alpha_A}{(\alpha_A+1)\Gamma(\alpha_A+1)\sin\pi\alpha_A}\right]^2\Big[|\sin\theta_t|^2(\alpha_A+1)|^2|\mathcal{T}_{\pi\rho}|^2$$
$$\times|\gamma_{10;\frac{1}{2}\frac{1}{2}}{}^A(t)|^2(s/s_0)^{2\alpha_A-2}+[1+(\cos\theta_t)^2]|t^{1/2}\mathcal{T}_{\pi\rho}|^2|\gamma_{10;\frac{1}{2}-\frac{1}{2}}{}^A(t)|^2(s/s_0)^{2\alpha_A-2}\Big]\Bigg\},\quad(19)$$

where the γ's are smoothly varying functions of t for $t<0$. Notice that the contribution from π vanishes as the π trajectory passes -1. For high enough energy, the contribution from π trajectory may be neglected. The γ^A's here are related to that of the $\pi N\to\eta N$ interaction through the following equation:

$$\frac{\beta_{10;\frac{1}{2}\frac{1}{2}}{}^A(\pi\rho\leftarrow N\bar{N})}{\beta_{10;\frac{1}{2}-\frac{1}{2}}{}^A(\pi\rho\leftarrow N\bar{N})}=\frac{\beta_{00;\frac{1}{2}\frac{1}{2}}{}^A(\pi\eta\leftarrow N\bar{N})}{\beta_{00;\frac{1}{2}-\frac{1}{2}}{}^A(\pi\eta\leftarrow N\bar{N})}.\quad(20)$$

For non-charge-exchange $\pi N\to\rho N$ interaction, the ω trajectory can also be exchanged. It appears coherently with the A_2.

G. $\pi N\to\rho\Delta(1236)$

Again in this case, A_2 and π can be exchanged. Even though the π trajectory is lower than the A_2, there is still a possibility that the nearness of the π pole to the s-channel physical region makes the contribution from π important up to moderately high energies. In that case we have to consider both π and A_2 exchange. However as energy becomes really high, eventually A_2 will take over. Here we consider only A_2 exchange. The kinematics here are similar to those for $\pi N\to\omega\Delta$ (1236). The differential cross section is in a form exactly the same as Eq. (4) except that the signature factor is $[1+\exp(-i\pi\alpha)]$, and the α and β's refer respectively to the trajectory and the residue functions of A_2. The kinematic factors of the β's for A_2 are the same as given by Eqs. (5a), (5b), and (5c), except for an additional $\alpha(\alpha+1)$ factor, assuming as usual that A_2 couples to nonsense-nonsense amplitudes. The final form of the differential cross section is

$$\frac{d\sigma}{dt}=\frac{1}{\mathcal{S}_{\pi N}{}^2}|1+\exp(-i\pi\alpha)|^2\left[\frac{\alpha}{(\alpha+1)\Gamma(\alpha+1)\sin\pi\alpha}\right]^2\Big\{[1+(\cos\theta_t)^2]|[t-(m_\Delta-m_N)^2]^{-1/2}\mathcal{T}_{\pi\rho}|^2[|\gamma_{10;\frac{3}{2}\frac{1}{2}}(t)|^2$$
$$+|\gamma_{10;\frac{1}{2}-\frac{1}{2}}(t)|^2](s/s_0)^{2\alpha-2}+|t^{-1/2}\sin\theta_t|^2[1+(\cos\theta_t)^2](\alpha-1)^2|\mathcal{T}_{\pi\rho}{}^2[t-(m_\Delta+m_N)^2]^{1/2}|^2|\gamma_{10;\frac{3}{2}-\frac{1}{2}}(t)|^2$$
$$\times(s/s_0)^{2\alpha-4}+|t^{-1/2}\sin\theta_t|^2(\alpha+1)^2|[t-(m_\Delta-m_N)^2]^{-1/2}\mathcal{T}_{\pi\rho}|^2|\gamma_{10;\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha-2}\Big\},\quad(21)$$

where the γ's are defined through Eqs. (4) and (21) and they are smoothly varying functions of t for $t<0$. The γ's here are related to those of $\pi N\to\eta\Delta$ in Eq. (16) through the following equation:

$$\frac{\beta_{10;\Delta\bar{N}}(\pi\rho\leftarrow\Delta\bar{N})}{\beta_{10;\Delta'\bar{N}'}(\pi\rho\leftarrow\Delta\bar{N})}=\frac{\beta_{00;\Delta\bar{N}}(\pi^\eta\leftarrow\Delta\bar{N})}{\beta_{00;\Delta'\bar{N}'}(\pi^\eta\leftarrow\Delta\bar{N})}.$$

Evidently many of the formulas of πN interactions considered here can be used for KN scattering.

III. NN INTERACTIONS

The NN scattering formulas are more complicated than those of πN scattering due to the presence of more particles with spin and more trajectories which can be exchanged. Even though we can write down formulas for all quasi-two-body NN interactions, there will in general be so many arbitrary parameters as to make phenomenological fitting hopeless. We consider only three especially simple cases and for each keep only the highest trajectory that can be exchanged: $NN\to N\Delta(1236)$ with ρ exchange, and $NN\to NN^*(1500,\frac{3}{2}^-)$ and $NN\to NN^*(1480,\frac{1}{2}^+)$ with P exchange. The P' trajectory contributes exactly in the same way as P.

A. $NN\to N\Delta(1236)$

In addition to ρ,π can also be exchanged. [For the same reasons as given for the reaction $\pi N\to\rho\Delta(1236)$, the π contribution may be very important at intermediate energies.] Even with ρ alone there are eight residue functions. We shall see that we can combine some of the residue functions and reduce the number of arbitrary functions from eight to five in practical fitting. The unmodified differential cross section is

$$
\begin{aligned}
\frac{d\sigma}{dt}=&\frac{1}{4\pi s p_{NN}^2}\times\frac{1}{(2+\frac{1}{2}+1)(2\times\frac{1}{2}+1)}\times 2\times\tfrac{1}{4}\{2|\sin\theta_t|^2|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{3}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2\\
&+2|(\sin\theta_t)^2|^2|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)E_{02}{}^{\alpha,+}(\cos\theta_t)|^2+2|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\\
&\times\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)E_{00}{}^{\alpha,+}(\cos\theta_t)|^2+2|\sin\theta_t|^2|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2\\
&+2[1+(\cos\theta_t)^2]|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)E_{11}{}^{\alpha,+}(\cos\theta_t)|^2+2|\sin\theta_t|^2[1+(\cos\theta_t)^2]\\
&\times|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)E_{12}{}^{\alpha,+}(\cos\theta_t)|^2+2|\sin\theta_t|^2|[1-\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\\
&\times\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)E_{10}{}^{\alpha,+}(\cos\theta_t)|^2+2[1+(\cos\theta_t)^2]|[1-\exp(-i\pi\alpha)]\\
&\times[(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)E_{11}{}^{\alpha,+}(\cos\theta_t)|^2\}. \qquad (22)
\end{aligned}
$$

The kinematic factors of the residue functions are[9]

$$\beta_{\frac{3}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t),\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t),\text{ and }\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\propto[\alpha(\alpha+1)]^{1/2}[t-(m_\Delta-m_N)^2]^{-1/2}(p_{\bar{N}N}p_{\bar{N}\Delta})^{\alpha-1}, \qquad (23a)$$

$$\beta_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)\propto[\alpha(\alpha+1)(\alpha-1)(\alpha+2)]^{1/2}(t-4m_N{}^2)^{1/2}[t-(m_\Delta+m_N)^2]^{1/2}(p_{\bar{N}N}p_{\bar{N}\Delta})^{\alpha-2}, \qquad (23b)$$

$$\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)\propto(t-4m_N{}^2)^{-1/2}[t-(m_\Delta+m_N)^2]^{-1/2}[t-(m_\Delta-m_N)^2]^{-1}(p_{\bar{N}N}p_{\bar{N}\Delta})^{\alpha}, \qquad (23c)$$

$$\beta_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}\text{ and }\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\propto\alpha(\alpha+1)t^{1/2}[t-(m_\Delta-m_N)^2]^{-1/2}(p_{\bar{N}N}p_{\bar{N}\Delta})^{\alpha-1}, \qquad (23d)$$

$$\beta_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\propto\alpha(\alpha+1)[(\alpha-1)(\alpha+2)]^{1/2}t^{1/2}(t-4m_N{}^2)^{1/2}[t-(m_\Delta+m_N)^2]^{1/2}(p_{\bar{N}N}p_{\bar{N}\Delta})^{\alpha-2}. \qquad (23e)$$

The final form of the differential cross section is

$$
\begin{aligned}
\frac{d\sigma}{dt}=&\frac{1}{s(s-4m_N{}^2)}|1-\exp(-i\pi\alpha)|^2\left[\frac{1}{\Gamma(\alpha+1)\sin\pi\alpha}\right]^2\{|\sin\theta_t|^2\alpha^2|[t-(m_\Delta-m_N)^2]^{-1/2}|^2[|\gamma_{\frac{3}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)|^2+|\gamma_{\frac{1}{2}-\frac{1}{2}\,\frac{1}{2}\frac{1}{2}}(t)|^2\\
&+|\gamma_{\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)|^2](s/s_0)^{2\alpha-2}+|(\sin\theta_t)^2|^2\alpha^2(\alpha-1)^2|(t-4m_N{}^2)^{1/2}[t-(m_\Delta+m_N)^2]^{1/2}|^2|\gamma_{\frac{3}{2}-\frac{1}{2}\frac{1}{2};\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha-4}\\
&+|[t-(m_\Delta-m_N)^2]^{-1}[t-(m_\Delta+m_N)^2]^{-1/2}(t-4m_N{}^2)^{-1/2}|^2|\gamma_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha}+[1+(\cos\theta_t)^2]\alpha^4\\
&\times|t^{1/2}[t-(m_\Delta-m_N)^2]^{-1/2}|^2[|\gamma_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)|^2+|\gamma_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)|^2](s/s_0)^{2\alpha-2}+[1+(\cos\theta_t)^2]|\sin\theta_t|^2\alpha^4(\alpha-1)^2\\
&\times|t^{1/2}(t-4m_N{}^2)^{1/2}[t-(m_\Delta+m_N)^2]^{1/2}|^2|\gamma_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}|^2(s/s_0)^{2\alpha-4}\}, \qquad (24)
\end{aligned}
$$

where the γ's are defined through Eqs. (22) and (24). Notice the α factor in all terms of Eq. (24) except the third term. Thus if $\gamma_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)$ turns out not too big, we can observe a minimum in $d\sigma/dt$ at $\alpha(t)=0$. The γ's here are related to those of $\pi N\to\pi\Delta(1236)$ and $\pi N\to\pi N$ by

$$\frac{\beta_{\bar{N}N;\bar{N}\Delta}(\bar{N}N\leftarrow\bar{N}\Delta)}{\beta_{\bar{N}N;\bar{N}'\Delta'}(\bar{N}N\leftarrow\bar{N}\Delta)}=\frac{\beta_{00;\bar{N}\Delta}(\pi\pi\leftarrow\bar{N}\Delta)}{\beta_{00;\bar{N}'\Delta'}(\pi\pi\leftarrow\bar{N}\Delta)}, \qquad (25a)$$

and

$$\frac{\beta_{\bar{N}N;\bar{N}\Delta}(\bar{N}N\leftarrow\bar{N}\Delta)}{\beta_{\bar{N}'N';\bar{N}\Delta}(\bar{N}\mathrm{N}\leftarrow\bar{N}\Delta)}=\frac{\beta_{\bar{N}N;00}(\bar{N}N\leftarrow\pi\pi)}{\beta_{\bar{N}'N';00}(\bar{N}N\leftarrow\pi\pi)}. \qquad (25b)$$

B. $NN\to NN^*(1500,\frac{3}{2}^-)$

Here almost all nonstrange trajectories can be exchanged. Due to the availability of very high energy data for this interaction, hopefully the consideration of only the P trajectory will be adequate. The formalism for this case is quite the same as that of $NN\to N\Delta$ (1236), except that due to the difference in parity the kinematic factors of the residue functions β's are different. They are[14]

$$
\begin{aligned}
&\beta_{\frac{3}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t),\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)\quad\text{and}\quad\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\propto[\alpha(\alpha+1)]^{1/2}\\
&\qquad\times[t-(m_{N^*}+m_N)^2]^{-1/2}(p_{\bar{N}N}p_{\bar{N}N^*})^{\alpha-1}, \qquad (26a)
\end{aligned}
$$

$$
\begin{aligned}
&\beta_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)\propto[\alpha(\alpha+1)(\alpha-1)(\alpha+2)]^{1/2}(t-4m_N{}^2)^{1/2}\\
&\qquad\times[t-(m_{N^*}-m_N)^2]^{1/2}(p_{\bar{N}N}p_{\bar{N}N^*})^{\alpha-2}, \qquad (26b)
\end{aligned}
$$

$$
\begin{aligned}
&\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)\propto\alpha(\alpha+1)(t-4m_N{}^2)^{-1/2}[t-(m_{N^*}-m_N)^2]^{-1/2}\\
&\qquad\times[t-(m_{N^*}+m_N)^2]^{-1}(p_{N\bar{N}}p_{\bar{N}N^*})^{\alpha}, \qquad (26c)
\end{aligned}
$$

$$
\begin{aligned}
&\beta_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\quad\text{and}\quad\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\propto t^{1/2}[t-(m_{N^*}+m_N)^2]^{-1/2}\\
&\qquad\times(p_{\bar{N}N}p_{\bar{N}N^*})^{\alpha-1}, \qquad (26d)
\end{aligned}
$$

$$
\begin{aligned}
&\beta_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\propto[(\alpha-1)(\alpha+2)]^{1/2}t^{1/2}(t-4m_N{}^2)^{1/2}\\
&\qquad\times[t-(m_{\bar{N}^*}-m_N)^2]^{1/2}(p_{\bar{N}N}p_{\bar{N}N^*})^{\alpha-2}. \qquad (26e)
\end{aligned}
$$

The final form of the differential equation is

$$\frac{d\sigma}{dt}=\frac{1}{s(s-4m_N{}^2)}|1+\exp(-i\pi\alpha)|^2\left[\frac{\alpha}{(\alpha+1)\Gamma(\alpha+1)\sin\pi\alpha}\right]^2\Big\{|\sin\theta_t|^2(\alpha+1)^2|[t-(m_{N^*}+m_N)^2]^{-1/2}|^2[|\gamma_{\frac{3}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)|^2$$
$$+|\gamma_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)|^2+|\gamma_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)|^2](s/s_0)^{2\alpha-2}+|(\sin\theta_t)^2|^2(\alpha-1)^2(\alpha+1)^2|(t-4m_N{}^2)^{1/2}[t-(m_{N^*}-m_N)^2]^{1/2}|^2$$
$$\times|\gamma_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha-4}+(\alpha+1)^4|(t-4m_N{}^2)^{-1/2}[t-(m_{N^*}-m_N)^2]^{-1/2}[t-(m_{N^*}+m_N)^2]^{-1}|^2|\gamma_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha}$$
$$+[1+(\cos\theta_t)^2]|t^{1/2}[t-(m_{N^*}+m_N)^2]^{-1/2}|^2[|\gamma_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)|^2+|\gamma_{\frac{1}{2}-\frac{1}{2};\frac{3}{2}-\frac{1}{2}}(t)|^2](s/s_0)^{2\alpha-2}+|\sin\theta_t|^2[1+(\cos\theta_t)^2]$$
$$\times(\alpha-1)^2|t^{1/2}(t-4m_N{}^2)^{1/2}[t-(m_{N^*}-m_N)^2]^{1/2}|^2|\gamma_{\frac{3}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha-4}\Big\}. \quad (27)$$

The γ's are related to β's by Eqs. (22) and (27).

C. $NN\to NN^*(1480,\frac{1}{2}^+)$

Here we consider only the contribution from the P trajectory.

$$\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{NN}{}^2}\frac{1}{(2\times\frac{1}{2}+1)(2\times\frac{1}{2}+1)}\times2\times\tfrac{1}{4}\Big\{2|[1+\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)E_{00}{}^{\alpha,+}(\cos\theta_t)|^2$$
$$+2|\sin\theta_t[1+\exp(-i\pi\alpha)][(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)E_{10}{}^{\alpha,+}(\cos\theta_t)|^2+2|\sin\theta_t[1+\exp(-i\pi\alpha)]$$
$$\times[(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)E_{01}{}^{\alpha,+}(\cos\theta_t)|^2+2[1+(\cos\theta_t)^2]|[1+\exp(-i\pi\alpha)]$$
$$\times[(2\alpha+1)/\sin\pi\alpha]\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)E_{11}{}^{\alpha,+}(\cos\theta_t)|^2\Big\}. \quad (28)$$

The kinematic factors of the β's are[14]

$$\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)\propto\alpha(\alpha+1)(t-4m_N{}^2)^{-1/2}[t-(m_{N^*}+m_N)^2]^{-1/2}(p_{N\bar{N}}p_{\bar{N}N})^{\alpha}, \quad (29a)$$

$$\beta_{\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\quad\text{and}\quad\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)\propto[\alpha(\alpha+1)]^{1/2}[t-(m_{N^*}-m_N)^2]^{1/2}(p_{\bar{N}N}p_{\bar{N}^*N})^{\alpha-1}, \quad (29b)$$

$$\beta_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)\propto t^{1/2}[t-(m_{N^*}-m_N)^2]^{1/2}(p_{\bar{N}N}p_{\bar{N}N})^{\alpha-1}. \quad (29c)$$

The final form of the differential cross section is

$$\frac{d\sigma}{dt}=\frac{1}{s(s-4m_N{}^2)}|1+\exp(-i\pi\alpha)|^2\left|\frac{\alpha}{(\alpha+1)\Gamma(\alpha+1)\sin\pi\alpha}\right|^2\Big\{(\alpha+1)^4|(t-4m_N{}^2)^{-1/2}[t-(m_{N^*}+m_N)^2]^{-1/2}|^2$$
$$\times|\gamma_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha}+|\sin\theta_t|^2(\alpha+1)^2|[t-(m_{N^*}-m_N)^2]^{1/2}|^2[|\gamma_{\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)|^2+|\gamma_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}\frac{1}{2}}(t)|^2](s/s_0)^{2\alpha-2}$$
$$+[1+(\cos\theta_t)^2]|t^{1/2}[t-(m_{N^*}-m_N)^2]^{1/2}|^2|\gamma_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}(t)|^2(s/s_0)^{2\alpha-2}\Big\}, \quad (30)$$

where the γ's are defined through Eqs. (28) and (30).

Obviously these formulas for NN interactions can also be used for $N\bar{N}$ interactions.

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APPENDIX I

In Ref. 5, the author did not write down explicitly the kinematic singularity and zero-free helicity amplitudes for the case of $m_a=m_b$ and $m_c\neq m_d$, which turns out to be the most useful case here. We write the result here as a supplement to the paper. In the case of $m_a=m_b$, $m_c\neq m_d$; or $m_a=m_b=m_c\neq m_d$, the following helicity amplitudes are free of kinematic singularities and zeros:

$$[\bar{f}_{cd;ab}{}^s\pm\bar{f}_{-c-d;ab}{}^s][s-4m_a{}^2]^{\frac{1}{2}\alpha_1}[s-(m_c+m_d)^2]^{\frac{1}{2}\beta_1}\times[s-(m_c-m_d)^2]^{\frac{1}{2}\beta_2}s^{\frac{1}{2}\alpha_2'},$$

where

$$\alpha_2'=\max(\mp)\eta_{ab}\quad\text{of}\quad[J_a+J_b-\tfrac{1}{2}(v_a+v_b)]+\tfrac{1}{2}(v_a+b_b),$$

for $v_a=v_b=1$;

$$\alpha_2'=\max(\pm)\eta_{ab}\quad\text{of}\quad[J_a+J_b],\quad\text{for}\quad v_a=v_b=0.$$

All the notations here are defined the same way as in Ref. 5.

APPENDIX II

It is an interesting phenomenon that purely because of the presence of some kinematic factor of α in the amplitude,[1,2,5] the contribution of a Regge pole to certain amplitude can vanish and it thus produces a minimum in the differential cross section. We illustrate this point by observing the simplest case of no spin.[18] The contribution of a Regge pole to the amplitude is

$$f(s,t)\approx\{1\pm\exp[-i\pi\alpha(t)]\}\frac{2\alpha(t)+1}{\sin\pi\alpha(t)}\times\beta(t)E_{00}{}^{\alpha(t),+}(\cos\theta_t)\,, \quad \text{(II.1)}$$

$$\approx[1\pm\exp(-i\pi\alpha)]\frac{2\alpha+1}{\sin\pi\alpha}\beta(t)\frac{\Gamma(\alpha+\frac{1}{2})}{(\pi)^{1/2}\Gamma(\alpha+1)}\times\left(\frac{s}{p_tp_t'}\right)^{\alpha}. \quad \text{(II.2)}$$

Now let us focus our attention on the factors containing α in, say, the case of positive signature:

$$\frac{[1+\exp(-i\pi\alpha)]}{\sin\pi\alpha}\frac{(2\alpha+1)\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}. \quad \text{(II.3)}$$

At $\alpha=-1$, -2, and -3, etc., the $1/\Gamma(\alpha+1)$ has zeros that are nonsense (unphysical) values of α. These zeros are to cancel the pole of $1/\sin\pi\alpha$ at these nonsense values of α. However, at the wrong signature values of α, i.e., $\alpha=-1$, -3, -5, etc., the signature factor $[1+\exp(-i\pi\alpha)]$ also vanishes. Thus the amplitude vanishes at these values of α. Similar arguments apply also to the case of negative signature. We see that *the amplitude always vanishes at nonsense and wrong-signature values of* α.[19] For cases with spins, the details of deriving this conclusion are more complicated; however, the principle is the same. The interactions considered in this paper provide an illustration. The poles of $(2\alpha+1)\Gamma(\alpha+\frac{1}{2})$ at $\alpha=-\frac{3}{2}$, $-\frac{5}{2}$, $\cdots$ in Eq. (II.3) are annihilated by zeros in the residue function.[10]

So when the spins of external particles are high, for some amplitudes even $\alpha(t)=0$, 1, 2, $\cdots$ can become nonsense values and the amplitudes can vanish at those value of $\alpha(t)$ with the wrong signature. Of course only those t-channel zeros at $\alpha(t)<1$ of the amplitudes will have an effect on the s-channel differential cross section.

[18] J. D. Stack has also observed this phenomenon for the case of no spin.

[19] I owe this nice summarizing statement to Professor G. F. Chew.

APPENDIX III

The kinematic factors $[\sin(\frac{1}{2}\theta_t)]^{|\lambda'-\mu'|}[\cos(\frac{1}{2}\theta_t)]^{|\lambda'+\mu'|}$ of $f_{cA;Db}$,[5] with $\lambda'\equiv D-b$, $\mu'\equiv c-A$, are 1 or 0 in the forward direction of the s channel for production interactions (i.e., $m_a\neq m_b$ or $m_c\neq m_d$; or both), since at this point $\sin\theta_t=0$ and $|\cos\theta_t|=1$. We use $a+b\to c+d$ to denote an s-channel interaction and $D+b\to c+A$ to denote t-channel interaction. Thus the contribution of a Regge pole is diminished for those amplitudes with $\lambda'\neq0$ or $\mu'\neq0$, or $\lambda'\neq0$ and $\mu'\neq0$ compared with those having both $\lambda'=0$, $\mu'=0$. For example, if particles c and c' can be produced through the interactions $a+b\to c+d$ and $a+b\to c'+d$, respectively, with the same highest trajectory exchange, while at the same time c can couple to the t-channel helicity amplitude with $\lambda'=0$ and $\mu'=0$ but c' cannot, then the relative production rate of c will be greater than that of c'. Whether the particle can couple to the t-channel helicity amplitude with $\lambda'=0$ and $\mu'=0$ depends on its spin and parity relative to that of the particle a; therefore this may constitute a way of determining the spin and parity of the produced particle, in case the spin and parity of the particle a are known.[20]

Sufficient conditions that would prevent a Regge pole with (J parity)$\times$(parity)$=+$ from coupling to the t-channel helicity amplitude with $\mu'=0$ are

(a) One of the particles A or c has spin zero. Let us choose this to be the particle a.

(b) The spins and intrinsic parities of A and c are such that $\eta_A\eta_c(-)^{s_A+s_c}=-1$. The η is the intrinsic parity.

When conditions (a) and (b) are satisfied, the (J-parity)$\times$parity$=+$helicity state of A and c with $\mu'=0$ does not exist. A similar argument applies for the amplitudes with $\lambda'=0$. We see that there is no selection rule at any fermion-fermion vertex. An example of the above is that the relative production rate of $A_2(2^+)$ in the interaction $\pi N\to A_2N$ will be more diminished than that of $A_1(1^+)$ in the interaction $\pi N\to A_1N$. The P trajectory cannot contribute with full strength in the interaction $\pi N\to A_2N$.

If the highest trajectory exchanged has (J parity) $\times$(parity)$=-$, the particle c with $\eta_A\eta_c(-)^{s_A+s_c}=1$ will have a smaller relative prouction rate at high energies.

[20] I would like to thank Dr. A. Goldhaber for bringing my attention to this selection rule. The same conclusion is obtained in coherent productions in heavy nuclei by A. Goldhaber and M. Goldhaber, in *Preludes in Theoretical Physics*, edited by A. de-Shalit, H. Feshbach, and L. Van Hove (John Wiley & Sons, Inc., New York, 1966).